Indiana Educational Series

Complete Arithmetic

Indiana School Book Co.

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INDIANA EDUCATIONAL SERIES.

COMPLETE

ARITHMETIC

COMBINING

ORAL AND WRITTEN EXERCISES.

INDIANAPOLIS, IND.:
INDIANA SCHOOL BOOK CO.

INTRODUCTION.

In the preparation of the Complete Arithmetic, practical or business methods have received unusual attention. Those portions designed to qualify pupils for the actual business of life are amply treated and made clear. Principles have been fully illustrated by abundant exercises and problems, while difficult questions, without principle or purpose, are omitted altogether.

Subjects of secondary importance have received only the attention to which they are entitled. The Elementary and Complete Arithmetics have been built on the theory that the average pupil, under the guidance of the intelligent teacher, is able to master the principles of business arithmetic, and that growth in mathematical knowledge does not come from aimless repetition so much as from the gradual and thorough mastery of new subjects. The mind, like the body, demands nutritious and readily assimilated food.

The unity of plan and execution promised in the Elementary is fully carried out in the Complete Arithmetic. The progressive drills, as well as the definitions, analyses, principles, and rules of the smaller work are reproduced in the larger. Manifestly there is one form for all these which is the best, and that having been secured there can be no reason for variation in either book.

For country schools, either the Elementary or Complete Arithmetic, with its mental exercises and requirement of original problems, will be found sufficient, and there is enough under every branch to establish proper forms of expression and model solutions.

The publishers cannot deny themselves the privilege of expressing their great obligations to the principals of the St. Louis schools and others, whose experience as educators enabled them to render invaluable assistance in the preparation of both works.

CONTENTS.

SUGGESTIONS TO TEACHERS.

THE progressive drills, the clear, rigid analyses and logical methods insisted upon in the Elementary Arithmetic should be continued through the Complete Arithmetic.

1. Never pass a lesson until it is fully understood. Encourage pupils to ask questions, and thoroughly test their knowledge of each principle.

2. Whenever practicable, require pupils to illustrate each definition and principle by original examples. You will observe that such requirements are an original feature of the Standard Arithmetics. In some cases, only partial answers are given—enough to afford the pupil all the guidance necessary.

3. Too great weight cannot be attached to the oral exercises. In every instance state the problem clearly and but once. Then call upon some pupil to solve it, who should immediately repeat the example, and reason it out to a correct result. If he is unable to state it correctly, he should be charged with a failure. Attention will thus be fostered, memory strengthened, a rigidly logical method of solution acquired, and pupils will soon do creditable and even brilliant work.

4. In solving problems on the blackboard, the same method should be insisted upon, until the analysis is thoroughly understood. He who formulated the first rule for the addition of numbers must have done so from investigation, and such should be the course of every student of arithmetic.

5. Frequent reviews are more profitable to the pupil than additional drill in new subjects, since by the former that which is imperfectly understood becomes clear and is permanently fixed in the mind.

6. In the preparation of original problems, require pupils to make their examples consistent. Thus a dollar should not be divided into ninths or elevenths, nor a pound into thirds, nor a yard into fifths, etc.

THE COMPLETE ARITHMETIC.

DEFINITIONS.

ART. 1.—A **Unit** is one, or a single thing; as one, one cent, one bushel.

ART. 2.—A **Number** is a unit, or a collection of units; as one, five, seven boys, nine girls.

ART. 3.—**Arithmetic** treats of numbers and their uses.

ART. 4.—A **Concrete Number** is a number applied to one or more objects; as three pounds, five eggs.

ART. 5.—An **Abstract Number** is a number not applied to any object; as three, four, five, nine.

ART. 6.—**Like** or **Similar Numbers** are those whose units are of the same kind; as six girls, three girls; eleven miles, four miles.

ART. 7.—**Unlike** or **Dissimilar Numbers** are those whose units are of different kinds; as six girls, three miles; five pounds, three boys.

ART. 8.—An **Integer** or **Whole Number** is composed of whole units; as eight, ten, fourteen.

ART. 9.—A **Problem** is a question offered for solution.

ART. 10.—A **Solution** in arithmetic is the process of obtaining the required result.

ART. 11.—The **Proof** of a solution is the process by which its correctness is tested.

ART. 12.—A **Rule** is a statement of the process by which a problem is solved.

ART. 13.—A **Principle** is a general truth.

Notation and Numeration.

ART. 14. **Notation** is the art of writing numbers.

ART. 15. **Numeration** is the art of reading numbers.

ART. 16. Numbers may be expressed in three ways:

1. By words; as four, five, eight, etc.
2. By figures, called the Arabic Method.
3. By letters, called the Roman Method.

ART. 17.—The **Arabic Notation** requires ten characters, called figures, to express numbers.

They are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

ART. 18. The first, called **naught** or **cipher,** when standing alone, expresses no value.

ART. 19.—The other nine characters are called **significant figures**, because they always express value.

ART. 20. By combining these figures, any number can be expressed.

ART. 21. -The law governing the combination of figures is—*In any whole number the first figure on the right expresses units; the second, tens; the third, hundreds; the fourth, thousands, etc.*

EXERCISES.

ART. 22. 1. How many tens and units in 37? 73? 65? 84? 91? 83? 77? 88? 90?

2. How many hundreds, tens and units in 329? 763? 907? 378? 593? 105? 810? 503?

3. How many thousands, hundreds, tens and units in 1886? 9230? 4376? 8295? 7034? 2010? 6352? 1101? 7839? 6354? 1878?

ART. 23. For convenience in writing and reading numbers, they are separated into groups of three figures each, called periods, as shown below:

Quadrillions' Period.	Trillions' Period.	Billions' Period.	Millions' Period.	Thousands' Period.	Units' Period.
Hundreds of Quadrillions. Tens of Quadrillions. Quadrillions.	Hundreds of Trillions. Tens of Trillions. Trillions.	Hundreds of Billions. Tens of Billions. Billions.	Hundreds of Millions. Tens of Millions. Millions.	Hundreds of Thousands. Tens of Thousands. Thousands.	Hundreds. Tens. Units.
					6
					48
				7,	327
				31,	232
			5,	420,	549
			74,	341,	321
		3,	429,	214,	418
		38,	152,	412.	325
	7,	315,	225,	513,	328
	45,	219,	412,	524,	415
9,	681,	305,	548,	417,	318
52,	186,	400,	462,	321,	618
937,	514,	815,	532,	517,	417
625,	329,	921,	654,	416,	898
415,	865,	325,	543,	928,	319

ART. 24. Write the following in figures:

1. 5 tens 7 units; 4 tens 3 units.

2. 9 tens 0 units; 6 tens 4 units.

3. 8 tens 7 units; 1 ten 1 unit.

4. 6 tens 6 units; 7 tens 9 units.

5. 2 hundred 5 tens 8 units.

6. 4 hundred 1 ten 0 units.

7. One thousand one hundred seventy-four.

8. One thousand eight hundred eighty five.

9. One thousand seven hundred seventy six.

10. Five thousand three hundred nineteen.

11. Twelve thousand three hundred twenty-one.

12. One hundred thousand one hundred one.

13. Seven thousand seven.

14. Seventy thousand seven.

15. Seven hundred thousand seven.

16. One million.

17. One hundred million one hundred one.

18. Five quadrillion five billion five million five.

19. One million ten.

20. Six hundred twenty-eight thousand sixty.

21. Write on the blackboard and read, a number consisting of three periods; of four periods.

22. Write on the blackboard and read, a number consisting of five periods.

ART. 25.—Rule for Notation. *Begin at the left, write the figures of each period in their proper order, filling all vacant periods and places with ciphers.*

ART. 26.—Rule for Numeration. *Begin at the right and point off the number into periods of three figures each.*

Begin at the left and read each successive period containing one or more significant figures as though it stood alone, giving the name to each period except that of units.

EXERCISES.

ART. 27.—Copy and read the following numbers:

1. 360674	**11.** 632043	**21.** 317014
2. 451892	**12.** 701216	**22.** 700312
3. 303642	**13.** 430158	**23.** 5000164
4. 216475	**14.** 603149	**24.** 7900040
5. 215842	**15.** 276367	**25.** 3241631
6. 412282	**16.** 500236	**26.** 5800314
7. 571667	**17.** 400048	**27.** 7001693
8. 312596	**18.** 301074	**28.** 43000585
9. 402389	**19.** 520017	**29.** 300561407
10. 310548	**20.** 783008	**30.** 7010907063

ART. 28.—Write the following numbers in figures:

1. Six hundred forty-eight.

2. Three thousand five hundred twenty.

3. Twenty-four thousand two hundred sixty-four.

4. Thirty-eight thousand thirty-eight.

5. One hundred eight thousand five hundred five.

6. Six hundred forty-nine thousand nine hundred one.

7. One million eight hundred thousand.

8. Nine million nine thousand two hundred twenty-four.

9. Twelve million ten thousand sixteen.

10. Fifty eight million forty one thousand eighteen.

11. Forty million thirty thousand twenty.

12. Eight hundred million seven hundred eighteen thousand thirteen.

13. Nine billion eight million six thousand three

Roman Notation.

In the **Roman Method** of **Notation** seven letters are employed. The method is rarely used, except to number chapters and divisions of books, and on the dials of clocks and watches.

Letters:	I.,	V.,	X.,	L.,	C.,	D.,	M.
Values:	1,	5,	10,	50,	100,	500,	1000.

These letters are combined according to the following principles:

Principles.—1. *Repeating a letter repeats its value:* as XX. = 20; XXX. = 30.

2. *When a letter is placed before another of greater value, its value is taken from that of the greater;* as IX. = 9; XL. = 40.

3. *When a letter is placed after another of greater value, their values are added;* as XI. = 11; LX. = 60.

4. *A bar placed over a letter multiplies its value a thousand times;* as $\overline{V}$. = 5,000; $\overline{L}$. = 50,000; $\overline{M}$. = 1,000,000.

Write the following numbers in Roman notation: 5, 8, 6, 4, 10, 15, 18, 20, 35, 88, 121, 348, 610, 450, 832, 1,321, 2,453, 6,145, 3,853, 9,009.

Change the following numbers to the Arabic notation: III., VI., VIII., IX., XI., LV., LXII., LXIX., XC., CX., CXX., CCXX., CCXXV., CXXXI., CCCXXXVIII., DCCX., DCCXIX., DCCCXLIII., MDCXXXVII., $\overline{V}$., $\overline{M}$., MDCCCLXXXVI.

Addition.

Progressive Oral and Written Drills.

SUGGESTION.—The pupil should be taught to count with and without objects. After counting, the first process in arithmetic is to add **the** digits to a given number. Let the given number be **2** and add to it 2, 5, 8; 1, 4, 7; 3, 6, 9 respectively.

SLATE AND BLACKBOARD EXERCISES.

2	2	2	2	2	2	2	2	2	2
2	5	8	1	4	7	3	6	9	0

2	2	2	2	2	2	2	2	2	2
12	15	18	11	14	17	13	16	19	10

☞ Count by 2's to 20. Begin with 1 and count by 2's to 20.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
2	1	2	1	2	2	1	2	1	1
1	2	1	1	2	0	2	0	2	2
0	2	1	2	1	1	1	2	2	1
2	1	2	2	2	2	2	1	2	0
1	2	2	1	2	1	2	2	1	2

☞ Begin with 0, 7, 4, 11, 8, 12, 9, 6, or 13, and add those columns up and down.

SUGGESTION.—Take **3** and add to it 1, 4, 7; 3, 6, 9; 2, 5, 8 respectively. Three combinations are enough for a lesson. Illustrate each fact, so that the child may think the result clearly.

SLATE AND BLACKBOARD EXERCISES.

3	3	3	3	3	3	3	3	3	3
2	5	8	1	4	7	9	6	3	0

3	3	3	3	3	3	3	3	3	3
12	15	18	11	14	17	19	16	13	10

☞ Count by 3's to 30. Begin with 1 and count by 3's to 30. Begin with 2 and count by 3's to 30.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
3	2	2	2	3	2	3	3	2	3
2	3	1	3	1	3	1	1	3	3
1	2	3	2	2	1	3	2	1	2
3	1	2	2	2	3	3	0	2	3
2	3	3	3	3	3	2	2	1	1

☞ Begin with 0, 8, 4, 11, 5, 13, 16, 9, 12, or 21, and add these columns up and down.

SUGGESTION.—After adding the digits to 4, add 12 and 4, 22 and 4, 32 and 4; 15 and 4, 25 and 4, 35 and 4; and so on.

SLATE AND BLACKBOARD EXERCISES.

4	4	4	4	4	4	4	4	4	4
2	5	8	1	4	7	3	6	9	0

4	4	4	4	4	4	4	4	4	4
12	15	18	11	14	17	13	16	19	10

☞ Count by 4's to 40. Begin with 1, 2, or 3, and count by 4's to 40.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
3	4	4	1	3	4	4	4	4	2
4	1	3	4	2	3	2	3	3	4
2	4	4	2	4	2	3	2	4	3
3	3	3	4	1	4	4	4	2	4
4	4	2	3	4	3	3	2	1	2

☞ Add each of the above columns up and down, beginning with 0, 2, 13, 8, 17, 4, 16, 9, 15, 7, or 19.

SUGGESTION.—Abstract examples like the following, containing no figure greater than 4, should be dictated by the teacher:

(1)	(2)	(3)	(4)
123244	342431	243143	344243
344432	434241	434432	411402
412343	340413	142244	324143
344234	414043	434422	443404
421443	342414	342244	342244
344324	424123	414422	224434
412432	341234	341144	443043
344343	412432	424422	344234

☞ For explanation, see **Article 36.**

SUGGESTION.—The process with the **5's** is the same as with **2, 3** and **4.** After adding the digits to 5, add 16 and 5, 36 and 5, 26 and 5, 46 and 5, and so on, until all combinations up to 49 and 5 are mastered.

BLACKBOARD DRILLS.

(1, 2, 3, 4) 5		(2, 3, 4) 5		(3, 4) 5		(4) 5		5	
3	.	13	.	23	.	33	.	43	.
5	.	15	.	25	.	35	.	45	.
9	.	19	.	29	.	39	.	49	.
2	.	12	.	22	.	32	.	42	.
7	.	17	.	27	.	37	.	47	.
1	.	11	.	21	.	31	.	41	.
6	.	16	.	26	.	36	.	46	.
4	.	14	.	24	.	34	.	44	.
8	.	18	.	28	.	38	.	48	.
0	.	10	.	20	.	30	.	40	.

Explanation.—The teacher or a pupil points to the dots on the right of the vertical line, while the pupils in turn name the results obtained by adding the number on the left of this line to the given number at the top. This exercise fixes the attention of each pupil, and may be introduced with the **2's**, **3's**, and **4's** and also with the **6's**, **7's**, **8's** and **9's**. ☞ The numbers in parentheses should be used as review drills.

☞ Count by 5's to 50. Begin with 1, 2, 3, or 4 and count by 5's to 50.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
5	4	5	4	5	4	3	5	2	4
4	5	3	5	4	5	5	4	5	5
1	5	4	5	3	5	1	5	5	2
5	4	3	4	5	3	5	3	2	5
3	5	5	5	2	4	3	5	4	5

☞ Add each of the above columns up and down, beginning with 0, 7, 9, 18, 4, 22, 6, 17, 29, 8, or 14.

DICTATION.

Let the teacher dictate examples containing no figure greater than 5. There should not be more than twelve numbers, nor should any number be greater than hundreds of thousands.

SUGGESTION.—The treatment of addition of 6's is similar to the treatment of the other numbers, and involves three distinct drills: 1st. The addition of the digits to 6. 2d. The addition of all numbers up to 59 to 6. SEE BLACKBOARD DRILLS for the 5's. 3d. Counting by 6's, beginning with 0, 1, 2, 3, 4, or 5.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
6	6	5	4	6	6	6	6	6	4
3	5	4	5	6	1	4	6	6	4
6	4	6	6	4	6	6	4	6	5
2	6	3	3	6	6	6	6	1	6
5	1	6	4	5	2	3	3	6	6

☞ Add each column up and down, beginning with 0, 5, 19, 4, 27, 6, 15, 23, 8, 22, or 11.

DICTATION.

Let the teacher dictate examples containing no figure greater than **6**. There should not be more than twelve numbers, nor should any number be greater than hundreds of thousands. The numbers should be irregular, that is, not all contain the same number of figures.

SUGGESTION.—Treat the **7's** in the same way as the **5's** and **6's**, observing the three distinct kinds of sums up to 69. Another good written drill is this: Let the teacher arrange numbers and require pupils to add some given number to each of them. For instance, the teacher says, Add 7 to 8, 14, 6, 9, 24, 28, 16, 29, 44, 53. The pupils write 15, 21, 13, 16, 31, 35, 23, 36, 51, 60, if they add readily and accurately.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
7	6	7	6	7	5	7	6	7	7
6	7	6	7	5	[illegible]	7	7	4	1
7	7	7	5	4	4	4	7	6	5
5	2	4	7	7	3	5	5	7	7
2	4	1	2	1	2	1	7	2	3

☞ Add each of the above columns, beginning with 0, 5, 10, 4, 27, 6, 15, 23, 8, 22, or 11.

REMARK.—The units' figure of the sum of any number and a given number is always the same, whether the number is a digit, or simply fills the units' place. See DRILL on 5's.

BLACKBOARD DRILLS.

1		**4**	**6**		**4**	**6**		**9**
	5			**5**			**5**	
3		**2**	**3**		**7**	**8**		**7**

REMARK.—The above arrangement of numbers in circles gives endless columns, and is an excellent device for utilizing the spare minutes in concert drills. The teacher points to the numbers and the pupils add silently until the teacher calls for results. Go slowly at first, but gradually increase in rapidity as pupils acquire facility in adding.

DICTATION.

Let the teacher dictate examples containing no figure greater than 7. No number should exceed millions.

SUGGESTION.—The 8's and 9's are treated in the same manner as the other numbers.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
8	7	6	8	7	6	7	6	7	8
7	6	8	6	8	8	8	8	8	7
8	5	8	8	8	7	8	7	8	8
5	8	5	2	5	8	7	8	8	6
4	6	3	8	8	5	8	6	7	8

☞ Add each of the above columns up and down, beginning with 0, 17, 8, 35, 7, 29, 54, 39, 6, 15, or 26.

DICTATION.

Let the teacher dictate numbers containing no figure greater than 8. The numbers may be as great as hundreds of millions.

EXERCISES IN ADDING COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
8	9	7	4	8	9	8	9	7	8
9	8	8	9	9	8	7	6	9	9
6	6	6	5	9	9	9	9	6	7
7	9	5	9	6	6	8	7	9	8
9	8	9	2	9	9	6	9	5	9

☞ Add each of the above columns up and down, beginning with 0, 11, 17, 13, 18, 14, 12, 19, 10, 16, or 21.

DICTATION.

Let the teacher dictate examples of numbers containing any figure and not more than four periods. Let the numbers be irregular and contain ciphers.

☞ These inductive exercises are reviews of the facts of addition in the concrete. The pupil should name results without giving an analysis of his process.

INDUCTIVE EXERCISES.

ART. 29.—1. How many are 3 boys and 2 boys? 3 boys and 3 boys?

2. How many are 3 cows and 4 cows? 3 cows and 5 cows?

3. If an inkstand cost 5 cents and a lead pencil 6 cents, what is the cost of both?

4. In a field are 8 apple-trees and 9 peach-trees: how many trees are there in the field?

5. Jane bought a bonnet for 8 dollars and a dress for 12 dollars: how much did both cost?

6. A farmer had 6 horses and bought 11: how many horses had he then?

7. If you pick 4 apples from one tree, 3 from another, and 2 from a third, how many do you pick?

8. A man in one week planted 6 acres with corn, in the second week 3 acres, and in the third week 4 acres: how many acres did he plant?

9. A boy paid 20 cents for a book and 30 cents for some writing paper: how much did he pay for both?

10. A farmer sold some corn for 15 dollars and some wheat for 35 dollars: how much did he receive?

11. Thomas is 7 years old and his father is 24 years older: what is the age of his father?

12. If you pay 3 cents ferriage, 15 cents for lunch, and 10 cents for car fare, how much do you pay in all?

13. Three boys bought 4 oranges apiece: how many oranges were bought by all?

14. A certain house has 3 windows in front, 3 in the rear, and 4 on each of its two sides: how many windows in the house?

15. There are 3 eggs in one nest, 5 in another, and 6 in a third: how many eggs in the three nests?

16. Harry had 5 marbles, bought 4, and found 2: how many marbles had he then?

17. Anna read 4 pages in the morning, 6 during the afternoon, and 3 in the evening: how many pages did she read?

18. Mr. Brown has 3 gray horses, 5 brown horses, and 2 sorrel horses: how many horses has he?

19. A boy spent 4 cents on each of three days: how many cents did he spend?

20. Mr. Smith paid 7 dollars for a pair of trousers, 4 dollars for a vest, and 10 dollars for a coat: how much did his suit cost?

21. What is the total cost of a ton of coal at 6 dollars a ton, and a cord of wood at 4 dollars?

22. Henry has 7 marbles, William 6, John 5, George 4, and Edward 6: how many marbles do the five boys have?

23. Jane bought 6 yards of ribbon, Mary 8, Julia 4, Fanny 7, and Ellen 9: how many yards of ribbon did they all buy?

24. A man had seven sons: to the first he gave 7 pears, to the second 6 pears, to the third 3 pears, to the fourth 8 pears, to the fifth 7 pears, to the sixth 5 pears, and to the seventh 9 pears: how many pears did he give to his seven sons?

25. There is a school of six rooms, from which the absence was as follows: No. 1, 8 absent, No. 2, 7 absent, No. 3, 5 absent, No. 4, 9 absent, No. 5, 6 absent, and No. 6, 8 absent: how many pupils were absent?

26. How many nuts are 6 nuts + 3 nuts + 7 nuts + 5 nuts + 8 nuts + 2 nuts + 9 nuts?

DEFINITIONS AND PRINCIPLES.

ART. 30. Addition is the process of uniting two or more numbers into one.

ART. 31. The **Sum** or **Amount** is the number obtained by addition. Thus, when 6 and 5 are added, the *sum* or *amount* is 11.

ART. 32. The **Sign of Addition** (+) is called *plus*, and means *more*. When placed between two numbers, it shows that they are to be added. Thus, 7 + 6 is read 7 *plus* 6, and means that 7 and 6 are to be added; 9 + 3 + 2 is read 9 *plus* 3 *plus* 2, and means that 9, 3 and 2 are to be added.

ART. 33.—The **Sign of Equality** (=) is read *equals*. It shows that the expression on the left of the sign is equal to the expression on the right of that sign. Thus, 10 + 6 = 16, is read 10 plus 6 *equals* 16, and means that when 10 and 6 are added, the sum is 16; 12 + 9 = 21, is read 12 plus 9 *equals* 21, and means that when 12 and 9 are added, the sum is 21.

The *dollar sign* ($) is placed before the number to which it belongs. Thus, for convenience, 275 dollars is written $275.

Cents written either alone or with dollars, occupy two places on the right of dollars, from which they are separated by a period, called the decimal point. Thus, 24 dollars 36 cents, is written $24.36; 75 cents is written $.75.

ART. 34.—**Principles.** 1. *Only like numbers can be added.*

2. *The sum or amount is of the same kind as the numbers added.*

ORAL EXERCISES.

ART. 35.—1. Richard paid $8 for a coat and $7 for a pair of trousers: how much did the garments cost?

Solution.—They cost the sum of $8 and $7, or $15.

2. Susan paid $14 for a sacque and $3 for a doll: how much did the two cost?

3. Samuel paid $90 for a bicycle and $8 for his knickerbockers: how much did he pay for both?

4. Johnson gave $30 to the missionary cause and $25 to the tract society: what was the amount of his contributions?

5. A horse cost $120 and a carriage $80: what was the cost of both?

6. Thomas paid $.50 for a book and $.75 for a knife: how much did the two cost?

7. James rode 15 miles on the cars and 20 miles on the steamboat: how far did he ride?

8. If there are 40 lines on one page of a book, how many lines are on two such pages?

9. It is 14 miles from A to B and 18 miles beyond B is C: what is the distance from A to C?

10. A merchant sold 24 yards of cloth on Monday and 36 yards on Tuesday: how many yards did he sell in the two days?

11. John gave to his mother 8 apples, to his brother 6, and to his sister 5: how many did he give to the three?

12. George has 10 cents in one pocket, 15 cents in another, and 20 cents in another: how many cents has he?

13. Helen spelled 14 words on Wednesday, 14 on Thursday, and 12 on Friday: how many words did she spell?

14. Horace studied 15 minutes one afternoon, 25 another, and 30 another: how many minutes did he study during the three afternoons?

15. Jenkins spent \$.25 in one store, \$.35 in another, and \$.45 in a third: how much did he spend?

16. James earned 18 cents on Monday, 15 cents on Tuesday, and 16 cents on Wednesday: how many cents did he earn?

17. A farmer had three grape-vines. The first produced 32 pounds of grapes, the second 40 pounds, and the third 48 pounds: how many pounds did all produce?

18. A drover bought 37 sheep of one man, 35 of another, and 38 of a third: how many sheep did he buy?

19. The price of a horse was \$98, of a carriage \$72, and of a harness \$20: what was the cost of all?

20. An orchard contains 60 peach trees, 35 plum trees, and 25 pear trees: how many trees in the orchard?

21. A lady paid \$38 for a dress, \$18 for a shawl, and \$8 for a bonnet: how much did she pay for all?

22. A pole is 22 feet in the air, 19 feet in the water, and 11 feet in the earth: what is the length of the pole?

23. There are 45 pupils in one school-room, 48 in another, and 53 in a third: how many pupils in the three rooms?

24. Mr. Harper was 47 years old 12 years ago: how old will he be 9 years hence?

25. A horse was sold for \$120, which was \$15 less than it cost: for what should it have been sold to gain \$25?

26. John has 45 cents and Mary 35 cents: if each had 15 cents more, what would be the sum possessed by both?

WRITTEN EXERCISES.

ART. 36.—1. What is the sum of 482, 324, 473, and 638?

Process

```
 482
 324
 473
 638
1917
```

Analysis. We write the numbers so that units of the same order stand in the same column, draw a line underneath, and begin at the right to add.

Beginning with the order of units, we add each column separately. Thus, 8 + 3 + 4 + 2 = 17 units, equal to 1 ten and 7 units. We write 7 in the units' place and add the 1 ten with the column of tens.

One ten from the units' column + 3 + 7 + 2 + 8 = 21 tens, equal to 2 hundreds and 1 ten. We write 1 in the tens' place and add the 2 hundred with the column of hundreds.

Two hundred from the tens column + 6 + 4 + 3 + 4 = 19 hundreds, equal to 1 thousand and 9 hundreds, which we write in their proper places.

Therefore, the sum of 482, 324, 473, and 638 is 1917.

NOTE.—In adding name results only. Thus, instead of saying 8 and 3 are 11 and 4 are 15 and 2 are 17, say 8, 11, 15, 17.

ART. 37. Rule for Addition.—*Write the numbers so that units of the same order stand in the same column, draw a line underneath and begin at the right to add.*

If the sum of any column is less than 10, write it under the column added; and if 10 or more, write the units' figure under the column added, and add the ten or tens with the next column. Write the total sum of the last column.

Proof.—*Begin at the top and add each column downward. If the sums agree, the work is probably correct.*

Add and prove the following:

2.	3.	4.	5.	6.	7.
$472.25	$723.47	$385.54	$425.31	38777	$385.24
364.56	384.38	412.38	846.74	23648	486.06
542.38	453.74	246.38	360.48	4235	555.49
641.54	621.38	521.42	405.38	63838	731.22
0. 3	8 .9	5.7	19 7	8	5 .0

8.	9.	10.	11.	12.	13.
348562	712341	708506	2452038	84836	59482
27849	205704	50136	478324	3714	41325[illegible]
30505	388	98472	560472	695	31009
48092	2476	3847	9845	38	4712
7453	794538	29	3260	2184	1040
28	4101	6397	2090747	720473	129395
4 9	9 4	3	9 2	79	8 8

14. Add seven thousand forty-nine; eighty-eight thousand six hundred; three million four hundred thirty-two thousand ninety-nine; sixteen million six hundred thousand six hundred.

15. Add nineteen million thirty-eight thousand twenty-five; four hundred thousand eight hundred eight; one billion one hundred twenty-four thousand ninety-two; three hundred thousand two hundred eighty-five.

16. What was the total amount of taxes received by the city of Trenton during the week in which the receipts were as follows: Monday, $13,845.50, Tuesday, $9,842.28; Wednesday, $5,584.90; Thursday, $11,694; Friday, $10,356.51; Saturday, $17,328.48?

17. The mariner's compass was invented in China 1120 years before Christ, and America was discovered 1492 years after Christ: how many years between the dates?

18. A drover sold 564 cattle to one dealer, 785 to a second, 546 to a third, 764 to a fourth, and to a fifth as many as he sold to the first and second together: how many cattle did he sell to the five dealers?

19. The corn crop of this country in 1882 was 812,771,000 bushels; in 1883, 852,666,000; and in 1884, 1,297,000,000: what was the yield during the three years?

20. Mr. Thompson gave $1.25 to William, $1.36 to Mary, as much to Susan as he gave to William and Mary, and as much to George as he gave to William, Mary, and Susan: how much money did he give to all?

21. Five men formed a partnership. A furnished $3,254; B, $2,848; C, $1,865; D, $1,080; and E, $1,590: what was the amount of their capital?

22. The prices received for four farms were: for the first, $8,350; for the second, $9,875; for the third, $9,240; and for the fourth, $10,350: what was the amount received for all?

23. A gentleman owns a house worth $9,200, a store worth $4,800, and a farm worth $11,250: how much are the three worth?

24. A man owing a debt paid $393 at one time, at another $417, at another $319, at another $211, and there was $419 still unpaid: what was the amount of the debt?

25. Mr. B. owns a house worth $11,250, a store worth $6,300, a lot worth $4,200, and has $12,500 personal property: how much is he worth?

26. A man left to his wife $12,500, to each of his three sons $15,200, and to each of his two daughters $8,500: how much did he leave to all?

27. Illustrate by an original problem addition of numbers.

Subtraction.

Progressive Oral and Written Drills.

SUGGESTION.—Subtraction is the process of finding the difference between two numbers. Take 2 from 5, 8, 11, 10, 4, 7, 3, 6, 9 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	5	8	11	4	10	7	3	6	9	2
Take	2	2	2	2	2	2	2	2	2	2

From	4268	74684	6834246756
Take	2122	12212	2221222112

REMARK.—Dictate one figure at a time to be written in line.

SUGGESTION.—Take 3 from 5, 8, 11; 10, 4, 7; 12, 9, 6 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	5	8	11	10	4	7	12	9	6	3
Take	3	3	3	3	3	3	3	3	3	3

From	7643	98675	9746543687
Take	3213	31323	3323133233

SUGGESTION.—Take 4 from 5, 8, 12; 11, 9, 6; 7, 10, 13 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	5	8	12	11	9	6	7	10	13	4
Take	4	4	4	4	4	4	4	4	4	4

From	4874	68797	8745867498
Take	4234	42434	3243423434

SUGGESTION.—Take 5 from 7, 13, 6; 11, 14, 9; 8, 12, 10 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	7	10	6	11	14	10	18	13	9	5
Take	5	5	5	5	5	5	5	5	5	5

From	7402434643	9164506471
Take	4435245345	3554155445

For explanation, see **Article 46.**

SUGGESTION.—Take 6 from 12, 15, 8, 10, 14, 7, 13, 9, 11 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	12	15	8	10	14	7	13	9	11	6
Take	6	6	6	6	6	6	6	6	6	6

From	9064874654	8402648735
Take	5646616556	5466366556

SUGGESTION.—Subtract 7 from 12, 15, 9, 11, 14, 10, 8, 13, 16 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	12	15	9	11	14	10	13	16	8	7
Take	7	7	7	7	7	7	7	7	7	7

From	9740787465	8640346561
Take	7467477467	7463665667

SUGGESTION.—Subtract 8 from 12, 15, 10, 17, 11, 14, 9, 13, 16 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	12	15	10	17	11	14	9	13	16	8
Take	8	8	8	8	8	8	8	8	8	8

From	9482716387	8741087647
Take	[illegible]	[illegible]

SUGGESTION.—Subtract 9 from 11, 14, 17, 10, 13, 16, 12, 15, 18 respectively.

SLATE AND BLACKBOARD EXERCISES.

From	11	14	17	10	13	16	12	15	18	[illegible]
Take	9	9	9	9	9	9	9	9	9	9

From	9876543210	1234567890
Take	9687938789	989489889

☞ [illegible] *exercises* are [illegible] of the facts of subtraction. The pupil should name results without giving an analysis of this process.

INDUCTIVE EXERCISES.

ART. 38. **1.** Mary had 6 pins and lost 2: how many pins had she left?

2. A grocer had 7 barrels of flour and sold 4: how many remained?

3 John bought 6 cakes and ate 3: how many were left?

4. If Samuel earns $10 a month and saves $5, how many dollars does he spend each month?

5. If 7 birds were on a limb and 4 flew away, how many remained?

6. Sarah, having 10 cents, gave 5 for a lead pencil: how many cents had she left?

7. Maria had [illegible] peaches and gave her father 7: how many peaches had she left?

8. If Richard is 11 years old and his brother is 4 years younger, what is the age of his brother?

9. From a class of 11 pupils, 7 went out to play: how many remained in the class?

10. There were 12 boys in one class and 9 in another: how many more were in the first than in the second class?

11. Henry had 11 marbles and lost 7: how many had he left?

12. George wrote 13 lines in his copy book, and Harry wrote 8 less: how many lines did Harry write?

13. Jennie had 15 chicks, and 6 of them died: how many had she left?

14. John bought 14 eggs and broke 8: how many were left?

15. From a piece of cloth containing 16 yards, 8 yards were cut for a suit: how many yards remained?

16. Julia having 13 cents, gave a beggar 10: how much had she left?

17. Alfred picked 11 pears and gave away 9: how many pears had he left?

18. A man paid $4 for a hat: how much change should he receive from a ten-dollar bill?

19. Wilson had 16 pigeons and sold 9: how many remained?

20. Warren had 18 cents and spent 10 for candy: how much had he left?

21. If I had 14 books and sold 11, how many were left?

22. A watch was bought for $14 and sold for $9: how much was the loss?

23. From a class numbering 18 pupils, 10 were absent: how many pupils were present?

24. Albert gave $12 for a donkey and sold it for $17: how much was the gain?

25. From a cask of cider containing 17 gallons, 13 leaked away: how many gallons were left?

26. Two hunters fired into a flock of 14 birds and killed 9: how many escaped?

27. A hen had 18 chicks and a rat killed 7 of them: how many were left?

28. There were 19 passengers in a street car and 11 got out: how many remained?

29. Twenty words were given to Thomas and he missed 7: how many did he spell correctly?

30. A farmer had 20 cows and sold 18: how many did he keep?

DEFINITIONS AND PRINCIPLES.

ART. 39.—Subtraction is the process of finding the difference between two numbers. 8 less three is 5.

ART. 40.—The **Minuend** is the number from which another number is to be subtracted. 8 is the minuend.

ART. 41. The **Subtrahend** is the number to be subtracted. 3 is the subtrahend.

ART. 42.—The **Difference** is the number obtained by subtraction. 5 is the difference.

ART. 43.—The **Sign of Subtraction** ($-$) is read *minus*. When placed between two numbers, it shows that the number on the right is to be subtracted from the number on the left. Thus, $8-3$ is read 8 *minus* 3, and means that 3 is to be subtracted from 8.

ART. 44.—Principles. 1. *Only like numbers can be subtracted.*

2. *The sum of the subtrahend and difference equals the minuend.*

3. *If the minuend and subtrahend are equally increased or diminished, the difference is unchanged.*

Thus: $7-4=3$. If 2 be added to both the minuend and the subtrahend, we have $9-6=3$; or, if 2 be subtracted from both the minuend and the subtrahend, we have $5-2=3$.

ORAL EXERCISES.

ART. 45.—1. Newton had 13 apples and gave away 9: how many had he left?

Solution.—He had left the difference between 13 apples and 9 apples, or 4 apples.

2. Thomas had 14 cents and spent 9: how many had he left?

3. In a school numbering 35 pupils, 15 were absent: how many were present?

4. A watch which was bought for $23 was sold for $18: what was the loss?

5. John solved 21 examples, and Harry solved 11: how many more did John solve than Harry?

6. If a man earns $27 a week and his expenses are $15, how much does he save?

7. Fritz earns $55 a month; he spends $15 for board and $12 for other expenses: how much does he save?

8. Harry bought a hat for $2 and a book for $1: how much change was due him from a five-dollar bill?

9. From a piece of cloth containing 65 yards, a merchant sold 15 yards to one customer and 20 yards to another: how many yards were left?

10. Anna's father gave her 10 cents, her mother gave her 12 cents, and her grandmother gave her 15 cents. She spent 37 cents: how much was left?

11. Amos sold a horse for $120, which was $15 more than he gave for it: what was the cost of the horse?

12. Mr. Glenn bought a carriage for $75, paid $10 for repairing it, and then sold it for $88: how much did he gain?

13. The difference is 10 and the subtrahend 10: what is the minuend? The difference is 12 and the subtrahend 45, what is the minuend?

14. The minuend is 36; subtrahend 14: what is the difference? The minuend is 49, difference 34: what is the subtrahend?

15 The difference is 40; minuend 60: what is the subtrahend? The minuend is 35; subtrahend 18: what is the difference? The difference is 21; minuend 52: what is the subtrahend?

WRITTEN EXERCISES.

ART. 46.—1. From 8756 take 4894.

Process.
8756
4894
3862

Analysis.—We write the subtrahend under the minuend, placing units of the same order in the same column, and begin at the right to subtract.

Four units from 6 units leave 2 units, which we write in the units' place. Nine tens cannot be subtracted from 5 tens, so we add 10 tens to 5 tens in the minuend making 15 tens. Nine tens from 15 tens leave 6 tens, which we write in the tens' place. Since we increased the minuend by 10 tens, we must increase the subtrahend by 1 hundred, equal to 10 tens. Nine hundreds cannot be taken from 7 hundreds, so we add 10 hundreds to 7 hundreds in the minuend, making 17 hundreds. Nine hundreds from 17 hundreds leave 8 hundreds, which we write in the hundreds' place. Since we increased the minuend by 10 hundreds, we must increase the subtrahend by 1 thousand, equal to 10 hundreds. Five thousands from 8 thousands leave 3 thousand, which we write in the thousands' place.

Therefore, 4894 subtracted from 8756 leaves 3862.

Proof.—4894 + 3862 = 8756.

ART. 47.—Rule for Subtraction. *Write the subtrahend under the minuend, placing units of the same order in the same column, and begin at the right to subtract.*

Subtract each order in the subtrahend from the order above it, placing the remainder beneath. If any order in the subtrahend is greater than the order above it, add 10 *to the latter and subtract. In such case increase the next subtrahend order by* 1 *before subtracting.*

Proof.—*Add the subtrahend to the remainder, and if the sum equals the minuend the work is correct.*

Solve and prove the following:

	2.	**3.**	**4.**
From	6405621402	7204325621	5024301382
Take	3233313323	2323632313	2233223323

	5.	**6.**	**7.**
From	7403420133	6216432643	7142036342
Take	4254143424	32343[illegible]2434	3432344334

	8.	**9.**	**10.**
From	6250436243	8423046342	7021564342
Take	4511131215	4423454335	4345545335

	11.	**12.**	**13.**
From	7420434242	5432042364	8024314624
Take	5616363456	4625043654	5454316616

	14.	**15.**	**16.**
From	8240672043	7302416734	8024670771
Take	6665771567	56472[illegible]7667	4564647[illegible]7

	17.	**18.**	**19.**
From	8064724367	8671204326	8630421467
Take	5674624778	7868147818	7826780858

	20.	**21.**	**22.**
From	9028914678	9981704368	9490646768
Take	8878814369	8978950959	8[illegible]89

23. A contractor received $18,250 for building a house that cost him $16,119: how much was his profit?

24. A gained $1,785.75 in one year, and B gained $2,348.90 in one year: how much more did B gain than A?

25. In a storm at sea, a ship was obliged to throw overboard $23,250 worth of goods. If the value of the cargo was $89,784, how much was saved?

26. A gentleman bequeathed $38,475 to his two sons,

To one he gave $10,300: how much did he give to the other?

27. If a man's income is $2,875 and his expenses $1,976, how much does he save?

28. The population of the United States in 1790 was 3,929,214, and in 1800 it was 5,308,483: how much was the increase in 10 years?

29. In 1880, there were in Missouri 1,557,631 persons not less than 10 years old, of whom 138,818 could not read: how many were able to read?

30. The number of people in the United States who were not less than 10 years of age in 1880, was 28,761,607; of these 4,923,451 could not read: how many were able to read?

31. The minuend is $4,750.80, and the difference $3,240.66: what is the subtrahend?

32. The subtrahend is $66,580.05, and the difference $29,333.08: what is the minuend?

33. The difference is $17,482.04, and the subtrahend $23,541.09: what is the minuend?

34. From ten thousand six hundred fifty four take eight thousand three hundred ninety six.

35. From ten thousand ten take five thousand five.

36. From four million four thousand fourteen take two million six hundred thousand eighteen.

37. From seventeen thousand four hundred twenty-five dollars take eleven thousand fifty dollars.

38. From one hundred twenty-five cents take eighty cents.

39. From one thousand one dollars take eleven dollars.

40. From one million one thousand dollars take one million nine dollars.

41. Illustrate by an original problem the subtraction of one number from another.

Review Problems.

ORAL EXERCISES.

ART. 48.—1. A merchant gave $32 for goods, and paid $5 for freight: at what price must he sell them to gain $7?

Solution.—The goods with the freight added cost the sum of $32 and $5, or $37; and to gain $7, he must sell them for the sum of $37 and $7, or $44.

2. John had $15, earned $7, and then spent $11: how much had he left?

3. A girl having 13 apples, bought 8, and then gave away 12: how many remained?

4. A man bought a cow for $40, and paid $9 for her keeping: for what must he sell her to gain $12?

5. A lady bought needles for 12 cents and thread for 18 cents: what change should she receive from 50 cents?

6. Susan having 15 cents, spent 5 cents, and her mother gave her 6 cents: how much had she then?

7. John having 9 pears, picked 11, and then gave his mother 14; how many had he left?

8. A gentleman bought a watch for $25 and a chain for $8. He sold them both for $42: how much did he gain?

9. A man bought a vest for $6 and a pair of trousers for $12: what change should he receive from a twenty-dollar bill?

10. A street car contained 18 passengers; 10 got out; after which 8 entered: how many were then in the car?

11. A man earned $15 one week and $13 another week. His expenses for the two weeks were $17: how much did he save?

12. How much less than 38 pounds is the sum of 14 pounds and 7 pounds?

13. How much less than 60 feet is the sum of 25 feet and 30 feet?

14. What number must be added to 32 to make the sum 6 less than 49?

15. From the sum of 12 plus 10 subtract 13.

16. From 10 plus 5 plus 4 subtract 4.

17. From 8 plus 6 plus 5 subtract 13.

18. From 7 plus 4 plus 6 subtract 3 plus 8.

19. How much is 8 plus 6 minus 5 plus 4 minus 3?

20. How much is 14 plus 4 minus 7 plus 5 minus 2?

WRITTEN EXERCISES.

ART. 49.—1. Easter in 1886 fell on April 25, its latest possible date. It will not so fall again until 1943: how great is the interval?

2. Mr. Jones bought a house for $8,000. He paid $1,500 for alterations and then sold it for $9,400: did he gain or lose, and how much?

3. A horse cost $1,500; a set of harness $200; a carriage $275: how much does the amount lack of being $2,000?

4. A merchant paid $75 for some goods and they cost $6 for transportation: for what must he sell them so as to gain $13?

5. George Washington was born in 1732; how many years was that before you were born?

6. The area of France is 200,705 square miles and the area of Texas is 265,780 square miles: how many more square miles does Texas contain than France?

7. A merchant's profits on his sales for one year were $8,450; out of this he paid $1,000 rent, $120 insurance, for salaries $2,800, and his incidental expenses were $148: how much had he left?

8. An estate worth $28,000 has two mortgages upon it, one for $9,000, and the other for $6,500: what is it worth above encumbrances?

9. Mr. Harris deposited in bank $785 on Monday, $385 on Tuesday, and $650 on Wednesday. During the same time he checked out $1,184: how much did his deposits exceed the amount checked out?

10. A gentleman distributed $4,800 among his three children. To one he gave $1,400, and to another $1,500: how much did he give to the third?

11. A merchant having $2,368 on deposit, gave checks for $385, for $428, and for $299: how much remained on deposit?

12. A man having $11,284, invested $4,200 in railroad stock, $1,800 in a house, $3,800 in bank stock, and $1,300 in government bonds: how much had he left?

13. What number is that from which if you take 38,526, the remainder will be 13,250 less than 16,250?

14. The sum of three numbers is 24,256; the first is 8,256, and the second is 894 more than the first: what is the third?

15. The area of Dakota is 149,100 square miles, of Indian Territory 64,690 square miles, and of New Mexico 122,580 square miles: how much does their united area lack of equaling the area of all the territories, which is 1,509,770 square miles?

16. A man paid $3,250 for a lot, on which he erected a house costing $5,713. He sold his property for $9,224: how much did he gain?

17. Mr. Brown bought a watch for $103.50 and a chain for $27.25. He gave his note for $100 and paid the rest of the debt in cash: how much cash did he pay?

18. Illustrate by an original problem addition and subtraction of numbers.

Multiplication.

Progressive Oral and Written Drills.

SUGGESTION.—Multiplication grows out of addition, and is a short method of adding a number to itself. The numbers only that result from adding one to itself are called prime numbers, such as 1, 2, 3, 5, 7, 11, 13, 17, 19, etc. When a number larger than one is added to itself, the resulting number is called a multiple of that number. The multiples that arise from multiplying a digit by a digit, are the facts to be learned in multiplication, of which there are 36 only, as will be shown in the following drills.

DEVELOPMENT OF THE TABLE.

☞ Take columns of 2's and bring out the 8 facts of the multiplication of 2's. Thus:

2	2	2	2	2	2	2	2			×2
2	2	2	2	2	2	2	2		4	.
4	2	2	2	2	2	2	2	$2 \times 2 = 4$	8	.
	6	2	2	2	2	2	2	$3 \times 2 = 6$	2	.
		8	2	2	2	2	2	$4 \times 2 = 8$	6	.
			10	2	2	2	2	$5 \times 2 = 10$	3	.
				12	2	2	2	$6 \times 2 = 12$	7	.
					14	2	2	$7 \times 2 = 14$	1	.
						16	2	$8 \times 2 = 16$	5	.
							18	$9 \times 2 = 18$	9	.

SLATE AND BLACKBOARD EXERCISES.

2122	12212	21220	21102	12221	22121
1	2	3	4	5	6

21221221212	122121121212	221021212222
7	8	9

REMARK.—The examples are graded, so that the multiplicand has no higher figure than the number treated, but the multiplier may be any of the nine digits. The pupil must be taught how to carry when the product is more than 9.

SUGGESTION.—Develop the table of 3's from columns of 3's. Thus

3	3	3	3	3	3	3		× 3
3	3	3	3	3	3	3		7 .
3	3	3	3	3	3	3		4
9	3	3	3	3	3	3	$3 \times 3 = 9$	8 .
	12	3	3	3	3	3	$4 \times 3 = 12$	2 .
		15	3	3	3	3	$5 \times 3 = 15$	9 .
			18	3	3	3	$6 \times 3 = 18$	3 .
				21	3	3	$7 \times 3 = 21$	6 .
					24	3	$8 \times 3 = 24$	5 .
						27	$9 \times 3 = 27$	1 .

REMARK.—As 3×2 is the same as 2×3, this combination educes no new fact, and it is therefore omitted. Only the seven new combinations with 3 are given.

SLATE AND BLACKBOARD EXERCISES.

2233	23213	231123	231233	233221	123332
1	2	3	4	5	6

231323231132	323103132332	321332323213
7	8	9

REMARK.—It is an excellent exercise to write a long line of 1's, 2's and 3's, and require pupils to multiply in class.

SUGGESTION.—Since 2×4 is the same as 4×2, and 3×4 as 4×3, there are only six new facts to be learned. Thus:

4	4	4	4	4	4		× 4
4	4	4	4	4	4		7
4	4	4	4	4	4		2 .
4	4	4	4	4	4		9
16	4	4	4	4	4	$4 \times 4 = 16$	5 .
	20	4	4	4	4	$5 \times 4 = 20$	8 .
		24	4	4	4	$6 \times 4 = 24$	1 .
			28	4	4	$7 \times 4 = 28$	4 .
				32	4	$8 \times 4 = 32$	6 .
					36	$9 \times 4 = 36$	3 .

SLATE AND BLACKBOARD EXERCISES.

342442434241	424434343243	434144413424
1(2)	3(4)	5(6)
432432434424	243423344443	432143414424
7	8	9

REMARK.—Dictate a long line of figures containing no figure greater than 4, and multiply the number so written by all the digits. Such exercises fix the facts learned and assure certain and steady advancement in arithmetical knowledge.

SUGGESTION.—Since 5 × 2 = 2 × 5; 5 × 3 = 3 × 5, and 5 × 4 = 4 × 5, these combinations need not be repeated. There are only five new facts to be learned. Thus

5	5	5	5	5		×5
5	5	5	5	5		3
5	5	5	5	5		6
5	5	5	5	5		4
5	5	5	5	5		1
25	5	5	5	5	5 × 5 = 25	8
	30	5	5	5	6 × 5 = 30	5
		35	5	5	7 × 5 = 35	9
			40	5	8 × 5 = 40	2
				45	9 × 5 = 45	7

SLATE AND BLACKBOARD EXERCISES.

543515452453	454525543452	515454354521
1(2)	3(4)	5(6)
545325525355	453542253545	535454515455
7	8	9

SUGGESTION.—Count by 6 up to 54, and then ask, How many 6's make 42? 30? 48? 24? 18? and so on. Learn the four new combinations, 6 × 6, 7 × 6; 8 × 6, 9 × 6.

SLATE AND BLACKBOARD EXERCISES.

636266624653	563646465362	461645656245
1(2)	3(4)	5(6)
645653646366	646564566466	646345656636
7	8	9

SUGGESTION.—There are only three new combinations to be learned with the 7's, viz.: 7×7; 8×7; 9×7.

SLATE AND BLACKBOARD EXERCISES.

74764737374676754	67674676764677675
2(3,4,5)	6(7,8,9)

REMARK.—It is a very profitable exercise to multiply a long example by each digit in turn.

SUGGESTION.—There are only two new combinations to be learned with the 8's, viz.: 8×8 and 9×8.

SLATE AND BLACKBOARD EXERCISES.

87887487868758678	87686878786868487
2(3,4,5)	6(7,8,9)

SUGGESTION.—There is but one combination to be learned with the 9's, viz.: $9 \times 9 = 81$.

SLATE AND BLACKBOARD EXERCISES.

98969497897896978	89879989698497987
2(3,4,5)	6(7,8,9)

REVIEW OF FACTS.

2×2, 3×2, 4×2, 5×2, 6×2, 7×2, 8×2, 9×2.

3×3, 4×3, 5×3, 6×3, 7×3, 8×3, 9×3.

4×4, 5×4, 6×4, 7×4, 8×4, 9×4.

5×5, 6×5, 7×5, 8×5, 9×5.

6×6, 7×6, 8×6, 9×6.

7×7, 8×7, 9×7.

8×8, 9×8.

9×9.

REMARK.—Keep up a continual oral review of the facts as learned.

The graded work in the fundamental rules, as presented in the "Progressive Oral and Written Drills," is the outcome of the best modern teaching, and, if carried out faithfully, will secure the great ends of (1) accuracy, (2) facility, and (3) rapidity in arithmetical operations. When the pupil has been taught to think all the primary combinations, advancement in applying numbers will be assured. Time spent in fixing the facts, will be a tenfold gain finally.

☞ These *inductive exercises* are reviews of the facts of multiplication in the concrete.

The pupil should name results without giving an analysis of his process.

INDUCTIVE EXERCISES.

ART. 50. **1.** How much do 3 two-cent postage stamps cost?

2. How much do 3 three-cent postage stamps cost?

3. Harry bought 4 peaches at 2 cents each: how much did he pay?

4. A drover bought 5 sheep at $3 a head: how many dollars did they cost?

5. How much do 4 cakes cost at 4 cents apiece?

6. There are 3 feet in one yard: how many are there in 4 yards?

7. I bought 4 pencils at 5 cents each: how much did they cost me?

8. If a quart of currants costs 6 cents, what do 3 quarts cost?

9. There are 7 days in one week: how many days are there in 3 weeks?

10. William recited 5 lessons a day for 5 days: how many lessons did he recite?

11. Horace rode 5 miles an hour on his bicycle for 6 hours: how many miles did he ride?

12. There are 4 pecks in a bushel: how many pecks are there in 7 bushels?

13. How much do 6 copy-books cost at 6 cents each?

14. At 8 cents a yard, how much do 3 yards of muslin cost?

15. Moore piled his books in 4 piles, placing 8 books in each pile: how many books had he?

16. How much do 6 hats cost at $5 each?

17. A dime is worth 10 cents: how much are 2 dimes worth?

18. What do 4 pairs of trousers cost at $7 a pair?

19. If there are 5 school days in 1 week, how many school days are there in 7 weeks?

20. At 3 cents a mile, what does it cost to ride 9 miles?

21. How much do 3 dozen eggs cost at 12 cents a dozen?

22. There are 8 quarts in 1 peck: how many quarts are there in 4 pecks?

MULTIPLICATION TABLE.

2 × 1 = 2	3 × 1 = 3	4 × 1 = 4	5 × 1 = 5	6 × 1 = 6
2 × 2 = 4	3 × 2 = 6	4 × 2 = 8	5 × 2 = 10	6 × 2 = 12
2 × 3 = 6	3 × 3 = 9	4 × 3 = 12	5 × 3 = 15	6 × 3 = 18
2 × 4 = 8	3 × 4 = 12	4 × 4 = 16	5 × 4 = 20	6 × 4 = 24
2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25	6 × 5 = 30
2 × 6 = 12	3 × 6 = 18	4 × 6 = 24	5 × 6 = 30	6 × 6 = 36
2 × 7 = 14	3 × 7 = 21	4 × 7 = 28	5 × 7 = 35	6 × 7 = 42
2 × 8 = 16	3 × 8 = 24	4 × 8 = 32	5 × 8 = 40	6 × 8 = 48
2 × 9 = 18	3 × 9 = 27	4 × 9 = 36	5 × 9 = 45	6 × 9 = 54
2 × 10 = 20	3 × 10 = 30	4 × 10 = 40	5 × 10 = 50	6 × 10 = 60
2 × 11 = 22	3 × 11 = 33	4 × 11 = 44	5 × 11 = 55	6 × 11 = 66
2 × 12 = 24	3 × 12 = 36	4 × 12 = 48	5 × 12 = 60	6 × 12 = 72

7 × 1 = 7	8 × 1 = 8	9 × 1 = 9	10 × 1 = 10	11 × 1 = 11
7 × 2 = 14	8 × 2 = 16	9 × 2 = 18	10 × 2 = 20	11 × 2 = 22
7 × 3 = 21	8 × 3 = 24	9 × 3 = 27	10 × 3 = 30	11 × 3 = 33
7 × 4 = 28	8 × 4 = 32	9 × 4 = 36	10 × 4 = 40	11 × 4 = 44
7 × 5 = 35	8 × 5 = 40	9 × 5 = 45	10 × 5 = 50	11 × 5 = 55
7 × 6 = 42	8 × 6 = 48	9 × 6 = 54	10 × 6 = 60	11 × 6 = 66
7 × 7 = 49	8 × 7 = 56	9 × 7 = 63	10 × 7 = 70	11 × 7 = 77
7 × 8 = 56	8 × 8 = 64	9 × 8 = 72	10 × 8 = 80	11 × 8 = 88
7 × 9 = 63	8 × 9 = 72	9 × 9 = 81	10 × 9 = 90	11 × 9 = 99
7 × 10 = 70	8 × 10 = 80	9 × 10 = 90	10 × 10 = 100	11 × 10 = 110
7 × 11 = 77	8 × 11 = 88	9 × 11 = 99	10 × 11 = 110	11 × 11 = 121
7 × 12 = 84	8 × 12 = 96	9 × 12 = 108	10 × 12 = 120	11 × 12 = 132

12 × 12 = 144

DEFINITIONS AND PRINCIPLES.

ART. 51. Multiplication is the process of repeating one number as many times as there are units in another number. Thus: $32 + 32 + 32 + 32 = 4$ times 32, or 128.

ART. 52. The **Multiplicand** is the number repeated. $4 \times 12 = 48$; 12 is the multiplicand.

ART. 53. The **Multiplier** is the number showing the number of times the multiplicand is repeated. 4 is the multiplier.

ART. 54. The **Product** is the result of the multiplication. 48 is the product.

The **Factors of a Number** are the numbers which, when multiplied together, produce that number.

The multiplicand and multiplier are factors of the product. 4 and 12 are factors of 48.

ART. 55. The **Sign of Multiplication** ($\times$) is read *times* or *multiplied by,* and when placed between two numbers, means that they are to be multiplied together. Thus: 2×5 is read 2 *times* 5, or 2 *multiplied by* 5.

ART. 56. The **Parenthesis** ( ) signifies that the inclosed expression is to be considered as one number. Thus: $32 - (8 + 3)$ signifies that the sum of 8 and 3 is to be taken from 32.

ART. 57. Principles. 1. *The multiplicand and product are like numbers.* 6 times 7 pens = 42 pens.

2. *The multiplier is always an abstract number.*

3. *The product divided by either of its factors gives the other factor.* 42 pens $\div$ 6 = 7 pens; 42 pens $\div$ 7 pens = 6.

ORAL EXERCISES.

ART. 58.—1. What is the cost of 3 penknives at 25 cents each?

Solution.—The cost is 3 times 25 cents, or 75 cents.

2. What are 7 ducks worth at 80 cents apiece?

3. What is the price of 6 horses at $120 each?

4. How much will 8 cows cost at $30 apiece?

5. What is the price of 24 melons at 7 cents each?

6. Morgan earns $8 a week and pays $3 for his board: how much remains in 7 weeks after his board is paid?

7. Charles walked 4 miles a day for 6 days, and Samuel walked 5 miles a day for 7 days: how much farther did Samuel walk than Charles?

8. A certain orchard contains 12 rows of 16 trees each: how many trees are in the orchard?

9. Belle and Jennie were engaged 4 hours in picking cherries; Belle picked 3 quarts an hour and Jennie picked 5 quarts an hour: how many quarts were picked by both?

10. Albert receives $40 a month, and his expenses are $12: how much can he save in 5 months?

11. Peter bought 3 horses at $150 each, and sold them for $165 apiece: how much did he make?

12. Fannie missed 3 words and Carrie 6 for each of 3 days: how many more did Carrie miss than Fannie?

13. Mrs. Dobbins sold her grocer 5 dozen eggs at 30 cents a dozen; she bought 8 yards of calico at 12 cents a yard and received the remainder in cash: how much cash did she receive?

14. A drover bought 9 horses at $150 each and sold them at a gain of $20 each: how much did he receive?

15. Two men walk in opposite directions, one going 4 miles an hour and the other 5 miles an hour: how far apart will they be in 8 hours?

16. A gentleman bought 72 oranges and gave 9 to each of 7 children: how many had he left?

17. John earns $7 a week, which is $2 less than Harry earns: how much can Harry earn in 8 weeks?

18. John earns $9 a week, which is $3 more than George earns: how much can George earn in 9 weeks?

19. A farmer exchanges 7 tons of hay worth $9 a ton for 8 barrels of flour worth $7 a barrel: how much is still due him on the trade?

20. A drover bought 9 sheep at $3 a piece, and sold them for $36: did he gain or lose, and how much?

21. How many more or fewer bushels are there in [illegible] sacks of wheat, each containing 3 bushels, than in 8 sacks of bran, each containing 4 bushels?

22. A baker bought 9 barrels of flour at $6 a barrel and had $8 left: how much money had he at first?

23. A lady had six five dollar bills: she paid $9 for a cloak, $8 for a bonnet, $6 for a shawl, and $3 for a pair of shoes: how much money had she left?

24. James has 8 marbles and John has 7 times as many: how many marbles have they both?

25. A chain is worth $7 and a watch 6 times as much: how much more is the watch worth than the chain?

26. How much less is the cost of 8 oranges at 7 cents each than the cost of 9 lemons at 7 cents each?

27. A steamboat left St. Louis at 7 o'clock A.M. and went down the river at the rate of 8 miles an hour. At 2 o'clock P.M. the boat turned and came back at the rate of 6 miles an hour. How far from St. Louis was the boat at 10 o'clock P.M.?

ART. 59.—To Multiply by a Number Not Greater than 12

WRITTEN EXERCISES.

1. Multiply 586 by 8.

Process.

$$\begin{array}{r} 586 \\ 8 \\ \hline 4688 \end{array}$$

Analysis.—We write the multiplier under the multiplicand, and begin to multiply at the right.

Eight times 6 units are 48 units, equal to 4 tens and 8 units. We write 8 in the units' place and add the tens to the product of tens. Eight times 8 tens are 64 tens, and 4 tens added are 68 tens, equal to 6 hundreds and 8 tens. We write 8 in the tens' place and add 6 to the product of hundreds. Eight times 5 hundreds are 40 hundreds, and 6 hundreds added are 46 hundreds, equal to 4 thousands and 6 hundreds. We write 6 in the hundreds' place and 4 in the thousands' place. Therefore, 586 × 8 = 4688.

Proof.—586 + 586 + 586 + 586 + 586 + 586 + 586 + 586 = 4688.

2. Multiply 64534 × 6.	**10.** Multiply 69875 × 10.
3. " 74625 × 7.	**11.** " 84689 × 11.
4. " 86485 × 8.	**12.** " 98647 × 12.
5. " 76479 × 9.	**13.** " 76437 × 11.
6. " 89684 × 8.	**14.** " 84968 × 12.
7. " 76987 × 7.	**15.** " 78497 × 10.
8. " 98768 × 9.	**16.** " 68979 × 11.
9. " 79697 × 8.	**17.** " 86498 × 12.

18. A man bought 11 horses at the average price of $148: how much did they cost?

19. At $387 an acre, what will 12 acres of land cost?

20. If a garrison consume 3,482 pounds of beef in one week, how much will they consume in 11 weeks?

21. Allowing 365 days to a year, how many days has a boy lived, who is 12 years old?

ART. 60.—To Multiply by a Number Greater than 12.

1. Multiply 546 by 24.

Process.
546
24
2184
1092
13104

Analysis.—We write the multiplier under the multiplicand, so that units of the same order stand in the same column, and begin at the right to multiply.

Four times 6 units are 24 units, equal to 2 tens and 4 units. We write 4 in the units' place and add the tens to the product of tens. Four times 4 tens are 16 tens and 2 tens added are 18 tens, equal to 1 hundred and 8 tens. We write 8 in the tens' place and add the hundreds to the product of hundreds. Four times 5 hundreds are 20 hundreds, and 1 hundred added are 21 hundreds, equal to 2 thousands and 1 hundred. We write 1 in the hundreds' place and 2 in the thousands' place.

Twenty (or 2 tens) times 6 units are 120 units, equal to 1 hundred, 2 tens. We write 2 in the tens' place and add 1 hundred to the product of hundreds. Twenty (or 2 tens) times 4 tens are 8 hundreds, and 1 hundred added are 9 hundreds, which we write in the hundreds' place. Twenty (or 2 tens) times 5 hundreds are 10 thousands, equal to 1 ten thousand and 0 thousand, which we write in their proper places. Adding these partial products, we have 13104.

Therefore, $546 \times 24 = 13104$.

ART. 61.—Rule for Multiplying when the multiplier is greater than 12. *Write the multiplier under the multiplicand, so that units of the same order stand in the same column.*

Multiply each order of the multiplicand by each order of the multiplier successively, beginning with the units. Write the first figure of each partial product under the multiplier that produced it, add the partial products, and the sum is the product required.

Solve the following:

2. 456 × 13.	**24.** $1945 × 64.
3. 675 × 15.	**25.** $71921 × 48.
4. 697 × 17.	**26.** $12305 × 56.
5. 794 × 16.	**27.** $34175 × 64.
6. 534 × 18.	**28.** $41220 × 72.
7. 4362 × 19.	**29.** $37319 × 84.
8. 6534 × 21.	**30.** $14863 × 132.
9. 8564 × 23.	**31.** $81231 × 223.
10. 3654 × 26.	**32.** $24852 × 305.
11. 57354 × 27.	**33.** $54208 × 423.
12. 60932 × 28.	**34.** $31723 × 321.
13. 35408 × 29.	**35.** $47328 × 604.
14. 56008 × 33.	**36.** $72309 × 423.
15. 80008 × 37.	**37.** $66648 × 508.
16. 100001 × 49.	**38.** $30439 × 294.
17. 356732 × 54.	**39.** $53821 × 452.
18. 547390 × 57.	**40.** $27638 × 704.
19. 203940 × 59.	**41.** $53575 × 813.
20. 573982 × 63.	**42.** $69833 × 808.
21. 718302 × 89.	**43.** $392038 × 532.
22. 367098 × 124.	**44.** $516324 × 789.
23. 938465 × 492.	**45.** $820406 × 798.

46. If a steamer sails 453 miles a day, how far will she sail in 79 days?

47. A farm containing 135 acres cost $78 an acre: what did the farm cost?

48. If a bicyclist averaged 63 miles a day for 68 days, how far did he travel?

49. If a man spends $95 annually for cigars, what will the amount be in 18 years?

50. How much will 69 cords of wood cost at $4.50 a cord?

51. If a railroad engineer travels 794 miles a week, how far will he travel in 48 weeks?

52. Mr. Moore sold 1,324 acres of land at $193 an acre: how much did he receive?

53. How much will it cost to build 734 miles of railroad at $48,356 per mile?

54. How much pension money do 679 soldiers receive, if each receives $2,495?

55. What is the cost of 185 city lots at $3,125 each?

56. A butcher bought 48 sheep at $7 a head, and 32 barrels of beef at $17 a barrel: how much more did he pay for the beef than for the sheep?

57. What is the cost of 3,150 acres of prairie land at $38 an acre, and 2,542 acres of woodland at $46 an acre?

58. Mr. Robinson's yearly income is $7,250, and his average daily expenditure $7: how much can he save in a year of 365 days?

59. A certain field contains 4,238 rows of corn; each row contains 3,102 hills, and each hill has 4 stalks: how many stalks of corn are in the field?

60. A builder used 7 loads of brick a day for 139 days. If each load contained 1,244 bricks, how many bricks did he use?

61. Since a day contains 24 hours and each hour contains 60 minutes and each minute 60 seconds, how many seconds are there in 19 days?

62. A certain street in a city contains 48 houses, each house 24 windows, and each window 16 panes: how many window panes are in the 48 houses?

63. Illustrate by an original problem the multiplication of one number by another.

ART. 62. To Multiply when the Multiplier is a Composite Number.

1. Multiply 342 by 18.

Process.

$$\begin{array}{r} 342 \\ 6 \\ \hline 2052 \\ 3 \\ \hline 6156 \end{array}$$

Analysis.—Since 18 is composed of the factors 3 and 6, it is evident that if the multiplicand be multiplied by either of them and that product by the remaining factor, the result is the same as that obtained by multiplying by 18.

2. Multiply 643 by 16.
3. Multiply 584 by 22.
4. Multiply 963 by 24.
5. Multiply 4765 by 50.
6. Multiply 7649 by 32.
7. Multiply 678 by 48.
8. Multiply 7654 by 64.
9. Multiply 3087 by 72.
10. Multiply 6009 by 81.
11. Multiply 908070 by 144.

ART. 63. To Multiply by 10, 100, 1000, etc.

1. Multiply 34 by 10; by 100; by 1000.

It is evident that when any number is multiplied by 10, it is moved one order to the left; when multiplied by 100, it is moved two orders to the left, and when multiplied by 1000, it is moved three orders to the left.

$34 \times 10 = 340$; $34 \times 100 = 3400$; $34 \times 1000 = 34000$.

Therefore, *To multiply any number by* 10, 100, 1000, *etc., annex as many ciphers to the number as there are ciphers in the multiplier.*

2. Multiply 368 by 10; by 100; by 1000.
3. Multiply 459 by 100; by 10; by 1000.
4. Multiply 1324 by 1000; by 100; by 10.
5. Multiply 70406 by 10; by 1000; by 100.
6. Multiply 35600 by 10; by 1000; by 100.

Division.

Progressive Oral and Written Drills.

SUGGESTION.—Division is the reverse of multiplication, and is the process of finding how many times one number is contained in another. Take the multiples of 2, and ask, How many 2's in 4? 12? 16? 6? 8? 14? 10? 18? Illustrate division by separating objects into groups of 2. If odd numbers are taken, there will always be one over, which may be written over the divisor, giving rise to the fractional form $\frac{1}{2}$.

SLATE AND BLACKBOARD EXERCISES.

2)168246　　2)864691　　2)604826　　2)842697

For explanation, see **Article 76.**

REMARK.—The dividend may contain any figure, but the divisor must not be larger than the digit which is the object of study.

What is $\frac{1}{2}$ of 4? 6? 8? 10? 12? 14? 16? 18?

SUGGESTION.—Count by 3 to 27. Ask, How many 3's in 9? 18? 12? 6? 21? 15? 24? 27?

Begin at 1 and count by 3's. The numbers thus obtained will contain a certain number of 3's with one over, which is $\frac{1}{3}$ of 3. How many 3's in 7? 13? 19? 25? 16? 28? 10? 22?

Begin at 2 and count by 3's. These numbers will all [illegible] 2 over, which is $\frac{2}{3}$ of 3. How many 3's in 8? 14? 20? 26? 17? [illegible] 23?

SLATE AND BLACKBOARD EXERCISES.

3)345963　　3)6481583　　3)6378457　　3)4579864

REMARK.—When there is 1 or 2 over with the last division, write what remains over the divisor.

What is $\frac{1}{3}$ of 6? 9? 12? 15? 18? 21? 24? 27?

What is $\frac{2}{3}$ of 6? 9? 12? 15? 18? 21? 24? 27?

NOTE.—The process of finding the fractional parts of multiples, is an excellent review exercise in division and multiplication.

SUGGESTION.—Count by 4's up to 36. Ask, How many 4's in 16? 24? 20? 32? 12? 36? 8? 28? Illustrate by groups of 4.

Begin with 1 and count by 4's up to 37. These numbers give a certain number of 4's and 1 over, which is $\frac{1}{4}$ of 4.

Begin with 2 and count by 4's up to 38. These numbers contain a certain number of 4's and 2 over, which is $\frac{2}{4}$ of 4.

Begin with 3 and count by 4's up to 39. These numbers contain a certain number of 4's, and 3 over, which is $\frac{3}{4}$ of 4.

SLATE AND BLACKBOARD EXERCISES

4)4845286476 4)8070842676 4)6880462416

REMARK.—Dictate additional examples.

What is $\frac{1}{4}$ of 4? 8? 12? 16? 20? 24? 28? 32? 36?

☞ Find $\frac{2}{4}$ and $\frac{3}{4}$ of the above numbers.

SUGGESTION.—Begin with 0, 1, 2, 3, 4, and count by 5. Tell how many 5's and how many over there are in each number.

SLATE AND BLACKBOARD EXERCISES.

5)4056849686 5)8402868405 5)6847967245

What is $\frac{1}{5}$ of 10? 30? 45? 20? 35? 15? 40? 25?

☞ Find $\frac{2}{5}$, $\frac{3}{5}$ and $\frac{4}{5}$ of the above numbers.

SUGGESTION.—After beginning with 0, 1, 2, 3, 4 or 5, and counting by 6's let pupils take such columns as the following and tell how many 6's and how many over there are in each number. Thus, In 14 there are two 6's and 2 over, and so on.

14	34	54	4	24	44
39	59	9	29	49	19
50	0	20	40	10	30
8	28	48	18	38	58
21	41	11	31	51	1
47	17	38	57	7	27
12	32	52	2	22	42
36	56	6	26	46	16
53	63	23	43	13	33
5	25	45	15	35	55

SLATE AND BLACKBOARD EXERCISES.

6)7264816743 6)8463948765 6)4376798268

REMARK.—Dictate additional examples.

What is $\frac{1}{6}$ of 18 ? 36 ? 54 ? 42 ? 24 ? 12 ? 30 ? 48 ?

☞ Find $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$ and $\frac{5}{6}$ of the above numbers.

SUGGESTION.—Begin with 0, 1, 2, 3, 4, 5, or 6, and count by 7's up to 69. After the pupil can do this with facility, let him tell how many 7's and how many over there are in each of the following numbers:

14	34	54	4	24	44	64
39	59	9	29	49	69	19
50	0	20	40	60	10	30
8	28	48	68	18	38	58
21	41	61	11	31	51	1
47	67	17	37	57	7	27
62	12	32	52	2	22	42
16	36	56	6	26	46	66
33	53	3	23	43	63	13
55	5	25	45	65	15	35

REMARK.—This table contains all numbers from 0 to 70.

What is $\frac{1}{7}$ of 14 ? 21 ? 28 ? 35 ? 42 ? 49 ?

☞ Find $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$ of the above numbers.

SLATE AND BLACKBOARD EXERCISES.

7)8964068976 7)8604964843 7)8774879543

REMARK.—Dictate additional examples.

SUGGESTION.—Begin with 0, 1, 2, 3, 4, 5, 6, or 7, and count by 8's up to 79. This exercise is not only a review of addition, but prepares the pupil for the analytic process of division.

☞ Tell how many 8's and how many over there are in each of the following numbers:

4	24	44	64	14	34	54	74
29	49	69	19	39	59	79	9
40	60	10	30	50	70	0	20
68	18	38	58	78	8	28	48
11	31	51	71	1	21	41	61
37	57	77	7	27	47	67	17
52	72	2	22	42	62	12	32
76	6	26	46	66	16	36	56
3	23	43	63	13	33	53	73
25	45	65	15	35	55	75	5

SLATE AND BLACKBOARD EXERCISES.

8)5487465346 8)9706843768 8)3807649876

What is $\frac{1}{8}$ of 16? 24? 32? 40? 48? 56? 64?

☞ Find $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$ and $\frac{7}{8}$ of the above numbers.

SUGGESTION.—Begin with 0, 1, 2, 3, 4, 5, 6 or 7, and count by 9's up to 89. Arrange the numbers as in the table of 8's, and tell how many 9's and how many over there are in each number.

SLATE AND BLACKBOARD EXERCISES

9)6436794230 9)9835264984 9)6031687648

REMARK.—Dictate additional examples, and divide by each of the digits.

What is $\frac{1}{9}$ of 18? 27? 36? 45? 54? 63? 72? 81?

☞ Find $\frac{2}{9}$, $\frac{3}{9}$, $\frac{4}{9}$, $\frac{5}{9}$, $\frac{6}{9}$, $\frac{7}{9}$ and $\frac{8}{9}$ of the above numbers.

REMARK.—Finding the fractional parts of the multiples of the digits, fixes the tables of multiplication and division. No other drill will accomplish as much with as little effort. If the plans suggested heretofore are faithfully followed out, the pupil will have acquired such a mastery over pure numbers as to enable him to perform the required operations with accuracy, facility, and rapidity.

☞ These *inductive exercises* are reviews of the facts of division. The pupil should name results without giving an analysis of his process.

INDUCTIVE EXERCISES.

ART. 64.—1. If an orange cost 2 cents, how many can be bought for 4 cents? How many 2's are there in 4?

2. In one quart there are 2 pints: how many quarts are there in 6 pints? How many 2's are there in 6?

3. If a man earns $2 a day, how many days will it take him to earn $8? How many 2's are there in 8?

4. There are 4 quarts in a gallon: how many gallons are there in 12 quarts? How many 4's are there in 12?

5. George bought 3 sticks of candy for 18 cents: what was the price of each stick? How many 3's are there in 18?

6. How many yards of calico at 5 cents a yard can be bought for 15 cents?

7. Caroline paid 16 cents for 4 pencils: what was the price of each pencil?

8. If 12 trees are arranged in 3 equal rows, how many trees are there in each row?

9. How many piles of books, with 4 books in each pile, can be made with 20 books?

10. If I pay 6 cents for a pencil, how many pencils can I buy for 24 cents?

11. At 7 cents a yard, how many yards of muslin car be bought for 28 cents?

12. At 8 cents a quart, how many quarts of milk can be bought for 24 cents?

13. At $6 a ton, how many tons of coal can be bought for $30?

14. Six men paid $36 rent for a room: how much did each pay?

15. The rent of Mr. Cooley's house for 3 months is \$30 : what is the rent per month?

16. Isaac gave 32 cents for 4 pounds of sugar: what was the price per pound?

17. If 3 yards of velvet cost \$27, what is the price per yard?

18. John received 50 cents for 5 hours' work: how much was he paid per hour?

19. Mary divided 42 cents equally among 7 girls: how much did she give each girl?

20. A grocer bought 6 barrels of flour for \$48: what was the price per barrel?

21. A farmer sold 7 cords of wood for \$49: what was the price per cord?

22. A merchant sold 4 stoves for \$36: how much did he get for each stove, the price of each being the same?

23. Mr. Burton received \$60 for 6 tons of hay: what was the price per ton?

24. Richard paid 40 cents for 4 copy books: what did he pay for each?

25. How many yards of muslin, at 8 cents a yard, can be bought for 48 cents?

26. There are 7 days in a week: how many weeks are there in 21 days?

27. If a steamer goes 10 miles an hour, how many hours will it take her to go 40 miles?

28. There are 8 quarts in a peck: how many pecks are there in 32 quarts?

29. How many hats at \$4 apiece can be bought for \$28?

30. How many pairs of shoes at \$5 a pair can be bought for \$45?

DEFINITIONS AND PRINCIPLES.

ART. 65. Division is the process of finding how many times one number is contained in another like number, or of separating a number into equal parts. Thus: $5 + 5 + 5 + 5 = 20$; there are four 5's in 20.

ART. 66.—The **Dividend** is the number to be divided. Thus: $20 \div 4 = 5$; 20 is the dividend.

ART. 67. The **Divisor** is the number by which we divide. Thus: $20 \div 4 = 5$; 4 is the divisor and 5 is the quotient.

ART. 68. The **Quotient** is the number showing how many times the divisor is contained in the dividend.

ART. 69. The **Remainder** is what is left of the dividend when the division is not exact. Thus: $22 \div 4 = 5$ and 2 over; 2 is the remainder.

The divisor and quotient are factors of the dividend. Thus: $5 \times 4 = 20$.

ART. 70. The **Sign of Division** ($\div$) is read *divided by*, and, when placed between two numbers, it shows that the number on the left is to be divided by the number on the right. Thus, $12 \div 3$ is read 12 *divided by* 3. Division may also be expressed thus: $\frac{12}{3}$ and $3\overline{)12}$.

ART. 71. Division is the reverse of multiplication. Thus: 5 times 7 = 35; in 35 there are five 7's.

ART. 72. Principles. 1. *The dividend equals the product of the divisor and quotient, plus the remainder.*

2. *When the dividend and divisor are like numbers, the quotient is abstract.*

3. *If the divisor only is abstract, the quotient is of the same kind as the dividend.*

4. *Multiplying or dividing the divisor and dividend by the same number does not change the quotient.*

ORAL EXERCISES.

ART. 73.—1. How many lemons at 6 cents each can be bought for 18 cents?

Solution.—As many lemons can be bought for 18 cents as 6 cents is contained times in 18 cents. Hence, 3 lemons can be bought for 18 cents.

2. If John earns $2 a day, how long will it take him to earn $28?

3. A boy spent 24 cents in buying melons at 4 cents each: how many did he buy?

4. James divided 36 chestnuts equally among 6 of his friends: how many did he give to each?

5. Three girls shared 27 cherries equally: how many had each girl?

6. How many kegs of 7 gallons each can be filled from a hogshead of cider containing 63 gallons?

7. A man gave 20 sheep for 2 cows worth $40 apiece: what was the value of each sheep?

8. I paid $20 for 5 pigs and sold them at a gain of $10: what did I receive for each?

9. A dealer delivered 3 tons of coal to a shoemaker, for 6 pairs of shoes at $3 a pair: what was the price of the coal per ton?

10. The dividend is 36 and the quotient 9: what is the divisor?

11. The divisor is 5 and the quotient 8: what is the dividend?

12. The quotient is 7, and the divisor 9: what is the dividend?

13. The dividend is 96 and the quotient 16: what is the divisor?

When the division is performed by writing results only, the process is called *short division*.

ART. 74.—To Divide by Short Division.

WRITTEN EXERCISES.

1. Divide 8642 by 8.

Process.

$$\begin{array}{r} 8)8642 \\ \hline 1080\frac{2}{8} \end{array}$$

Analysis.—We write the divisor on the left of the dividend with a line between them, and begin at the left to divide.

Eight is contained in 8 thousands, 1 (thousand) times. We write 1 underneath the dividend in the thousands' place of the quotient. Eight is not contained in the hundreds' order of the dividend, hence we write 0 in the hundreds' place in the quotient and annex the 4 tens. Eight is contained in 64 tens 8 (tens) times. We write 8 in the tens' place of the quotient. Eight is not contained in the units' order of the dividend. We write 0 in the units' place, and annex the remainder, written over the divisor.

Therefore, $8642 \div 8 = 1080\frac{2}{8}$.

Proof.—$(1080 \times 8) + 2 = 8642$.

ART. 75. Rule for Short Division.—*Write the divisor at the left of the dividend with a line between them.*

Beginning at the left, divide each order of the dividend by the divisor, writing the proper quotient figure underneath. If there be a remainder, change it to the next lower order, add to it the figure in the dividend of that order, and divide as before.

When there is a final remainder, write the divisor beneath it with a line between and annex it to the quotient.

Proof.—*Multiply the divisor by the quotient and add the remainder, if any. If the sum equals the dividend the answer is correct.*

Solve and prove the following:

2.	43432 ÷ 3.	**14.**	90807060 ÷ 10.
3.	68464 ÷ 4.	**15.**	237697 ÷ 11.
4.	53636 ÷ 6.	**16.**	961056 ÷ 12.
5.	72865 ÷ 5.	**17.**	8759460 ÷ 10.
6.	46784 ÷ 6.	**18.**	66099088 ÷ 11.
7.	58742 ÷ 4.	**19.**	976586472 ÷ 10.
8.	79875 ÷ 5.	**20.**	68643284 ÷ 12.
9.	68097 ÷ 7.	**21.**	48637480 ÷ 10.
10.	87612 ÷ 8.	**22.**	789674 ÷ 12.
11.	40672 ÷ 8.	**23.**	67496002 ÷ 10.
12.	32769 ÷ 9.	**24.**	92070640 ÷ 11.
13.	7967540 ÷ 9.	**25.**	86745392 ÷ 12.

26. If 1 man can chop a certain pile of wood in 128 days, how long will it take 6 men to chop it?

27. A dealer paid $84 for 12 barrels of flour: what was the price per barrel?

28. A man sold 6 horses for $798: what was the price of each?

29. When coal is worth $7 a ton, how many tons can be bought for $8,680?

30. How many suits of clothes worth $12 each can be bought for $1,332?

31. If a barrel of beef is worth $11, how many barrels can be bought for $1,309?

32. If a man walks 9 miles a day, how many days will it take him to walk 981 miles?

33. Illustrate short division by an original problem.

When the division is performed by writing the steps in the solution, the process is called *Long Division.*

ART 76.—To Divide by Long Division.

WRITTEN EXERCISES.

1. Divide 4235 by 24.

Process.

```
24)4235(176 11/24
   24
   183
   168
    155
    144
     11 rem.
```

Proof.

$(176 \times 24) + 11 = 4235.$

Analysis. We write the divisor on the left of the dividend with a line between them and a line on the right for the quotient. 24 is contained in 42 hundreds, 1 (hundred) time. We write 1 in the hundreds' place of the quotient, multiply the divisor 24 by the quotient figure 1 (hundred) and place the product 24 hundreds underneath the partial dividend 42 hundreds. Subtracting the product from the partial dividend gives a remainder of 18 hundreds. To this remainder we annex 3 tens of the dividend. 24 is contained in 183 tens, 7 tens, times. Writing 7 in the tens' place of the quotient, we multiply the divisor 24 by 7 tens, and subtract the product 168 tens from the partial dividend, and to the remainder annex the units' figure of the dividend. 24 is contained in 155 units 6 times with a remainder of 11. We express the remainder in the quotient by writing it over the divisor and annexing it to the quotient. Therefore, $4235 \div 24 = 176\frac{11}{24}$.

ART. 77. Rule for Long Division.—*Write the divisor on the left of the dividend, with a line between them.*

Find how many times the divisor is contained in the fewest left hand orders of the dividend which will contain it, and write that number for the first figure of the quotient.

Multiply the divisor by this quotient figure, subtract the product from the orders divided, and annex to the remainder the next figure of the dividend. Divide as before until all the orders of the dividend are used.

If any partial dividend does not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend and proceed as before.

Write the final remainder over the divisor and annex it to the quotient.

Proof. *The proof is the same as in short division.*

SUGGESTION.—When the divisor is large, use its left-hand figure as a trial divisor and one or two of the left-hand figures of the dividend as a trial dividend. See how many times this trial divisor is contained in the trial dividend and multiply the divisor by the quotient thus obtained. If the product is larger than the partial dividend, diminish the quotient figure by 1 and multiply and compare again. If the product is equal or less than the partial dividend, subtract; but if the remainder is greater than the divisor, increase the quotient figure by 1 and multiply, compare and subtract as before. Thus, 829804 ÷ 9768. 9 is contained in 82 9 times. 9 × 9768 = 87912. The quotient figure is too large, so we diminish 9 by 1 and try 8.

Solve and prove the following:

2. 3228432 ÷ 14.
3. 4801643 ÷ 15.
4. 4688976 ÷ 13.
5. 5285432 ÷ 16.
6. 6306187 ÷ 18.
7. 6809841 ÷ 17.
8. 7747682 ÷ 18.
9. 9209384 ÷ 20.
10. 10566381 ÷ 22.
11. 12007289 ÷ 24.
12. 12756247 ÷ 25.
13. 14046008 ÷ 26.
14. 17988709 ÷ 31.
15. 21966908 ÷ 36.
16. 23317649 ÷ 37.
17. 24324087 ÷ 38.
18. 28806004 ÷ 40.
19. 31507040 ÷ 42.
20. 34326283 ÷ 44.
21. 38404970 ÷ 80.
22. 40598231 ÷ 123.
23. 56704706 ÷ 135.
24. 306508970 ÷ 225.
25. 450567648 ÷ 128.
26. 82944872 ÷ 256.
27. 100580684 ÷ 235.
28. 205000987 ÷ 328.
29. 175680486 ÷ 360.
30. 313088354 ÷ 452.
31. 271150987 ÷ 425.
32. 34650676 ÷ 350.
33. 108992974 ÷ 131.
34. 192474762 ÷ 222.
35. 460645978 ÷ 509.
36. 143901643 ÷ 177.
37. 343700892 ÷ 491.
38. 159528968 ÷ 391.
39. 104329487 ÷ 323.
40. 137700593 ÷ 425.
41. 162192644 ÷ 436.
42. 156772761 ÷ 509.
43. 308025904 ÷ 555.
44. 424854760 ÷ 328.
45. 634625890 ÷ 387.
46. 714739074 ÷ 433.
47. 613846304 ÷ 584.
48. 718479987 ÷ 628.
49. 5134562687 ÷ 721.

50. $(18 + 16 - 14) \div 9 = ?$

51. $(360 + 234 - 144) \div (48 - 20) = ?$

52. $(420 + 264 \times 4) \div (5 \times 4 - 4) = ?$

53. $(354 \times 5 - 200) \div (36 - 6 \times 5) = ?$

54. How many acres will produce 8,400 bushels of wheat at an average yield of 28 bushels to the acre?

55. The circumference of the earth is 25,000 miles: how long will it take a ship to sail around it, at the rate of 115 miles a day?

56. There are 320 rods in a mile: how many miles in 48,762 rods?

57. There are 144 square inches in a square foot: how many square feet in 785,643 square inches?

58. A railroad earned $2,456,384 in a year: what was the average earnings each day, reckoning 365 days to a year?

59. Illustrate long division by an original problem.

ART. 78.—To Divide by Factors, or when the Divisor has Ciphers on the Right.

1. Divide 38564 by 1000; by 5,000.

Process.

1|000)38|584

Ans. $38\frac{584}{1000}$

5|000)38|584

Ans. $7\frac{3584}{5000}$

Analysis. It has been shown (Art. 63) that any integer may be multiplied by 10, 100, 1000, etc., by annexing to the multiplicand as many ciphers as there are ciphers in the multiplier. It follows, therefore, that to divide any number by 10, 100, 1000, etc., as many figures should be cut off from the right of the dividend as there are ciphers in the divisor, such figures being placed over the divisor and annexed to the quotient as a fractional part of the same.

2. Divide 875692 by 10; by 100; by 1000.

3. Divide 38956 by 400; by 60; by 7000.

4. Divide 983480 by 5000; by 70; by 900.

5. Divide 483614 by 40; by 6000; by 80000.

6. Divide 82356 by 5000; by 700; by 9000.

7. Divide 588 by 12, using the factors 3 and 4.

Process.

3)588

4)196(**3's**)

Ans. 49(**12's**)

Proof.—$3 \times 4 \times 49 = 588$.

Analysis.—Dividing by 3, we find that there are 196 **3's** in 588, and since 4 **3's** make 12, there are as many **12's** in 588 as there are 4's in 196, which is 49. Hence to divide by the factors of a number, divide the given dividend by one of the factors, and the quotients thus arising consecutively by the others.

8. Divide 5084 by 63, using factors.

9. Divide 13516 by 32, using factors.

10. Divide 8759 by 84, using factors.

Process.

3)8759

4)2919(**3's**) + 2

7)729(**12's**) + 3(**3's**) = 9

104(**84's**) + 1(**12**) = 12

True rem. 23

Analysis.—Dividing by 3, we find that there are 2919 **3's** in 8759, and 2 units remaining. Dividing 2919 **3's** by 4, we find that there are 729 **12's** in 2919 **3's**, and 3 **3's** = 9 units remaining. Dividing 729 **12's** by 7, we find that there are 104 **84's** and 1 **12** = 12 units remaining. The sum of the remainders 2 + 9 + 12 = 23, which is the **true remainder.**

11. Divide 10206 by 72, using factors.

12. Divide 78460 by 108, using factors.

Review Problems.

ORAL EXERCISES.

ART. 79.—1. John labored 7 weeks at $9 a week; he took in payment 5 sheep at $7 each, and the rest in cash: how much cash did he receive?

Solution.—He received the difference between the sum earned and the cost of the sheep. He earned 7 times 9 dollars, or $63. The sheep cost 7 times 5 dollars, or $35. $63 − $35 = $28, cash received.

2. If 9 yards of cloth are worth $27, how much are 8 yards worth?

3. If 12 horses cost $3,600, how much do 9 horses cost at the same rate?

4. If 24 pounds of butter cost $7.20, how much will 18 pounds cost?

5. If 8 men can do a piece of work in 32 days, how long will it take 12 men?

6. I gave 10 barrels of potatoes, worth $2.50 a barrel for 5 pairs of shoes: what was the average price of the shoes a pair?

7. If 7 yards of cloth cost $28, what is the cost of 9 yards?

8. Isaac earned $72 in 12 weeks: how much did he earn in 11 weeks?

9. A man bought 12 pounds of beef, and handing the butcher a two-dollar bill, received 20 cents change: what was the price of the beef per pound?

10. If 6 vests can be made from 12 yards of cloth, how many vests can be made from 84 yards?

11. When 9 tons of coal cost $45, how many tons can be bought for $90?

12. If eggs are worth 15 cents a dozen, and butter is 24 cents a pound, how many dozen eggs are worth 10 pounds of butter?

13. A man bought 5 coats at $8 apiece, and 4 hats at $2 apiece. He paid for them with apples at $4 a barrel: how many barrels did it take?

14. Harry bought 14 bananas at 3 cents apiece, and after giving away ½, sold the rest at 4 cents each: did he gain or lose, and how much?

15 Arthur has 12 cents and Amos 11 times as many, lacking 11 cents: how much has Amos?

16. John paid 60 cents for a slate, a book and a ruler. The slate cost twice as much as the ruler, and the book as much as the slate and ruler. How much did he pay for each?

SUGGESTION.—The slate costs twice as much as the ruler, and the book three times as much as the ruler: hence they all cost six times as much as the ruler=60 cents.

17. Three boys have 84 marbles: George has three times as many as John, and Harry has twice as many as George and John have: how many has each?

WRITTEN EXERCISES.

ART. 80.—1. My horse cost $396, which was $12 more than 12 times the cost of the harness: how much did the harness cost?

2. Mr. Chapman rode 375 miles in one week and the next week he rode 6 times as far, lacking 987 miles: how far did he ride the second week?

3. The sum of two numbers is 8,769 and the larger is 5,674: what is the less number?

4. The minuend is 10,987 and the subtrahend 9,690: what is the remainder?

5. The subtrahend is 4,070 and the remainder 7,659: what is the minuend?

6. The minuend is 5,678 and the remainder 2,111: what is the subtrahend?

7. The multiplicand is 456 and the multiplier 607: what is the product?

8. The product is 8,704 and the multiplier 32: what is the multiplicand?

9. The dividend is 87,694 and the divisor 65: what is the quotient?

10. The dividend is 6,386 and the quotient 206: what is the divisor?

11. My farm cost $9,460, which lacks $1,800 of being twice as much as the cost of the house: what was the cost of both?

12. Grace is 13 years old and her brother Asa is 15 years old. Three times the sum of their ages lacks 6 years of being the age of their great-grandfather: how old is he?

13. Mr. Harris sold 7,600 bushels of corn; he afterwards bought 3,456 bushels and then had 5,700 bushels: how many bushels had he at first?

14. The number of envelopes made in the United States in one year is about 18,000,000. What is the average number made each day, counting 308 working days in a year?

15. The pyramids of Egypt are supposed to have been built 2,170 years before the birth of Christ: how many years since they were built?

16. A farm of 525 acres produced 45,675 bushels of potatoes: what was the average yield per acre?

17. Friction matches were first used in this country in 1829; 9 years previous, illuminating gas was introduced, and 30 years previous to that it was first used in England: what is the last-named date?

18. The Bank of North America was established in 1781, which was 87 years after the founding of the Bank of England. The first bank in Europe was established in Venice 523 years before the Bank of England: how many years from the establishment of the first bank in Europe to the death of General Grant in 1885?

19. The estimated number of Christians in the world is: Roman Catholics, 201,000,000; Protestants, 106,-000,000; Eastern Churches, 81,000,000. The number of Buddhists is 340,000,000; Mohammedans, 201,000,000; Brahmans, 175,000,000; followers of Confucius, 80,000,-

000; followers of Shinto, 14,000,000; Hebrews, 7,000,000. How many less Christians are there in the world than believers in other faiths?

20. Illustrate addition and subtraction by an original problem.

21. Illustrate subtraction and multiplication by an original problem.

22. Illustrate multiplication and division by an original problem.

REVIEW QUESTIONS.

What is addition? What is meant by the sum or amount? What is the sign of addition called? What does it mean? Illustrate.

How is the sign of equality read? Illustrate its use. Illustrate the use of the dollar mark. What places are occupied by cents? What are the principles of addition? Give the rule for addition. What is the proof?

What is subtraction? What is the minuend? The subtrahend? What is the difference? How is the sign of subtraction read? Illustrate its use. What are the principles of subtraction? Give the rule for subtraction. What is the proof?

What is multiplication? What is the multiplicand? The multiplier? The product? What are the factors of a number? What are the factors of the product? How is the sign of multiplication read? Illustrate its use.

Explain the use of the parenthesis. What are the principles of multiplication? Give the rule for multiplication. What is the proof?

What is division? What is the dividend? The divisor? The quotient? The remainder? How is the sign of division read? Illustrate its use. Of what is division the reverse? Give the principles of division. What is short division? Give the rule for short division. What is the proof?

What is long division? Give the rule for long division. How do you divide when the divisor has ciphers on the right? How do you divide by using factors? How is the true remainder found?

Properties of Numbers.

DEFINITIONS.

ART. 81.—An **Integer** is a whole number; as 1, 5, 6, 11.

Integers are either *Even* or *Odd*.

ART. 82.—An **Even Number** is a number that is exactly divisible by 2; as 4, 6, 8, 10, 14.

ART. 83. An **Odd Number** is a number that is not exactly divisible by 2; as 3, 5, 7, 9, 11.

Numbers are either *Composite* or *Prime.*

ART. 84. A **Composite Number** is one that is produced by multiplying together two or more numbers each greater than 1; as 9, 10, 15, 16, 24.

ART. 85. A **Prime Number** is one that cannot be produced by multiplying together two numbers each greater than 1; as 7, 11, 13, 19, 29.

Numbers are prime to each other when no number greater than 1 exactly divides them; as 5, 7; 4 and 9.

ART. 86. A **Prime Factor** is a factor that is a prime number; thus, 2, 3, and 7 are the prime factors of 42.

ART. 87. A **Common Divisor** of two or more numbers is any number greater than 1 that exactly divides each of them; thus, 4 is a common divisor of 12, 20, 24; and 5 is a common divisor of 15, 25, 30, 35.

ART. 88. The **Greatest Common Divisor** of two or more numbers is the greatest exact divisor of each of them; thus, 9 is the greatest common divisor of 18, 27, and 36; and 15 is the greatest common divisor of 30, 45, 60, 90.

ART. 89.—A Multiple of a number is any number of times that number; thus, 6, 9 and 12 are multiples of 3. A **Common Multiple** of two or more given numbers is any number exactly divisible by each of them; thus, 24 is a common multiple of 4, 6, 8 and 12.

ART. 90.—The **Least Common Multiple** of two or more numbers is the least number that is exactly divisible by each of them; thus, 30 is the least common multiple of 2, 3, 5, 10 and 15.

Factors.

ORAL EXERCISES.

ART. 91.—1. What are the factors of 9?

Solution.—The factors of 9 are 3 and 3, because 3 times 3 is 9.

What are the factors:

2. Of 15?	**6.** Of 34?	**10.** Of 48?
3. Of 16?	**7.** Of 38?	**11.** Of 60?
4. Of 25?	**8.** Of 40?	**12.** Of 75?
5. Of 28?	**9.** Of 44?	**13.** Of 100?

ART. 92.—Principles. 1. *Every prime factor of a number is an exact divisor of that number.*

2. *Every composite number equals the product of all its prime factors.*

14. What are the prime factors of 16?

Solution.—The prime factors of 16 are 2, 2, 2 and 2, because they are the only prime numbers whose product is 16.

What are the prime factors:

15. Of 10?	**20.** Of 24?	**25.** Of 35?
16. Of 12?	**21.** Of 27?	**26.** Of 39?
17. Of 15?	**22.** Of 30?	**27.** Of 44?
18. Of 20?	**23.** Of 32?	**28.** Of 49?
19. Of 21?	**24.** Of 33?	**29.** Of 57?

30. Of what number are 3, 2 and 3 the prime factors? 2, 3, 2 and 3? 3, 5 and 2? 3, 2 and 7?

WRITTEN EXERCISES.

ART. 93.—1. What are the prime factors of 480?

Process.

```
2)480
2)240
2)120
 2)60
 2)30
 3)15
    5
```

Analysis.—Dividing 480 by the prime number 2 gives 240 for a quotient, dividing again by 2, gives 120; dividing again by 2, gives 60; dividing again by 2, gives 30; dividing this by the prime number 2, gives 15; and dividing this by the prime number 3, gives the prime number 5 as the quotient. Since we have divided only by prime numbers, and since our last quotient is a prime number, it is evident that the several divisors and the last quotient must form the prime factors of 480.

Therefore, 2, 2, 2, 2, 2, 3 and 5 are the prime factors of 480.

ART. 94.—Rule for finding the Prime Factors of a Number. *Divide the number by the least prime number that will divide it; divide the quotient thus obtained by the least prime number that will divide it, and continue the process until the quotient is a prime number; the several divisors and the last quotient are the prime factors required.*

What are the prime factors:

2. Of 21?	**9.** Of 88?	**16.** Of 395?
3. Of 24?	**10.** Of 81?	**17.** Of 456?
4. Of 36?	**11.** Of 96?	**18.** Of 490?
5. Of 42?	**12.** Of 144?	**19.** Of 384?
6. Of 48?	**13.** Of 275?	**20.** Of 472?
7. Of 75?	**14.** Of 348?	**21.** Of 510?
8. Of 84?	**15.** Of 382?	**22.** Of 564?

23. Of 1284?	**26.** Of 31254?	**29.** Of 20482?
24. Of 3216?	**27.** Of 48762?	**30.** Of 39516?
25. Of 4124?	**28.** Of 51946?	**31.** Of 42856?

32. Name three numbers and find their prime factors.

DIVISORS.

ORAL EXERCISES.

ART. 95.—1. What is the common divisor of 9 and 12?

Solution.—Three is the common divisor of 9 and 12, because it is exactly contained in each of them.

2. What are the common divisors of 12 and 16? 20 and 25?

3. What are the common divisors of 18 and 21? 24 and 28?

ART. 96.—Principles. 1. *The product of all the prime factors common to two or more numbers is their greatest common divisor.*

2. *The greatest common divisor of two numbers is a divisor of their sum.*

3. *The greatest common divisor of two numbers is a divisor of their difference.*

WRITTEN EXERCISES.

ART. 97.—1. Find the greatest common divisor of 84 and 90.

FIRST METHOD.

Process.

$84 = 2 \times 2 \times 3 \times 7$
$90 = 2 \times 3 \times 3 \times 5$

Analysis. — By inspecting the prime factors of which 90 and 84 are composed, we observe that 2 and 3 are the only prime factors common to both numbers. Therefore, 2×3 or 6 must be the greatest common divisor of 84 and 90.

ART 98.—Rule for finding the Greatest Common Divisor. *Resolve each number into its prime factors; the product of the prime factors common to all the numbers is the greatest common divisor.*

SECOND METHOD.

2. Find the greatest common divisor of 72 and 96.

Process.

```
72)96(1
   72
   24)72(3
      72
```

Analysis.—The greatest common divisor of two or more numbers cannot be greater than the least number. 72 is not an exact divisor of 96, since there is a remainder of 24.

The difference (Prin. 3) contains the greatest common divisor. 24 is an exact divisor of 72, and is therefore the greatest common divisor of 72 and 96.

ART. 99. Rule for finding the Greatest Common Divisor. *Divide the greater number by the less, and, if there be a remainder, divide the divisor by it, and so continue to divide the last divisor by the last remainder until nothing remains. The last divisor is the greatest common divisor. If there are more than two numbers, find the greatest common divisor of two of them; then of this divisor, and a third number, and so on. The last divisor is the greatest common divisor required.*

Find by the first method the greatest common divisor of:

3. 20 and 66.

4. 42 and 82.

5. 36 and 78.

6. 120 and 132.

7. 125 and 175.

8. 64 and 114.

9. 70 and 180.

10. 150 and 275.

11. 126 and 264.

12. 640 and 960.

13. Find by the second method the greatest common divisor of 360, 484 and 684.

14. Find by the second method the greatest common divisor of 280, 640 and 1728.

15. Find by the second method the greatest common divisor of 542, 1084 and 1626.

16. I have three rooms, the first 15 feet wide, the second 24 feet wide, and the third 36 feet wide: what is the width of the widest carpet that will fit either room?

17. Mr. Emmons wishes to divide his three farms of 324 acres, 486 acres, and 4,293 acres respectively, into tracts of equal size: what is the greatest number of acres each tract may contain?

18. Mr. Jones shipped by rail in equal car-loads four lots of wood, the first containing 84, the second 96, the third 108, and the fourth 120 cords: how many cords in each car-load, the loads being the largest possible?

19. Illustrate by an original problem each method of finding the greatest common divisor.

Multiples.

ORAL EXERCISES.

ART. 100.—1. Of what numbers is 8 a multiple? 12? 15? 20? 24? 25? 32? 36? 48?

2. Name three multiples of 4; of 5; of 8; of 9; of 12; of 14; of 11; of 13; of 15.

3. Name a common multiple of 3 and 4; of 4 and 5; of 6 and 7; of 5 and 8.

4. Name a common multiple of 2, 3 and 4; of 3, 5 and 6; of 5, 6 and 8; of 2, 5 and 10.

5. Name the least common multiple of 2, 3 and 5; of 3, 4 and 6; of 3, 6 and 8.

6. Name the least common multiple of 2, 4 and 6; of 3, 5 and 6; of 1, 3 and 7.

7. Name the least common multiple of 3, 4 and 5; of 2, 5 and 6; of 4, 5 and 8.

8. Name the least common multiple of 5, 6 and 8; of 3, 7 and 8; of 5, 6 and 7.

WRITTEN EXERCISES.

ART. 101.—1. Find the least common multiple of 14, 24 and 36.

Process.

$$\begin{array}{r|rrr} 2 & 14, & 24, & 36 \\ \hline 2 & 7, & 12, & 18 \\ \hline 3 & 7, & 6, & 9 \\ \hline & 7, & 2, & 3 \end{array}$$

$2 \times 2 \times 3 \times 7 \times 2 \times 3 = 504$

Analysis.—Since 2 is a prime factor of each of the numbers, it is also a factor of their least common multiple. After dividing, there remain as the other factors of the numbers 7, 12 and 18. 2 is a prime factor of 12 and 18, and is therefore a prime factor of their least common multiple. Dividing by 2, there remain 7, 6 and 9. 3 is a prime factor of 6 and 9, and hence is another factor of their least common multiple. Dividing by 3, there remain 7, 2 and 3, which are prime to each other. Hence, the product of the factors 2, 2, 3, 7, 2 and 3, is the least common multiple required. **Proof.**—$504 \div 14 = 36$; $504 \div 24 = 21$; $504 \div 36 = 14$.

ART. 102.—Rule for finding the Least Common Multiple.—*Write the numbers in a horizontal line. Divide the given numbers by any prime number that is an exact divisor of two or more of them, and write the quotients and undivided numbers in a line beneath.*

Continue the division until the quotients and undivided numbers are prime to each other. The product of the divisors and the numbers in the last horizontal line is the least common multiple required.

NOTE.—When one of the numbers is an exact divisor of one of the others, it may be disregarded in finding the least common multiple.

Find the least common multiple of:

2. 26, 32 and 36.

3. 42, 48 and 54.

4. 120, 180 and 240.

5. 236, 284 and 320.

6. 482, 520 and 564.

7. 548, 624 and 680.

8. 148, 164, 248.

9. 308, 416 and 456.

10. 432, 528 and 564.

11. 18, 24, 48 and 56.

12. 32, 48, 60 and 72.

13. 28, 72, 84 and 96.

14. What is the least debt that can be paid with either 3 cent pieces, 5 cent pieces, 10 cent pieces, 25 cent pieces, or 50 cent pieces?

15. Find the least sum with which I can buy sheep at $5, cows at $36, or horses at $120 each.

16. What is the least number of bushels of potatoes that can be divided into heaps of 12, 18 or 20 bushels?

17. What is the shortest piece of rope that can be cut exactly into pieces either 15, 20 or 25 feet long?

18. What is the least number of acres in a farm that can be exactly divided into lots of 12, 16, 18 or 20 acres each?

19. What is the least number of gallons that can exactly be measured by either of four casks holding 12, 15, 25 and 45 gallons respectively?

20. What is the least number of oranges that a teacher can exactly divide among either of four classes of children containing 16, 18, 20 and 24 respectively?

21. What is the least sum of money that will exactly pay either for cows at $48, oxen at $64, or horses at $120 each respectively?

22. Five men start from the same point to walk around an island: the first can make the circuit in 3, the second in 4, the third in 6, the fourth in 8, and the fifth in 10 hours: in what time will they meet at the starting point?

23. Illustrate by an original problem the method of finding the least common multiple.

Cancellation.

ART. 103. **1.** How often is 2 times 4 contained in 4 times 4? 4 times 5 in 8 times 5? 4 times any number in 12 times that number?

2. How often is 3 times 7 contained in 9 times 7? 5 times 6 in 15 times 6? 5 times any number in 20 times that number?

3. How often is 7 times 8 contained in 14 times 7? 8 times 10 in 24 times 10? 8 times any number in 40 times that number?

4. What is the quotient of $(10 \times 12) \div (5 \times 12)$? Of $(36 \times 12) \div (12 \times 12)$?

5. What is the quotient of $(27 \times 30) \div (9 \times 30)$? Of $(63 \times 35) \div (9 \times 35)$?

6. What is the quotient of $(144 \times 120) \div (12 \times 120)$? Of $(160 \times 77) \div (40 \times 77)$?

7. What is the quotient of $(200 \times 129) \div (20 \times 129)$? Of $(360 \times 137) \div (36 \times 137)$?

In finding the quotient, what numbers may be omitted from the dividend and divisor?

Why does such omission not affect the value of the quotient? Art. 72, prin. 4.

ART. 104. **Cancellation** is the process of shortening arithmetical operations by striking out equal factors from the dividend and divisor.

ART. 105. — **Principle.** *Dividing both dividend and divisor by the same number does not affect the quotient.*

WRITTEN EXERCISES.

ART. 106.—1. Divide $4 \times 5 \times 12$ by $2 \times 3 \times 3$.

Process.

$$\frac{\overset{2}{\cancel{4}} \times 5 \times \overset{2}{\cancel{12}}}{\cancel{2} \times 3 \times \cancel{6}} = \frac{20}{3} = 6\frac{2}{3}$$

Analysis. — The divisor is written under the dividend with a line between.

Since the factor 2 in the divisor is a factor of 4 in the dividend, we strike it from both, leaving 2 in the dividend. Since the factor 6 in the divisor is a factor of 12 in the dividend, we strike it from both, leaving 2 in the dividend. The product of the remaining factors in the dividend is 20, and the divisor is 3. Hence, the quotient is $\frac{20}{3} = 6\frac{2}{3}$

ART. 107. Rule for Cancellation.—*Cancel from the dividend and the divisor all common factors, and divide the product of the remaining factors in the dividend by the product of the remaining factors in the divisor*

NOTE.—Should all the factors in the dividend and divisor be canceled, the quotient is 1 If the 1 appears in the dividend it must be retained; if in the divisor, it may be disregarded.

What is the quotient of:

2. $\overline{4 \times 8 \times 12} \div \overline{3 \times 4 \times 6}$?

3. $\overline{6 \times 5 \times 10} \div \overline{4 \times 3 \times 6}$?

4. $\overline{7 \times 8 \times 12} \div \overline{14 \times 2 \times 4}$?

5. $\overline{6 \times 5 \times 10} \div \overline{3 \times 2 \times 5}$?

6. $\overline{8 \times 9 \times 12} \div \overline{4 \times 6 \times 8}$?

7. $\overline{5 \times 6 \times 9} \div \overline{10 \times 2 \times 3}$?

8. $\overline{11 \times 7 \times 12} \div \overline{22 \times 14 \times 2}$?

9. $\overline{13 \times 8 \times 10} \div \overline{39 \times 2 \times 5}$?

10. $\overline{15 \times 9 \times 14} \div \overline{3 \times 6 \times 21}$?

11. How many dozen eggs worth 24 cents a dozen,

can be bought with 18 pounds of sugar worth 8 cents a pound?

12. I exchanged 24 yards of ribbon worth 12 cents a yard for apples worth 72 cents a bushel: how many bushels were obtained?

13. A farmer gave 16 bushels of potatoes worth 60 cents a bushel for rice at 24 cents a pound: how many pounds did he obtain?

14. A man paid for 12 tons of coal worth $5 a ton, with wood at $4 a cord: how many cords were required?

15. How many chickens at $.60 each, must be given for 4 bushels of apples worth $.90 a bushel?

16. A farmer exchanged 20 cows worth $35 each for 5 horses: what was the value of each horse?

17. A farmer bought three pieces of calico each containing 50 yards at 8 cents a yard and paid for it with apples at $4 a barrel: how many barrels were required?

18. A farmer's daughters put up seven tubs of butter, each containing 32 pounds at 28 cents a pound, and exchanged it for 4 patterns of silk of 16 yards each: how much was the silk a yard?

19. How many tons of hay worth $16 a ton will pay for 30 cords of wood worth $4 a cord?

20. From 5 pieces of cloth, each containing 25 yards, worth $4 a yard, how many suits can be made worth $20 a suit?

21. How many bushels of corn, worth 40 cents a bushel, must be exchanged for 80 pounds of butter worth 32 cents a pound?

22. A grocer sold 18 boxes of soap, each containing 64 pounds at 10 cents a pound, and received in payment 80 barrels of potatoes, each containing 3 bushels: what were the potatoes worth a bushel?

6

23. How many boxes of tea, each containing 28 pounds at 75 cents a pound, must be given for 25 firkins of butter of 56 pounds each, at 30 cents a pound?

24. How many bales of cloth, each containing 45 pieces, and each piece containing 32 yards, worth $2 a yard, must be given for 120 horses, worth $120 each?

25. How many days' work at $1.50 a day will pay for 20 visits of a physician, at $.75 a visit?

26. A grocer sold 6 boxes of soap, each containing 45 pounds at 8 cents a pound, and received in payment 15 barrels of apples, each containing 3 bushels: what were the apples worth a bushel?

27. How many bushels of oats at $.48 a bushel must be given in exchange for 336 yards of cotton cloth at 11 cents a yard?

28. A miller bought 8 loads of wheat, each containing 30 bags of 3 bushels each, worth $1.25 a bushel, and paid for it with flour worth $9 a barrel: how much flour was required?

29. A grocer exchanged sugar worth 6 cents a pound for 12 bushels of potatoes, at $.60 a bushel: how many pounds were required?

30. A bought 5 pieces of muslin, each containing 40 yards at 12 cents a yard, and paid for it with wheat at $2 per bushel: how many bushels were required?

31. A farmer sold 20 barrels of apples, each containing 3 bushels, at $.80 a bushel: how many pieces of calico, each containing 40 yards, at 10 cents a yard, were required to pay for the apples?

32. A grocer gave 10 boxes of sugar, each containing 120 pounds, for 6 loads of apples, each load containing 15 bags, and each bag 2 bushels, at 40 cents a bushel: what was the sugar worth a pound?

33. Illustrate cancellation by an original problem.

Common Fractions.

ART. 108. James being presented with a large apple, wishes to share it equally with a playmate. If he cuts the apple into two equal parts, what is each part called?

Now, suppose each boy divides his portion into two equal parts, how many are there of such parts? What is the name of each part? Of two parts? Of three? Of four?

If a pear is sliced into three equal pieces, what is each one called? Suppose that each piece is cut into two equal parts, how many are there? What is each called?

How many halves are there in a melon? In a banana? In a dollar? In a foot? In a mile? In anything? How many thirds? How many fifths? How many fourths? How many sevenths? How many tenths? How many fifteenths? How many twentieths?

You will observe that in writing the fraction one half, I place the figure 1 above a short, horizontal line, and the figure 2 below it. The 2 below the line shows into how many parts the apple, banana, dollar, or whatever we are talking about, is divided. It therefore gives the name or *denomination* to the fraction, and is called the *denominator*; as $\frac{1}{2}$.

The figure 1 above the line shows the number of the parts that we have taken, and is therefore called the *numerator*; as $\frac{1}{2}$.

ART. 109. In the following fractions, name the numerator and the denominator, and tell into how many parts the unit, or object we are considering, is divided, and how many of those parts are meant:

$\frac{5}{8}$, $\frac{4}{9}$, $\frac{7}{4}$, $\frac{5}{6}$, $\frac{6}{8}$, $\frac{6}{7}$, $\frac{8}{9}$, $\frac{7}{10}$,

$\frac{11}{12}$, $\frac{13}{15}$, $\frac{12}{13}$, $\frac{15}{17}$, $\frac{9}{11}$, $\frac{16}{17}$, $\frac{18}{20}$, $\frac{17}{18}$, $\frac{20}{21}$.

Write the following fractions:

1. Nine seventeenths; eleven nineteenths; three thirty-firsts.

2. Thirty eighty-fourths; forty-three seventy-ninths.

3. Seventy-two one-hundredths; eighty-eight one hundred forty-fifths.

4. One hundred thirty-three two hundred twenty fifths.

5. One hundred ninety-five two hundred thirty thirds.

6. Two hundred eighteen three hundred twenty-fourths.

7. Three hundred forty eight four hundredths.

8. Five hundred twenty-two five hundred eighty-sixths.

9. Six hundred eleven seven hundred twenty-ninths.

10. Eight hundred thirteen nine hundred twentieths.

ART. 110.—Mrs. Jones has 3 boys, to each of whom she gives $\frac{1}{6}$ of a melon. How many sixths does she give to all? How many times $\frac{1}{6}$ of a melon in $\frac{3}{6}$ of a melon?

If I write $\frac{1}{6}$ on the board, and wish to multiply it by 3, how can it be done? Then, if I multiply the numerator of a fraction, what effect has it on the value of the fraction? Thus: $\frac{1}{6} \times 3 = \frac{3}{6} = \frac{1}{2}$, $\frac{1}{6} \div 3 = \frac{1}{2}$.

How many sixths are there in anything? Three sixths is what part of anything?

We have seen that if the numerator of $\frac{1}{6}$ is multiplied by 3, the result is $\frac{3}{6}$ or $\frac{1}{2}$. Now, instead of multiplying the numerator of the fraction $\frac{1}{6}$ by 3, let us divide the denominator by 3. The result is $\frac{1}{2}$, which you perceive is the same as we obtained by multiplying the numerator by 3. We conclude, therefore, that *if the nu-*

merator of a fraction is multiplied by a number, the fraction itself is multiplied by that number; and also, if the denominator of a fraction is divided by a number, the fraction itself is multiplied by that number.

How many sixths of a melon did Mrs. Jones give to her 3 boys? Since she divided the $\frac{3}{6}$ of a melon among the 3 boys, how much did each boy receive? You observe, therefore, *that when the numerator of a fraction is divided by a number, the fraction itself is divided by that number.* Thus: $\frac{3}{6} \div 3 = \frac{1}{6}$; $\frac{3}{6} \div 3 = \frac{3}{18}$.

Suppose that instead of dividing the numerator, as we have just done, we multiply the denominator of the fraction $\frac{3}{6}$ by 3. What is the result? How many eighteenths are in anything? How many eighteenths in $\frac{1}{6}$ of anything? You perceive that $\frac{1}{6}$ and $\frac{3}{18}$ have the same value, and we conclude, therefore, that *if the denominator of a fraction be multiplied by any number, the fraction itself is divided by that number.*

If I multiply the numerator and denominator of $\frac{1}{6}$ by 3, what is the result? But we have just learned that $\frac{1}{6}$ and $\frac{3}{18}$ have the same value. We conclude, therefore, that *it does not change the value of a fraction to multiply the numerator and denominator by the same number.* Thus: $\frac{1 \times 3}{6 \times 3} = \frac{3}{18}$; $\frac{3 \div 3}{18 \div 3} = \frac{1}{6}$.

We will now divide the numerator and denominator of $\frac{3}{18}$ by 3. What is the result? You will notice that the fractions $\frac{3}{18}$ and $\frac{1}{6}$ have the same value. We conclude, therefore, that *it does not change the value of a fraction to divide its numerator and denominator by the same number.*

ANALYSIS OF FRACTIONS.

ART. 111.—All fractions, as well as all numbers, are derived from the unit 1: hence, all numbers, whether integral or fractional, are referred in analysis to the number 1.

ART. 112.—The unit 1 is the measure of all numbers, hence, 1 is the unit of a fraction; one of the equal parts into which the unit is divided is a fractional unit; and more than one of the equal parts is a collection of fractional units.

ART. 113.—To analyze a fraction is to name the unit of the fraction; the fractional unit; the number of fractional units taken; the factors of the fraction; and the value of the fraction.

Thus, in analyzing the fraction $\frac{3}{4}$ we say: The unit of the fraction is 1; the fractional unit, $\frac{1}{4}$; the number of fractional units taken, 3; the factors of the fraction, $\frac{1}{4}$ and 3; the value of the fraction, $\frac{3}{4}$.

In analyzing the fraction $\frac{4}{5}$: The unit of the fraction is 1; the fractional unit, $\frac{1}{5}$; the number of fractional units taken, 4; the factors of the fraction, $\frac{1}{5}$ and 4; the value of the fraction, $\frac{4}{5}$.

Analysis of the fraction $\frac{5}{7}$: The unit of the fraction is 1; fractional unit, $\frac{1}{7}$; number of fractional units, 5; factors of the fraction, $\frac{1}{7}$ and 5; value of the fraction, $\frac{5}{7}$.

ART. 114.—Analyze, according to the models given, the following fractions:

1. $\frac{5}{8}$, $\frac{4}{9}$, $\frac{7}{12}$.
2. $\frac{3}{13}$, $\frac{7}{18}$, $\frac{16}{21}$.
3. $\frac{21}{28}$, $\frac{29}{48}$, $\frac{37}{64}$.
4. $\frac{63}{41}$, $\frac{63}{81}$, $\frac{71}{84}$.
5. $\frac{28}{121}$, $\frac{39}{111}$, $\frac{73}{146}$.
6. $\frac{39}{94}$, $\frac{83}{101}$, $\frac{94}{124}$.
7. $\frac{99}{132}$, $\frac{105}{213}$, $\frac{121}{324}$.
8. $\frac{142}{398}$, $\frac{513}{685}$, $\frac{1934}{3242}$.

PRINCIPLES AND DEFINITIONS.

ART. 115.—*Multiplying the numerator of a fraction by any number multiplies its value by that number*, because it increases the number of fractional units. Thus: $\frac{1 \times 3}{2} = \frac{3}{2}$; $\frac{2 \times 4}{3} = \frac{8}{3}$; $\frac{3 \times 5}{4} = \frac{15}{4}$.

ART. 116.—*Dividing the denominator of a fraction by any number multiplies its value by that number*, because it increases the size of the fractional unit. Thus, $\frac{3}{4} \div 2 = \frac{3}{2}$; $\frac{5}{6} \div 2 = \frac{5}{3}$; $\frac{5}{8} \div 4 = \frac{5}{2}$.

ART. 117.—*Multiplying the denominator of a fraction by any number divides the value of the fraction by that number*, because it decreases the size of the fractional unit. Thus, $\frac{2}{3} \times 3 = \frac{2}{9}$; $\frac{5}{7} \times 2 = \frac{5}{14}$; $\frac{7}{8} \times 3 = \frac{7}{24}$.

ART. 118.—*Dividing the numerator of a fraction by any number divides the value of the fraction by that number*, because it decreases the number of fractional units. Thus, $\frac{2}{3} \div 2 = \frac{1}{3}$; $\frac{3}{4} \div 3 = \frac{1}{4}$; $\frac{5}{6} \div 5 = \frac{1}{6}$.

ART. 119. *Multiplying both numerator and denominator of a fraction by the same number does not change its value*, because the increase in the number of fractional units is equaled by the decrease in their size. Thus, $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$; $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$; $\frac{5}{6} \times \frac{5}{5} = \frac{25}{30}$.

ART. 120.—*Dividing both numerator and denominator of a fraction by the same number does not change its value*, because the decrease in the number of fractional units is equaled by the increase in their size. Thus, $\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$; $\frac{3}{6} \div \frac{3}{3} = \frac{1}{2}$; $\frac{25}{30} \div \frac{5}{5} = \frac{5}{6}$.

ART. 121. A **Fraction** is one or more of the equal parts of a unit. It is an expression of division.

ART. 122.—A fraction is expressed by two numbers, written one above a short horizontal line, and the other below it. It is a quotient.

ART. 123.—The **Denominator** of a fraction shows into how many equal parts the unit is divided, and is written below the line. It is the divisor.

ART. 124.—The **Numerator** of a fraction shows how

many of the equal parts of the unit or number are taken, and is written above the line. It is the dividend.

ART. 125.—The **Terms of a Fraction** are the numerator and denominator.

ART. 126.—A **Proper Fraction** is one whose numerator is less than its denominator; as, $\frac{3}{5}$, $\frac{6}{7}$, $\frac{18}{48}$.

ART. 127.—An **Improper Fraction** is one whose numerator is equal to or greater than its denominator, as, $\frac{8}{8}$, $\frac{17}{12}$, $\frac{19}{5}$.

ART. 128.—A **Simple Fraction** is one having a single integral numerator and denominator; as, $\frac{4}{5}$, $\frac{21}{30}$, $\frac{38}{47}$.

ART. 129. A **Complex Fraction** is one whose numerator, or denominator, or both, are fractional; as, $\frac{\frac{2}{3}}{7}$, $\frac{7}{\frac{4}{5}}$, $\frac{\frac{4}{7}}{\frac{3}{8}}$, $\frac{2\frac{1}{4} + 3\frac{1}{2}}{6 \div \frac{2}{3}}$.

ART. 130. A **Mixed Number** is a whole number and fraction united; as, $3\frac{1}{4}$, $5\frac{2}{3}$, $19\frac{5}{8}$.

ART. 131. The **Reciprocal of a Number** is 1 divided by that number. Thus, the reciprocal of 5 is $1 \div 5$ or $\frac{1}{5}$; of 11, it is $1 \div 11$ or $\frac{1}{11}$. The reciprocal of *a fraction* is the fraction inverted. Thus, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$; of $\frac{4}{3}$ is $\frac{3}{4}$.

ART. 132. To change Fractions to their Lowest Terms.

ART. 133.—A fraction is in its lowest terms or simplest form when no number greater than 1 will exactly divide its terms, as $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{12}$.

ORAL EXERCISES.

1. Change $\frac{9}{15}$ to its lowest terms.

Solution.—It is readily seen that 3 exactly divides both terms of the fraction. Dividing the terms by 3 we obtain $\frac{3}{5}$. We cannot divide the terms of $\frac{3}{5}$ by any number greater than 1; hence, $\frac{9}{15}$ changed to its lowest terms is $\frac{3}{5}$. Thus, $\frac{9}{15} \div \frac{3}{3} = \frac{3}{5}$.

Change to their lowest terms:

2. $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{8}$, $\frac{8}{14}$, $\frac{9}{18}$, $\frac{10}{12}$, $\frac{12}{16}$.

3. $\frac{7}{21}$, $\frac{13}{39}$, $\frac{14}{21}$, $\frac{15}{35}$, $\frac{16}{48}$, $\frac{17}{34}$, $\frac{19}{57}$.

4. $\frac{8}{15}$, $\frac{8}{16}$, $\frac{9}{21}$, $\frac{18}{21}$, $\frac{18}{24}$, $\frac{20}{25}$, $\frac{23}{46}$.

5. $\frac{15}{18}$, $\frac{18}{21}$, $\frac{12}{20}$, $\frac{18}{27}$, $\frac{40}{48}$, $\frac{30}{42}$, $\frac{49}{63}$.

ART. 134.—Rule for changing Fractions to their Lowest Terms. 1. *Find the greatest common divisor of the terms of the fraction, and* 2. *Divide both terms of the fraction by their greatest common divisor.*

WRITTEN EXERCISES.

Change:

1. $\frac{17}{51}$, $\frac{5}{25}$, $\frac{27}{36}$, $\frac{48}{56}$, to their lowest terms.

2. $\frac{50}{75}$, $\frac{24}{36}$, $\frac{35}{63}$, $\frac{28}{84}$, to their lowest terms.

3. $\frac{56}{72}$, $\frac{60}{70}$, $\frac{44}{77}$, $\frac{120}{360}$, to their lowest terms.

4. $\frac{64}{128}$, $\frac{48}{84}$, $\frac{68}{82}$, $\frac{49}{147}$, to their lowest terms.

5. $\frac{64}{140}$, $\frac{55}{165}$, $\frac{65}{260}$, $\frac{40}{70}$, to their lowest terms.

6. $\frac{72}{84}$, $\frac{84}{96}$, $\frac{96}{168}$, $\frac{128}{144}$, to their lowest terms.

7. $\frac{54}{90}$, $\frac{88}{100}$, $\frac{99}{110}$, $\frac{95}{285}$, to their lowest terms.

8. $\frac{120}{180}$, $\frac{148}{172}$, $\frac{180}{230}$, $\frac{175}{255}$, to their lowest terms.

9. $\frac{144}{1728}$, $\frac{160}{250}$, $\frac{164}{492}$, $\frac{245}{600}$, to their lowest terms.

10. $\frac{800}{1000}$, $\frac{864}{982}$, $\frac{124}{360}$, $\frac{288}{494}$, to their lowest terms.

11. $\frac{328}{1284}$, $\frac{1030}{1500}$, $\frac{1136}{1260}$, $\frac{1275}{2550}$, to their lowest terms.

ART. 135.—To change Mixed Numbers to Improper Fractions.

ORAL EXERCISES.

1. How many fourths are in $12\frac{3}{4}$?

Solution.—Since there are $\frac{4}{4}$ in 1 unit, in 12 units there are 12 times $\frac{4}{4}$ or $\frac{48}{4}$, and $\frac{3}{4}$ added make $\frac{51}{4}$. **Proof:** $51 \div 4 = 12\frac{3}{4}$.

Change the following mixed numbers to improper fractions:

2. $2\frac{1}{2}$, $4\frac{1}{7}$, $3\frac{5}{6}$, $5\frac{4}{5}$, $6\frac{1}{8}$, $8\frac{2}{7}$, $11\frac{2}{3}$.

3. $3\frac{1}{3}$, $6\frac{2}{5}$, $5\frac{3}{4}$, $4\frac{5}{6}$, $7\frac{3}{8}$, $8\frac{9}{10}$, $13\frac{3}{4}$.

4. $5\frac{7}{8}$, $5\frac{3}{8}$, $4\frac{5}{7}$, $3\frac{2}{9}$, $6\frac{3}{10}$, $7\frac{4}{5}$, $12\frac{4}{5}$.

5. $4\frac{3}{4}$, $7\frac{1}{9}$, $6\frac{5}{8}$, $5\frac{5}{6}$, $9\frac{1}{2}$, $10\frac{9}{10}$, $14\frac{5}{6}$.

ART. 136.—Rule for changing Mixed Numbers to Improper Fractions.—*Multiply the integer by the denominator of the fraction, add the numerator to the product and write the sum over the given denominator.*

WRITTEN EXERCISES.

Change the following mixed numbers to improper fractions:

1. $7\frac{4}{7}$, $8\frac{2}{9}$, $9\frac{4}{5}$, $11\frac{2}{3}$, $13\frac{3}{8}$, $12\frac{4}{5}$.

2. $14\frac{3}{8}$, $10\frac{1}{7}$, $12\frac{3}{4}$, $13\frac{1}{3}$, $14\frac{5}{6}$, $10\frac{3}{4}$.

3. $9\frac{5}{9}$, $13\frac{2}{5}$, $14\frac{3}{7}$, $18\frac{5}{6}$, $9\frac{7}{8}$, $15\frac{4}{7}$.

4. $11\frac{5}{7}$, $12\frac{1}{9}$, $15\frac{3}{5}$, $16\frac{4}{7}$, $12\frac{3}{7}$, $17\frac{5}{9}$.

ART. 137.—To change Improper Fractions to Whole or Mixed Numbers.

ORAL EXERCISES.

1. How many units in $\frac{13}{5}$?

Solution.—Since there are $\frac{5}{5}$ in 1 unit, $\frac{13}{5}$ is equal to as many units as $\frac{5}{5}$ is contained times in $\frac{13}{5}$, which is $2\frac{3}{5}$ times. **Proof:** $13 \div 5 = 2\frac{3}{5}$.

Change the following to whole or mixed numbers:

2. $\frac{11}{3}$, $\frac{13}{4}$, $\frac{14}{3}$, $\frac{18}{5}$, $\frac{17}{4}$, $\frac{19}{3}$, $\frac{21}{6}$.

3. $\frac{24}{5}$, $\frac{40}{8}$, $\frac{19}{4}$, $\frac{20}{5}$, $\frac{24}{12}$, $\frac{17}{8}$, $\frac{33}{10}$.

4. $\frac{21}{6}$, $\frac{22}{11}$, $\frac{27}{3}$, $\frac{30}{8}$, $\frac{48}{6}$, $\frac{49}{7}$, $\frac{65}{8}$.

5. $\frac{23}{8}$, $\frac{59}{6}$, $\frac{37}{6}$, $\frac{53}{8}$, $\frac{63}{7}$, $\frac{81}{7}$, $\frac{99}{9}$.

ART. 138.—Rule for changing an Improper Fraction to a Whole or Mixed Number.—*Divide the numerator of the fraction by the denominator. If there be a remainder, write the denominator under it and annex the resulting fraction in its simplest form to the quotient.*

WRITTEN EXERCISES.

Change the following to whole or mixed numbers:

1. $\frac{19}{10}$, $\frac{17}{12}$, $\frac{18}{16}$, $\frac{26}{18}$, $\frac{58}{13}$.

2. $\frac{250}{25}$, $\frac{300}{30}$, $\frac{400}{150}$, $\frac{450}{200}$, $\frac{139}{101}$.

3. $\frac{144}{12}$, $\frac{160}{110}$, $\frac{170}{85}$, $\frac{220}{100}$, $\frac{350}{125}$.

4. $\frac{200}{100}$, $\frac{225}{120}$, $\frac{656}{164}$, $\frac{330}{140}$, $\frac{410}{135}$.

5. $\frac{300}{140}$, $\frac{340}{300}$, $\frac{320}{160}$, $\frac{850}{320}$, $\frac{666}{111}$.

6. $\frac{412}{69}$, $\frac{518}{21}$, $\frac{628}{101}$, $\frac{733}{133}$, $\frac{899}{350}$.

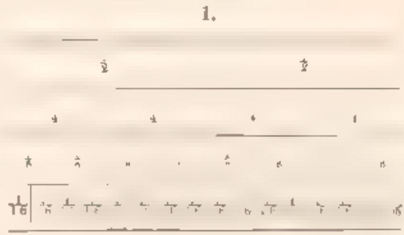
ART. 139.—To change a Fraction or an Integer to an Equivalent Fraction having a Given Denominator.

ORAL EXERCISES.

1. How many eighths are there in $\frac{1}{2}$?

Solution.—Since there are $\frac{8}{8}$ in 1, in $\frac{1}{2}$ of 1 there are $\frac{1}{2}$ of $\frac{8}{8}$, or $\frac{4}{8}$.

2. How many halves are there in 1?



3. How many fourths in 1?

4. How many eighths in 1?

5. How many sixteenths in 1?

6. How many sixteenths in one eighth? In one fourth? In one half?

7. How many eighths in $\frac{3}{4}$? In $\frac{2}{4}$? In $\frac{1}{4}$?

8. Change $\frac{1}{2}$ to fourths.

Solution.—It is readily seen that the denominator must be multiplied by 2 in order to make fourths, and since the value of a fraction is not changed by multiplying its terms by the same number, we multiply the terms of $\frac{1}{2}$ by 2; hence $\frac{1}{2}$ changed to fourths, is $\frac{2}{4}$.

9. Change $\frac{2}{3}$ to ninths; to twelfths; to fifteenths.

10. Change $\frac{1}{5}$ to twentieths; to thirtieths; to fortieths.

11. Change $\frac{3}{7}$ to twenty-firsts; to twenty-eighths.

12. Change $\frac{9}{10}$ to fortieths; to hundredths; to two hundred twentieths.

ART. 140.—Rule for changing a Fraction to an Equivalent Fraction having a Given Denominator. *Divide the given denominator by the denominator of the fraction, and multiply both its terms by the quotient thus obtained.*

NOTE.—An integer is changed to an equivalent fraction having a given denominator by multiplying it by the denominator required and writing the product over the given denominator.

WRITTEN EXERCISES.

Change:

1. $\frac{11}{14}$	to	98ths.	6. $\frac{17}{35}$	to	210ths.
2. $\frac{9}{16}$	to	96ths.	7. $\frac{35}{38}$	to	190ths.
3. $\frac{10}{11}$	to	121sts.	8. $\frac{43}{54}$	to	216ths.
4. $\frac{7}{15}$	to	105ths.	9. $\frac{61}{92}$	to	644ths.
5. $\frac{8}{13}$	to	104ths.	10. $\frac{73}{98}$	to	490ths.

ART. 141.—To change Fractions to Equivalent Fractions having a Common or a Least Common Denominator.

WRITTEN EXERCISES.

1. Change $\frac{2}{3}$ and $\frac{4}{7}$ to equivalent fractions having a common denominator.

Process.

$\frac{2}{3} \times \frac{7}{7} = \frac{14}{21}$

$\frac{4}{7} \times \frac{3}{3} = \frac{12}{21}$

Analysis.—The common denominator must be a common multiple of the denominators given. A multiple of 3 and 7 is their product, 21. Changing $\frac{2}{3}$ to 21sts by multiplying both terms by 7, we obtain the equivalent fraction $\frac{14}{21}$; changing $\frac{4}{7}$ to 21sts by multiplying both terms by 3, we obtain the equivalent fraction $\frac{12}{21}$

ART. 142.—Rule for Changing Fractions to Equivalent Fractions having a Common Denominator.—*Multiply both terms of each fraction by all the denominators except its own.*

Change to equivalent fractions having a common denominator:

2. $\frac{5}{6}$, $\frac{3}{8}$; $\frac{3}{5}$, $\frac{9}{10}$; $\frac{2}{3}$, $\frac{4}{9}$, $\frac{5}{6}$.

3. $\frac{4}{7}$, $\frac{3}{4}$, $\frac{1}{2}$; $\frac{1}{8}$, $\frac{5}{6}$, $\frac{4}{7}$, $\frac{1}{8}$.

4. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{7}$; $\frac{1}{5}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{3}$.

5. $\frac{5}{6}$, $\frac{4}{7}$, $\frac{5}{8}$; $\frac{3}{4}$, $\frac{3}{5}$, $\frac{8}{9}$, $\frac{1}{10}$.

6. $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{11}$; $\frac{4}{5}$, $\frac{6}{7}$, $\frac{1}{10}$, $\frac{3}{4}$.

ART. 143. Since it is necessary to change fractions to equivalent fractions having a common denominator in order to add or subtract them, it is convenient to have the common denominator as small as possible. Hence, we change the fractions to equivalent fractions with the *least common denominator*.

WRITTEN EXERCISES.

1. Change $\frac{3}{4}$, $\frac{3}{5}$, $\frac{5}{6}$, to equivalent fractions with the least common denominator.

Process.

$\frac{3}{4} \times \frac{15}{15} = \frac{45}{60}$

$\frac{3}{5} \times \frac{12}{12} = \frac{36}{60}$

$\frac{5}{6} \times \frac{10}{10} = \frac{50}{60}$

Analysis. The least common denominator of $\frac{3}{4}$, $\frac{3}{5}$, $\frac{5}{6}$, is the least common multiple of 4, 5 and 6, or 60.

To change each fraction to 60ths, we divide 60 by each denominator in turn, and multiply both terms of each fraction by the quotient thus obtained.

ART. 144.—Rule for finding the Least Common Denominator.—*Find the least common multiple of the denominators, divide this by each denominator in turn, and multiply both terms of each fraction by this quotient.*

Change the following to equivalent fractions having the least common denominator:

2. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$; $\frac{1}{2}$, $\frac{1}{5}$, $\frac{5}{6}$; $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{4}$.

3. $\frac{5}{6}$, $\frac{3}{8}$, $\frac{4}{5}$; $\frac{3}{4}$, $\frac{3}{5}$, $\frac{1}{7}$; $\frac{5}{6}$, $\frac{3}{7}$, $\frac{1}{14}$.

4. $\frac{1}{2}$, $\frac{3}{5}$, $\frac{3}{4}$; $\frac{5}{8}$, $\frac{2}{3}$, $\frac{1}{6}$; $\frac{4}{5}$, $\frac{9}{10}$, $\frac{1}{15}$.

5. $\frac{4}{7}$, $\frac{2}{3}$, $\frac{4}{5}$; $\frac{7}{9}$, $\frac{3}{8}$, $\frac{1}{4}$; $\frac{3}{7}$, $\frac{9}{14}$, $\frac{5}{21}$.

Addition of Fractions.

ORAL EXERCISES.

ART. 145.—1. Harry had $\frac{3}{4}$ of a dollar and his mother gave him $\frac{1}{4}$ of a dollar: how much, then, had Harry?

2. Mary paid $\frac{2}{5}$ of a dollar for a ribbon and $\frac{3}{5}$ of a dollar for a book: how much did she pay for both?

3. James gave $\frac{3}{10}$ of a dollar for a handkerchief and $\frac{5}{10}$ of a dollar for a knife: how much did he spend?

4. If a duck costs $\frac{3}{5}$ of a dollar and a chicken $\frac{2}{5}$ of a dollar, how much do both cost?

5. Kate gave $\frac{1}{4}$ of her money to her brother, $\frac{1}{4}$ to her sister, and $\frac{1}{3}$ to her cousin: how much did she give away?

ART. 146.—Principle. *Only like fractions can be added.*

6. George paid $\frac{1}{2}$ of a dollar for an arithmetic and $\frac{1}{5}$ of a dollar for a copy book: how much did he pay for both?

Solution.—He paid the sum of $\frac{1}{2}$ of a dollar and $\frac{1}{5}$ of a dollar. $\frac{1}{2} = \frac{5}{10}$ and $\frac{1}{5} = \frac{2}{10}$; $\frac{5}{10} + \frac{2}{10} = \frac{7}{10}$: he paid $\frac{7}{10}$ of a dollar.

7. One lot contains $\frac{1}{3}$ of an acre, and another $\frac{1}{4}$ of an acre: how many acres in both lots?

8. Mr. A. gave his son $\frac{3}{8}$ of a dollar and his daughter $\frac{1}{4}$ of a dollar: how much did he give to both?

9. John bought $\frac{1}{6}$ of a dozen of eggs at one store and $\frac{3}{4}$ of a dozen at another store: how many did he buy?

10. Thomas rode $\frac{2}{3}$ of a mile and walked $\frac{3}{4}$ of a mile: how far did he go?

11. Mary received $\frac{1}{4}$ of a dollar from her mother, $\frac{1}{5}$ from her father, and $\frac{1}{10}$ from her brother: how much did she receive?

12. William caught three fishes: the first weighed $\frac{1}{2}$ of a pound, the second $\frac{1}{3}$ of a pound, and the third $\frac{1}{4}$ of a pound: how much did they all weigh?

13. A merchant sold $\frac{1}{3}$ of a yard of velvet to one lady, $\frac{1}{4}$ of a yard to another, and $\frac{1}{6}$ of a yard to another: how much did he sell?

14. A slate cost $\frac{1}{5}$ of a dollar, a reader $\frac{1}{2}$ of a dollar, and a geography $\frac{3}{10}$ of a dollar: how much did all cost?

15. If I sold $\frac{1}{2}$ of an acre to Mr. A., $\frac{1}{4}$ to Mr. B., $\frac{1}{6}$ to Mr. C., and $\frac{1}{3}$ to Mr. D., how much did I sell to all?

16. John had $\frac{3}{4}$ of a dollar in one pocket, $\frac{1}{2}$ of a dollar in another pocket, and $\frac{1}{5}$ of a dollar in his hand: how much had he?

17. Mary gave $\frac{1}{4}$ of a cake to her sister, $\frac{1}{2}$ of a cake to her brother, and had $\frac{1}{4}$ of a cake left: how much had she at first?

18. A tailor cut $\frac{3}{4}$ of a yard from a piece of cloth for one customer, $\frac{2}{3}$ of a yard for another customer, and had $\frac{1}{4}$ of a yard left: how much cloth had he at first?

19. Thomas walked $\frac{2}{3}$ of a mile, Harry $\frac{3}{4}$ of a mile, and Samuel as far as both Thomas and Harry: how many miles were walked by the three?

Find the sum of:

20. $\frac{1}{2}$ and $\frac{1}{5}$; $\frac{2}{3}$ and $\frac{2}{7}$; $\frac{3}{8}$ and $\frac{3}{4}$; $\frac{3}{10}$ and $\frac{3}{4}$.

21. $\frac{1}{3}$ and $\frac{1}{5}$; $\frac{3}{4}$ and $\frac{1}{8}$; $\frac{8}{9}$ and $\frac{1}{6}$; $\frac{1}{12}$ and $\frac{5}{6}$.

22. $\frac{1}{4}$ and $\frac{1}{5}$; $\frac{3}{7}$ and $\frac{5}{6}$; $\frac{9}{10}$ and $\frac{1}{3}$; $\frac{5}{11}$ and $\frac{4}{5}$.

23. $\frac{1}{6}$ and $\frac{1}{9}$; $\frac{5}{8}$ and $\frac{1}{3}$; $\frac{4}{7}$ and $\frac{9}{11}$; $\frac{3}{13}$ and $\frac{1}{26}$.

24. $\frac{2}{3}$ and $\frac{3}{5}$; $\frac{5}{6}$ and $\frac{1}{9}$; $\frac{4}{9}$ and $\frac{1}{6}$; $\frac{1}{15}$ and $\frac{11}{30}$.

25. $\frac{1}{7}$ and $\frac{1}{6}$; $\frac{3}{4}$ and $\frac{5}{8}$; $\frac{2}{3}$ and $\frac{3}{10}$; $\frac{5}{12}$ and $\frac{3}{8}$.

WRITTEN EXERCISES.

ART. 147.—1. Add $\frac{3}{5}$, $\frac{3}{4}$, and $\frac{1}{10}$.

Process.

$\frac{3}{5} = \frac{24}{40}$ 40

$\frac{3}{4} = \frac{30}{40}$ $5 \mid 8 \times 3 = 24$

$\frac{1}{10} = \frac{4}{40}$ $4 \mid 10 \times 3 = 30$

  $10 \mid 4 \times 1 = \underline{\ \ 4}$

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$\frac{24}{40} + \frac{30}{40} + \frac{4}{40} = \frac{58}{40} = 1\frac{18}{40} = 1\frac{9}{20}$

Analysis.—Since only like fractions can be added, $\frac{3}{5}$, $\frac{3}{4}$ and $\frac{1}{10}$ must be changed to equivalent fractions having a common denominator. $\frac{3}{5} = \frac{24}{40}$, $\frac{3}{4} = \frac{30}{40}$, $\frac{1}{10} = \frac{4}{40}$, and the sum of these $= 1\frac{9}{20}$.

ART. 148. Rule for Addition of Fractions.—*Change the fractions to equivalent fractions having a common denominator, add their numerators and write their sum over the common denominator. When there are mixed numbers or integers, add the whole numbers and fractions separately, and then add their sums.*

NOTE. If the sum be an improper fraction, change it to a whole or mixed number and reduce fractional results to their lowest terms.

Find the sum of:

2. $\frac{3}{5} + \frac{4}{7} + \frac{1}{4}$.   4. $\frac{1}{5} + 2\frac{1}{2} + 4\frac{1}{4}$.

3. $\frac{3}{4} + \frac{5}{6} + \frac{11}{12}$.   5. $\frac{1}{7} + 2\frac{1}{3} + 11\frac{1}{11}$.

6. A owns three lots, the first containing $5\frac{1}{4}$ acres, the second $11\frac{1}{2}$ acres, and the third $13\frac{2}{3}$ acres: how many acres does he own?

7. A grocer has two casks of sugar, one containing $163\frac{7}{8}$ pounds, and the other $180\frac{3}{4}$ pounds: how many pounds in both?

8. If I pay $\$1,256\frac{3}{4}$ for a barn and expend $\$330\frac{2}{15}$ in repairs, what is the total cost?

9. A dealer purchased two boat-loads of coal. In the first there were $275\frac{1}{4}$ tons, and in the second $233\frac{1}{6}$ tons: how many tons did he purchase?

10. The sales made by a commercial traveler amounted to $\$54\frac{1}{4}$ in one day, $\$48\frac{9}{10}$ the second day, and $\$60\frac{4}{5}$ the third day: how much did he sell in the three days?

11. Three bins contain respectively $84\frac{7}{8}$ bushels, $88\frac{3}{4}$ bushels, and $90\frac{1}{2}$ bushels: how much in the three bins?

12. A farmer sold a horse for $\$175\frac{9}{10}$, a colt for $\$35\frac{3}{4}$, and a cow for $\$49\frac{4}{5}$: how much did he receive?

13. A man earned $\$39\frac{3}{4}$ the first month, $\$41\frac{1}{10}$ the second month, and $\$37\frac{4}{5}$ the third month: how much did he earn?

14. A grocer's sales amounted to $\$95\frac{1}{2}$ on Monday, $\$87\frac{1}{4}$ on Tuesday, $\$91\frac{4}{5}$ on Wednesday, and $\$88\frac{1}{10}$ on Thursday: what was the total amount?

15. A pedestrian walked $38\frac{1}{3}$ miles on Monday, $39\frac{3}{4}$ miles on Tuesday, $40\frac{3}{8}$ miles on Wednesday, and $42\frac{1}{2}$ miles on Thursday: how far did he walk?

16. A farmer received $\$27\frac{9}{10}$ for some hay, $\$16\frac{4}{5}$ for some oats, $\$25\frac{1}{3}$ for some wheat, and $\$15\frac{3}{4}$ for some corn: how much did he receive for all?

17. The value of a house is $\$4,250\frac{1}{2}$, of a lot $\$1,232$, of a barn $\$2,151\frac{9}{10}$, and of the other buildings $\$948\frac{1}{3}$: what is the value of all?

18. Mr. B. owns four farms, containing $111\frac{1}{8}$ acres, $119\frac{2}{3}$ acres, $205\frac{1}{5}$ acres, and $193\frac{5}{9}$ acres: how many acres does he own?

19. A clothier bought five pieces of cloth containing $21\frac{2}{3}$ yards, $33\frac{1}{2}$ yards, $37\frac{5}{6}$ yards, $39\frac{1}{4}$ yards, and $28\frac{7}{8}$ yards: how many yards did he buy?

20. A spent $\$19\frac{4}{5}$, B spent $\$11\frac{1}{2}$ more than A, and C spent $\$17\frac{1}{10}$ more than A and B: how much did they all spend?

21. Illustrate by an original problem addition of fractions.

Subtraction of Fractions.

ORAL EXERCISES.

ART. 149.—1. Albert having $\frac{4}{5}$ of a dollar spent $\frac{3}{5}$ of a dollar: how much had he left?

2. Susan having $\frac{5}{8}$ of a quart of cherries, gave away $\frac{3}{8}$ of a quart of cherries: how much had she left?

3. A man owned $\frac{11}{12}$ of an acre and gave away $\frac{7}{12}$ of an acre: how much had he left?

4. Wilson gathered $\frac{3}{4}$ of a bushel of pears and sold $\frac{1}{4}$ of a bushel: how much remained?

5. Jacob earned $\frac{9}{10}$ of a dollar and spent $\frac{3}{5}$ of a dollar: how much did he save?

ART. 150.—Principle.—*Only like fractions can be subtracted.*

6. Aaron paid $\frac{4}{5}$ of a dollar for a knife and sold it for $\frac{1}{2}$ of a dollar: what did he lose?

Solution.—He lost the difference between $\frac{4}{5}$ of a dollar and $\frac{1}{2}$ of a dollar. $\frac{4}{5} = \frac{8}{10}$ and $\frac{1}{2} = \frac{5}{10}$, $\frac{8}{10} - \frac{5}{10} = \frac{3}{10}$. He lost $\frac{3}{10}$ of a dollar.

7. If I pay $\frac{9}{10}$ of a dollar for a book and sell it for $\frac{1}{4}$ of a dollar, how much will I lose?

8. A man owning $\frac{3}{4}$ of a ship sold $\frac{1}{8}$ of it: what part of the ship did he still own?

9. From $\frac{9}{11}$ of a barrel of sugar, a grocer sold $\frac{2}{3}$ of a barrel: what part remained?

10. A farmer planted $\frac{5}{8}$ of an acre with corn and $\frac{1}{3}$ of an acre with potatoes: how much more did he plant with corn than with potatoes?

11. Thomas earns $\frac{3}{4}$ of a dollar a day and Jarvis $\frac{9}{10}$

of a dollar: how much more does Jarvis earn than Thomas?

From:

12. $\frac{2}{3}$ take $\frac{1}{4}$; $\frac{3}{4}$ take $\frac{1}{8}$; $\frac{5}{7}$ take $\frac{2}{5}$.

13. $\frac{5}{6}$ take $\frac{2}{3}$; $\frac{5}{6}$ take $\frac{1}{5}$; $\frac{7}{8}$ take $\frac{3}{5}$.

14. $\frac{4}{5}$ take $\frac{3}{4}$; $\frac{7}{10}$ take $\frac{2}{5}$; $\frac{9}{12}$ take $\frac{3}{4}$.

15. $\frac{6}{7}$ take $\frac{2}{5}$; $\frac{2}{3}$ take $\frac{1}{6}$; $\frac{19}{20}$ take $\frac{4}{5}$.

16. $\frac{10}{14}$ take $\frac{3}{7}$; $\frac{9}{14}$ take $\frac{4}{7}$; $\frac{11}{12}$ take $\frac{5}{8}$.

WRITTEN EXERCISES.

ART. 151.—1. From $\frac{5}{6}$ take $\frac{4}{5}$; from $13\frac{1}{3}$ take $11\frac{1}{2}$.

1st Process.

$\frac{25}{30} - \frac{24}{30} = \frac{1}{30}$

Analysis.—Reduce both fractions to equivalent fractions having a common denominator.

2d Process.

$13\frac{1}{3} = 13\frac{2}{6}$
$11\frac{1}{2} = 11\frac{3}{6}$
Ans. $1\frac{5}{6}$

Analysis.—Reduce the fractional parts $\frac{1}{3}$ and $\frac{1}{2}$ to equivalent fractions having a common denominator. As $\frac{3}{6}$ cannot be taken from $\frac{2}{6}$, take $1 = \frac{6}{6}$ from the integral part of the minuend and add it to $\frac{2}{6}$. Then $\frac{3}{6}$ from $\frac{8}{6}$ leaves $\frac{5}{6}$ and 11 from 12 leaves 1.

ART. 152.—Rule for Subtraction of Fractions.—1. *Change the fractions to equivalent fractions having a common denominator, subtract the numerator of the subtrahend from the numerator of the minuend, and write the difference over the common denominator.*

2. *If there are mixed numbers and the numerator of the subtrahend is greater than the numerator of the minuend, take 1 from the integral part of the minuend, reduce it to the required fractional form, add it to the fractional part of the minuend, and proceed as in 1 above.*

From:

2. $\frac{11}{12}$ take $\frac{5}{6}$.

3. $\frac{19}{21}$ take $\frac{5}{7}$.

4. $\frac{29}{30}$ take $\frac{8}{10}$.

5. $\frac{7}{8}$ take $\frac{4}{7}$.

6. $\frac{8}{9}$ take $\frac{5}{8}$.

7. $\frac{10}{11}$ take $\frac{4}{7}$.

8. $\frac{11}{12}$ take $\frac{8}{9}$.

9. $\frac{19}{20}$ take $\frac{9}{10}$.

10. $\frac{4}{5}$ take $\frac{7}{10}$.

11. $18\frac{1}{4}$ take $13\frac{1}{5}$.

12. $23\frac{1}{6}$ take $19\frac{1}{2}$.

13. $10\frac{1}{8}$ take $9\frac{1}{9}$.

14. $13\frac{5}{7}$ take $8\frac{2}{3}$.

15. $4\frac{1}{4}$ take $3\frac{1}{5}$.

16. $23\frac{5}{8}$ take $9\frac{2}{7}$.

17. $29\frac{1}{3}$ take $17\frac{4}{7}$.

18. $19\frac{5}{6}$ take $18\frac{3}{4}$.

19. $40\frac{1}{3}$ take $39\frac{4}{7}$.

20. From a bin containing $428\frac{1}{2}$ bushels of wheat, $317\frac{3}{4}$ bushels were sold: how many bushels remained?

21. A man bought a horse for $\$231\frac{4}{5}$ and sold it for $\$219\frac{3}{4}$: what was the loss?

22. A lady gave a clerk a twenty dollar bill in payment for 2 pairs of gloves at $\$1\frac{3}{4}$ a pair, some ribbon worth $\$7\frac{1}{2}$, and some lace worth $\$5\frac{2}{10}$: how much change did she receive?

23. From a piece of cloth containing $33\frac{1}{6}$ yards, a clothier cut $11\frac{1}{4}$ yards, $9\frac{1}{6}$ yards, and $12\frac{5}{8}$ yards: how much remained?

24. From $248\frac{3}{4}$ bushels, take the difference between $310\frac{3}{8}$ and $198\frac{1}{2}$ bushels.

25. A dealer owning $394\frac{3}{4}$ tons of coal, sold $94\frac{5}{6}$ tons to one customer, $113\frac{3}{5}$ tons to another, and $77\frac{3}{8}$ tons to a third: how much had he left?

26. Illustrate by an original problem subtraction of fractions.

Review Problems.

ORAL EXERCISES.

1. John having \$2, gave $\frac{1}{4}$ of a dollar for a reader and $\frac{1}{2}$ of a dollar for an arithmetic: how much had he left?

2. From a piece of cloth containing 12 yards, $3\frac{1}{2}$ yards were cut at one time, and $2\frac{3}{4}$ yards at another time: how much remained?

3. A man owning $10\frac{4}{5}$ acres, sold $3\frac{1}{4}$ to one neighbor and $2\frac{1}{3}$ acres to another neighbor: how many acres did he keep?

4. Mary's mother gave her $\frac{3}{4}$ of a pound of candy and her father gave her $\frac{4}{5}$ of a pound. She presented her sister with $\frac{1}{2}$ of a pound: how much did she retain?

5. Thomas having $\frac{9}{10}$ of a dollar, spent $\frac{1}{4}$ of a dollar for a ball and $\frac{3}{5}$ of a dollar for a bat: how much had he left?

6. Sarah bought $2\frac{3}{4}$ yards of ribbon and used $1\frac{1}{3}$ yards on one of her dolls and $1\frac{1}{4}$ yards on another doll: how much remained?

7. If William hands a clerk a five-dollar bill in payment for a hat costing $\$2\frac{1}{5}$ and a pair of gloves costing $\$2\frac{5}{10}$, how much change should he receive?

8. One fourth of the length of a pole is in the mud, $\frac{1}{6}$ in the water, and the rest in the air: what part of the pole is in the air?

9. A man owning a ship sold $\frac{1}{7}$ of it to one person and $\frac{5}{14}$ to another: what part of the ship did he still own?

10. From a cask containing $17\frac{3}{4}$ gallons of cider, $11\frac{3}{8}$ gallons were sold: how much remained?

11. A man left $\frac{1}{4}$ of his property to his daughter, $\frac{2}{3}$ to his son, and the remainder to his wife: what part of his property did his wife receive?

12. A farmer having $2\frac{1}{2}$ bushels of flax-seed, sold $1\frac{1}{8}$ bushels to one man and $1\frac{1}{4}$ bushels to another: how much did he keep?

13. Albert bought $\frac{3}{4}$ of a pound of raisins and gave $\frac{1}{4}$ of a pound to each of his two playmates: how much was left?

14. Thomas having \$3, earned $\$1\frac{4}{5}$, and then spent $\$2\frac{1}{2}$: how much had he left?

15. From a pile of coal containing 11 tons, a dealer sold $4\frac{3}{8}$ tons to one customer and $5\frac{3}{4}$ tons to another customer: how much remained?

16. From a barrel of flour weighing 196 pounds, a grocer sold $37\frac{1}{2}$ pounds to one person and $40\frac{3}{4}$ pounds to another: how much was left?

17. Mary having $\$2\frac{3}{4}$, bought a pair of gloves for $\$1\frac{1}{2}$ and some lace for $\frac{3}{4}$ of a dollar: how much had she left?

18. Richard walked $4\frac{1}{3}$ miles, Harry $5\frac{5}{6}$ miles, and Samuel $2\frac{1}{2}$ miles farther than both: how far did Samuel walk?

19. How much greater is $3\frac{1}{3} + 4\frac{3}{5}$ than $2\frac{3}{4} + 4\frac{1}{5}$?

20. What is the difference between $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ and $\frac{1}{2} + \frac{1}{4} + \frac{1}{6}$?

WRITTEN EXERCISES.

1. From the sum of $32\frac{1}{3}$ and $28\frac{3}{4}$, subtract the difference between $48\frac{1}{4}$ and $33\frac{1}{3}$.

2. From 305 bushels take the difference between $317\frac{1}{4}$ bushels and $229\frac{5}{8}$ bushels.

3. A man bought a horse for $\$175\frac{1}{2}$, a sleigh for $\$111\frac{9}{10}$, and sold them both for \$300: how much did he gain?

4. A farmer owning $248\frac{1}{2}$ acres of land, sold $109\frac{1}{3}$

acres to one man and $95\frac{3}{4}$ acres to another: how much was left?

5. From a piece of cloth containing $133\frac{1}{2}$ yards, a tailor cut $73\frac{5}{8}$ yards for one customer, and $49\frac{1}{4}$ yards for another customer: how many yards remained?

6. Maria having \$20, paid $\$2\frac{1}{4}$ for a pair of gloves, $\$11\frac{1}{5}$ for a bonnet, and $4\frac{7}{10}$ for some silk: how much had she left?

7. A has $\$11\frac{1}{4}$, B has $\$11\frac{4}{5}$, C has $\$11\frac{1}{10}$, and D has $\frac{9}{10}$ of a dollar more than all of them: how much money has D?

8. Mr. B bought $17\frac{4}{5}$ tons of coal. He burned $5\frac{1}{2}$ tons in January, $5\frac{1}{4}$ tons in February, and $5\frac{1}{5}$ tons in March: how many tons remained?

9. From a cask containing $47\frac{1}{2}$ gallons of molasses, a grocer sold to one customer $11\frac{1}{4}$ gallons, to another $10\frac{3}{8}$ gallons, and to another $8\frac{7}{8}$ gallons: how many gallons remained?

10. A father willed $\frac{1}{9}$ of his property to each of his three daughters, $\frac{1}{6}$ to each of his two sons, $\frac{1}{6}$ to his wife, and the remainder to his servant: what part of his property was received by his servant?

11. The sum of two fractions is $\frac{53}{60}$ and one of the fractions is $\frac{1}{4}$: what is the other?

12. The sum of two mixed numbers is $77\frac{7}{10}$ and one of them is $33\frac{3}{4}$: what is the other?

13. Mr. A paid $\$384\frac{4}{5}$ for a painting and $\$275\frac{1}{4}$ for another painting: for what sum must he sell the two so as to gain $\$113\frac{7}{10}$?

14. Three men joined in buying a load of coal containing $394\frac{3}{4}$ tons. A was entitled to $113\frac{1}{2}$ tons, and B to $131\frac{4}{5}$ tons: to how many tons was C entitled?

15. Illustrate by an original problem addition and subtraction of fractions.

Multiplication of Fractions.

ORAL EXERCISES.

General Note on Multiplication of Fractions.—Multiply $\frac{3}{4}$ by $\frac{5}{6}$. The multiplier is $\frac{5}{6}$ of 1, or $\frac{1}{6}$ of 5. Multiplying $\frac{3}{4}$ by the whole of 5, we get $\frac{15}{4}$. But we were not to multiply by 5, but by $\frac{1}{6}$ of 5. Having multiplied by a multiplier 6 times the true multiplier, the product is six times as large as it ought to be. Hence we divide by 6 to obtain the true product, which is done by multiplying the denominator of the multiplicand by 6. Thus: $\frac{3}{4} \times \frac{5}{6} = \frac{15}{24} = \frac{5}{8}$.

ART. 153. **1.** At $\frac{3}{8}$ of a dollar apiece, how much will 3 books cost?

Solution.—Three books will cost 3 times $\frac{3}{8}$ of a dollar, or $\frac{9}{8}$ of a dollar, equal to $1\frac{1}{8}$ dollars. *The result is obtained by multiplying the numerator of the fraction by the whole number and writing the product over the denominator.*

2. At $\frac{3}{4}$ of a dollar a yard, what is the cost of 7 yards of camel's hair cloth?

3. What is the cost of 5 turkeys at $\$\frac{9}{10}$ apiece?

4. At $6\frac{1}{2}$ cts. a yd., what is the cost of 6 yds. of calico?

5. What must I pay for 8 pounds of sugar at $6\frac{3}{4}$ cents a pound?

6. How much will 10 readers cost at $\$\frac{4}{5}$ apiece?

7. Warren walked $\frac{5}{8}$ of a mile an hour: how far did he walk in 7 hours?

8. What is the cost of 11 bushels of turnips at $\frac{3}{5}$ of a dollar a bushel?

9. What must I pay for 9 bbl. of flour at $\$8\frac{1}{2}$ a bbl.?

ART. 154.—**11.** John is 13 years old, and his sister is $\frac{3}{4}$ as old: what is her age?

Solution.—His sister's age is $\frac{3}{4}$ of 13 years, which is $\frac{39}{4}$ years, or $9\frac{3}{4}$ years. *The result is obtained by multiplying the whole number by the numerator of the fraction and writing the product over the denominator.*

The word "of" between fractions is equivalent to the sign $\times$. Such expressions are generally called *Compound Fractions.*

12. What is the cost of $\frac{3}{4}$ of a yard of silk velvet at \$5 a yard?

13. At \$3 a bushel, what is $\frac{3}{4}$ of a bushel of apples worth?

14. What is the cost of $\frac{3}{4}$ of a pound of sugar at 8 cents a pound?

15. What is $\frac{7}{8}$ of a gallon of molasses worth at $\frac{4}{5}$ of a dollar a gallon?

16. Multiply 5 by $\frac{5}{6}$; by $\frac{7}{10}$; by $\frac{2}{3}$; by $\frac{5}{9}$.

17. Multiply 6 by $\frac{1}{3}$; by $\frac{3}{8}$; by $\frac{3}{5}$; by $\frac{1}{4}$.

18. Multiply 8 by $\frac{3}{4}$; by $\frac{1}{6}$; by $\frac{1}{2}$; by $\frac{2}{3}$.

19. Multiply 7 by $\frac{4}{5}$; by $\frac{1}{6}$; by $\frac{2}{3}$; by $\frac{1}{7}$.

20. Multiply 4 by $\frac{6}{7}$; by $\frac{1}{6}$; by $\frac{3}{8}$; by $\frac{9}{10}$.

ART. 155. **21.** At $\frac{4}{5}$ of a dollar a day, how much does a boy earn in $\frac{3}{4}$ of a day?

Solution.—In $\frac{1}{4}$ of a day, he earns $\frac{1}{5}$ of a dollar. $\frac{1}{4}$ of $\frac{4}{5} = \frac{4}{20}$ and $\frac{3}{4} = \frac{12}{20}$, *equal to* $\frac{3}{5}$ *of a dollar. The result is obtained by multiplying the numerator of the multiplicand by the numerator of the multiplier and the denominator of the multiplicand by the denominator of the multiplier.* Thus, $\frac{4}{5} \times \frac{3}{4} = \frac{12}{20} = \frac{3}{5}$.

22. If James walked $\frac{2}{3}$ of a mile and his brother $\frac{3}{4}$ as far, how far did his brother walk?

23. Mary found $\frac{4}{5}$ of a dollar and gave away $\frac{3}{8}$ of it: how much did she give away?

24. What is the cost of $\frac{5}{6}$ of a yard of cloth at $\frac{9}{10}$ of a dollar a yard?

25. Henry owns $\frac{2}{3}$ of a sail boat, and he sold $\frac{1}{8}$ of his share to his cousin: how much did his cousin buy?

26. What is $\frac{1}{4}$ of $\frac{4}{5}$ of a ton of coal worth at $5 a ton?

27. Multiply $\frac{5}{8}$ by $\frac{7}{11}$; $\frac{4}{9}$ by $\frac{9}{10}$; $\frac{5}{11}$ by $\frac{5}{6}$; 8 by $\frac{3}{4}$.

28. Multiply $\frac{9}{13}$ by $\frac{1}{2}$; $\frac{3}{10}$ by $\frac{4}{11}$; $\frac{8}{9}$ by $\frac{5}{6}$; 7 by $4\frac{1}{4}$.

TO TEACHERS.—It is clear from the foregoing solutions that the usual "three cases" in multiplication of fractions, which from time immemorial have confused the beginner, should be treated as one case. This can be done by writing 1 as the denominator, when the multiplicand or the multiplier is an integer.

ART. 156.—Rule for Multiplication of Fractions.—*Change mixed numbers and integers to improper fractions, and then multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

WRITTEN EXERCISES.

NOTE.—In the following problems the pupils must use their ingenuity to discover the shortest methods of solution. Cancellation should be employed wherever possible. It is often, however, convenient to multiply whole and mixed numbers without reducing them to simple fractions.

1. Multiply $5\frac{3}{4}$ by 6; 135 by $16\frac{4}{11}$.

Thus: $5\frac{3}{4}$	135	**Explanation.**—Multiply the whole number by the numerator of the fraction and divide the result by the denominator. This gives the fractional part of the product, to which the integral part is added in a single operation, as indicated here.
6	$16\frac{4}{11}$	
4)18	11)540	
$4\frac{1}{2}$	$49\frac{1}{11}$	
30	810	
Ans. $34\frac{1}{2}$	135	
	$2209\frac{1}{11}$	

2. Multiply $17\frac{4}{5}$ by 8; $21\frac{4}{7}$ by 7.

3. Multiply $25\frac{3}{7}$ by 7; $18\frac{5}{8}$ by 5.

4. Multiply $89\frac{5}{8}$ by 4; $17\frac{4}{9}$ by 7.

5. Multiply $66\frac{7}{8}$ by 6; $31\frac{3}{8}$ by 8.

6. Multiply 644 by $2\frac{7}{8}$; 411 by $18\frac{3}{5}$.

7. Multiply 757 by $7\frac{5}{6}$; 89 by $24\frac{7}{8}$.

8. Multiply 989 by $11\frac{4}{5}$; 97 by $16\frac{6}{7}$.

9. Multiply 212 by $4\frac{7}{8}$; 612 by $9\frac{}{16}$.

10. Multiply 310 by $9\frac{10}{11}$; 915 by $18\frac{4}{15}$.

11. Multiply $18\frac{3}{4}$ by $\frac{4}{7}$; $18\frac{5}{9}$ by $\frac{3}{7}$.

12. Multiply $24\frac{6}{7}$ by $\frac{3}{5}$; $16\frac{5}{17}$ by $\frac{7}{11}$.

13. Multiply $33\frac{4}{5}$ by $\frac{7}{8}$; $4\frac{9}{13}$ by $\frac{8}{9}$.

14. Multiply $16\frac{7}{8}$ by $\frac{9}{16}$; $16\frac{8}{11}$ by $\frac{3}{7}$.

15. Multiply $89\frac{5}{6}$ by $\frac{11}{12}$; $21\frac{12}{17}$ by $\frac{4}{7}$.

16. Multiply $13\frac{1}{2}$ by $4\frac{2}{5}$; $617\frac{1}{4}$ by $2\frac{1}{4}$.

17. Multiply $25\frac{1}{7}$ by $6\frac{3}{4}$; $879\frac{4}{5}$ by $8\frac{2}{3}$.

18. Multiply $49\frac{4}{5}$ by $7\frac{5}{8}$; $710\frac{3}{4}$ by $9\frac{5}{6}$.

19. Multiply $\frac{3}{4}$ of $\frac{7}{8}$ by $\frac{4}{5}$ of $\frac{5}{8}$ of 15.

20. Multiply $3\frac{1}{8}$ times $\frac{5}{6}$ of 24 by $\frac{5}{8}$ of 4 times $\frac{1}{3}$ of 2.

21. A farmer sold $75\frac{1}{3}$ bushels of oats at $18\frac{3}{11}$ cents a bushel: how much did he receive?

22. At \$$19\frac{4}{13}$ a ton, how much are $17\frac{1}{2}$ tons of hay worth?

23. Samuel traveled $\frac{3}{5}$ of $14\frac{2}{3}$ miles, and Thomas traveled $\frac{5}{6}$ of that distance: how far did Thomas travel?

24. Mr. A bought $35\frac{1}{6}$ bushels of wheat at $87\frac{1}{5}$ cents a bushel, and sold $\frac{1}{4}$ at $88\frac{2}{3}$ cents, and the remainder at $89\frac{4}{7}$ cents a bushel: how much did he clear by the transaction?

25. Illustrate by an original problem, multiplication of fractions.

Division of Fractions.

ORAL EXERCISES

General Note on Division of Fractions.—Divide $\frac{3}{4}$ by $\frac{5}{6}$. The divisor is $\frac{5}{6}$ of 1, or $\frac{1}{6}$ of 5. Dividing $\frac{3}{4}$ by the whole of 5, we get $\frac{3}{20}$. But we were not to divide by 5, but by $\frac{1}{6}$ of 5. Having divided by a divisor 6 times the true divisor, the quotient is only $\frac{1}{6}$ as large as it ought to be. Hence we multiply by 6 to obtain the true quotient. Inspection shows that the terms of our fractional divisor are inverted. Thus: $\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$.

ART. 157.—**1.** Mary divided $\frac{4}{5}$ of a dollar equally among 8 girls: what part of a dollar did each girl receive?

Solution.—Each girl received $\frac{1}{8}$ of $\frac{4}{5}$ of a dollar. $\frac{1}{8}$ of $\frac{1}{5}$ of a dollar is $\frac{1}{40}$ of a dollar, and $\frac{1}{8}$ of $\frac{4}{5}$ of a dollar is $\frac{4}{40}$ of a dollar, equal to $\frac{1}{10}$ of a dollar, the part received by each girl. *The same result may be obtained by writing the divisor in the form of a fraction, inverting it, and proceeding as in multiplication.*

2. Andrew shared $\frac{3}{4}$ of a bushel of nuts with his 5 companions: how much did each receive?

3. If 6 books are worth $\frac{9}{10}$ of a dollar, what is one book worth?

4. A boy sold 8 fishes for $\frac{4}{5}$ of a dollar: how much did he receive apiece?

5. At \$3 a yard, what part of a yard of cloth can be bought for $\frac{3}{4}$ of a dollar?

6. At \$2 a load, what part of a load of kindling wood can be bought for $\frac{4}{5}$ of a dollar?

7. A tract of land containing $\frac{5}{8}$ of an acre was divided into 6 equal lots: what part of an acre did each lot contain?

8. A teacher paid $\frac{2}{15}$ of a dollar for 5 spellers: what did each speller cost?

9. Alfred divided $\frac{3}{4}$ of a dollar equally among [illegible] boys: how much did each receive?

10. If 6 yards of muslin cost $\frac{3}{4}$ of a dollar, what is the cost per yard?

ART. 158.—11. At $\frac{4}{5}$ of a dollar a pound, how many pounds of tea can be bought for \$3?

Solution.—As many pounds can be bought for \$3, as $\frac{4}{5}$ of a dollar is contained times in \$3. $\frac{1}{5}$ is contained in 3, 15 times, and $\frac{4}{5}$ is contained $\frac{1}{4}$ of 15 times, or $\frac{15}{4}$ times $= 3\frac{3}{4}$. Hence, $3\frac{3}{4}$ pounds can be bought for \$3. *The same result may be obtained by inverting the divisor and proceeding as in multiplication.* Thus, $3 \div \frac{4}{5} = 3 \times \frac{5}{4} = \frac{15}{4} = 3\frac{3}{4}$.

12. If $4\frac{1}{2}$ yards of cloth make a coat, how many coats can be made from 18 yards of cloth?

13. At $\$2\frac{1}{2}$ a day, how long will it take a man to earn \$15?

14. How long will \$52 pay my board at the rate of $\$3\frac{1}{4}$ a day?

15. At $3\frac{1}{3}$ miles an hour, how long will it take a man to walk 40 miles?

ART. 159.—16. At $\frac{4}{5}$ of a dollar a gallon, how many gallons of molasses can be bought for $\frac{9}{10}$ of a dollar?

Solution.—As many gallons can be bought as $\frac{4}{5}$ of a dollar is contained times in $\frac{9}{10}$ of a dollar. $\frac{1}{5}$ is contained in $\frac{9}{10}$, $\frac{45}{10}$ times, and $\frac{4}{5}$ is contained in $\frac{9}{10}$, $\frac{45}{40}$ times. Hence, $\frac{45}{40}$ or $1\frac{5}{40}$ or $1\frac{1}{8}$ gallons can be bought for $\frac{9}{10}$ of a dollar. *The same result may be obtained by inverting the divisor and proceeding as in multiplication.* Thus, $\frac{9}{10} \div \frac{4}{5} = \frac{9}{10} \times \frac{5}{4} = \frac{45}{40} = 1\frac{1}{8}$.

17. How many oranges at $\frac{3}{4}$ of a cent each, can be bought for 3 cents?

18. If a coat is worth $\$7\frac{1}{2}$, how many coats can be bought for $\$22\frac{1}{2}$?

19. At $\frac{3}{4}$ of a dollar a pound, how many pounds of tea can be bought for $\$9\frac{3}{4}$?

20. At $\$2\frac{1}{2}$ a yard, how many yards of cloth can be bought for $\$27\frac{1}{2}$?

21. Divide $\frac{4}{7}$ by 4; $\frac{5}{6}$ by 7; $\frac{8}{9}$ by 5.

22. Divide $\frac{5}{8}$ by 5; $\frac{3}{4}$ by 8; $\frac{3}{5}$ by $\frac{3}{8}$.

23. Divide 8 by $\frac{5}{8}$; $\frac{5}{6}$ by 8; $\frac{4}{7}$ by $\frac{3}{7}$.

TO TEACHERS.—It is clear from the foregoing solutions that the usual "three cases" in division of fractions, which from time immemorial have confused the beginner, should be treated as one case. This can be done by writing 1 as the denominator, when the dividend or the divisor is an integer.

ART. 160.—Rule for Division of Fractions.—*Change all the terms of the dividend and divisor to simple fractions, invert the divisor and proceed as in multiplication of fractions.*

NOTE.—Since the process in division of fractions is, after the inversion of the divisor, the same as in multiplication of fractions, the pupils should use the same methods for shortening their work.

WRITTEN EXERCISES.

1. Divide $\frac{7}{19}$ by 11.

2. Divide $\frac{3}{8}$ by 13.

3. Divide $\frac{5}{6}$ by 9.

4. Divide $\frac{9}{16}$ by 12.

5. Divide $\frac{33}{40}$ by 16.

6. Divide 8 by $\frac{2}{11}$.

7. Divide 13 by $\frac{5}{9}$.

8. Divide 17 by $\frac{20}{21}$.

9. Divide 16 by $\frac{8}{15}$.

10. Divide 24 by $\frac{11}{12}$.

11. Divide $\frac{6}{11}$ by $\frac{6}{13}$.

12. Divide $\frac{7}{8}$ by $\frac{9}{11}$.

13. Divide $\frac{8}{15}$ by $\frac{4}{7}$.

14. Divide $\frac{7}{10}$ by $\frac{10}{7}$.

15. Divide $\frac{9}{10}$ by $\frac{9}{13}$.

16. Divide $\frac{5}{6}$ by $10\frac{1}{4}$.

17. Divide $\frac{3}{5}$ by $11\frac{1}{5}$.

18. Divide $\frac{3}{8}$ of $\frac{4}{5}$ by $\frac{1}{2}$ of $\frac{5}{6}$.

19. Divide $\frac{3}{5}$ of $2\frac{1}{2}$ by $\frac{4}{9}$ of $11\frac{2}{3}$.

20. Divide $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ by $\frac{1}{3}$ of $\frac{5}{6}$ of $9\frac{2}{3}$.

21. At $11\frac{1}{2}$ cents a pound, how many pounds of sugar can be bought for 69 cents?

22. I paid $\$124\frac{4}{5}$ for $12\frac{1}{2}$ barrels of flour: what was the price per barrel?

23. A dealer paid $\$173\frac{9}{[illegible]}$ for $23\frac{1}{2}$ yards of broadcloth: what was the price per yard?

24. How many tons of hay at $\$[illegible]5\frac{3}{4}$ a ton can be bought for $\$97\frac{13}{20}$?

25. If John earns $\$1\frac{1}{5}$ a day, how long will it take him to earn $\$18\frac{3}{4}$?

26. A lady bought $13\frac{2}{3}$ yards of satin for $\$51\frac{[illegible]}{4}$: what was the price per yard?

27. Bought $13\frac{1}{2}$ baskets of peaches for $\$16[illegible]$: what was the price per basket?

28. How many pieces of lace $\frac{7}{10}$ of a yard long can be cut from $6\frac{3}{10}$ yards?

29. If a man spends $\frac{1}{4}$ of a dollar a day for tobacco, how long will it take him to spend $15?

30. How many suits containing $8\frac{2}{3}$ yards, can be made from $43\frac{1}{3}$ yards?

31. If Charles earns $\frac{3}{5}$ of a dollar a day, how long will it take him to earn $\$14\frac{1}{10}$?

32. A man chopped $30[illegible]$ cords of wood at the rate of $1\frac{2}{3}$ cords a day: how many days were required?

33. A grocer received $\$29[illegible]$ for coffee sold at $\frac{2}{[illegible]}$ of a dollar a pound: how many pounds did he sell?

34. How many tons of hay at $\$17\frac{3}{5}$ a ton can be bought for $\$281\frac{3}{5}$?

35. If [illegible] of a farm is worth $11,111, what is the value of the whole farm?

36. If $204[illegible]$ bushels of potatoes are raised from $3[illegible]$ acres, what is the average yield an acre?

37. Illustrate by an original problem division of fractions.

Relation of Numbers.

Numbers bear certain relations to each other and may be compared.

The unit 1 is the measure of all abstract numbers, and the fractional unit is the measure of all abstract fractions.

1. What part of 4 cents is 1 cent? 2 cents? 3 cents?

2. What part of 10 apples is 4 apples? 6 apples? 7 apples? 5 apples? 8 apples?

3. What part of 4 pounds is 1 pound? $\frac{1}{2}$ of 1 pound? $\frac{1}{4}$ of 1 pound? $\frac{2}{3}$ of 1 pound? $\frac{3}{4}$ of 1 pound?

4. What part of \$4 is \$2? \$3? $\frac{1}{2}$ of \$1? $\frac{1}{3}$ of \$1?

5. What part of \$8 is \$1? $\frac{1}{2}$ of \$1? $\frac{1}{3}$ of \$1? $\frac{3}{4}$ of \$1?

6. John had \$10 and spent $\frac{3}{4}$ of \$1: what part of his money did he spend?

7. William had \$12 and gave away $\frac{4}{6}$ of \$1: what part of his money did he give away?

8. Harry earned \$5 and Thomas \$3: what part of the whole did each earn?

9. What part of 5 is 3?

Solution. 1 is $\frac{1}{5}$ of 5, and 3 is 3 times $\frac{1}{5}$ or $\frac{3}{5}$ of 5.

What part:

10. Of 6 is 12?	**14.** Of 10 is 20?	**18.** Of 16 is 32?
11. Of 8 is 16?	**15** Of 12 is 36?	**19.** Of 18 is 40?
12. Of 7 is 18?	**16.** Of 11 is 44?	**20.** Of 17 is 30?
13. Of 9 is 30?	**17.** Of 15 is 18?	**21.** Of 25 is 45?

22. If 5 oranges cost 18 cents, how much will 25 oranges cost?

23. If 48 horses cost \$7,680, how much will 144 horses cost?

24. How much will 250 pounds of sugar cost, if 25 pounds cost \$2?

25. If 42 lemons cost \$1.26, how much will 36 lemons cost?

26. What part of 5 is $\frac{3}{4}$?

Solution.—1 is $\frac{1}{5}$ of 5, and $\frac{3}{4}$ is $\frac{3}{4}$ of $\frac{1}{5}$ of 5 $= \frac{3}{20}$ of 5.

What part:

27. Of 5 is $\frac{3}{5}$?
28. Of 7 is $\frac{5}{8}$?
29. Of 6 is $\frac{5}{6}$?
30. Of 8 is $\frac{6}{7}$?
31. Of 7 is $\frac{2}{3}$?
32. Of 9 is $\frac{4}{5}$?
33. Of 8 is $\frac{3}{4}$?
34. Of 10 is $\frac{7}{8}$?
35. Of 8 is $\frac{3}{8}$?
36. Of 11 is $\frac{4}{7}$?
37. Of 10 is $\frac{3}{7}$?
38. Of 12 is $\frac{8}{9}$?

39. If $\frac{3}{4}$ of a pound of tea cost 48 cents, how much will 4 pounds cost?

40. What is the value of 8 hats, if one hat is worth $\frac{7}{8}$ of \$1?

41. John having \$5, spent $\$1\frac{1}{4}$; what part of \$5 then remained?

42. Sarah paid 25 cents for 4 pencils; at the same rate, how much will 12 pencils cost?

43. How many times $\frac{2}{3}$ is 6?

Solution.—$6 = \frac{18}{3}$; $\frac{18}{3}$ is as many times $\frac{2}{3}$ as 2 is contained in 18, or 9 times $\frac{2}{3}$.

How many times:

44. $\frac{3}{4}$ is 5?
45. $\frac{4}{5}$ is 8?
46. $\frac{1}{5}$ is 7?
47. $\frac{5}{6}$ is 6?
48. $\frac{5}{8}$ is 9?
49. $\frac{4}{9}$ is 5?
50. $\frac{3}{7}$ is 7?
51. $\frac{3}{10}$ is 10?
52. $\frac{2}{7}$ is 9?
53. $\frac{5}{8}$ is 11?
54. $\frac{4}{7}$ is 10?
55. $\frac{9}{10}$ is 12?

56. If a yard of cloth cost $\frac{4}{5}$ of a dollar, how many yards can be bought for \$15?

57. How many times the distance that John can walk in $\frac{2}{3}$ of an hour can he walk in 7 hours?

58. A book cost $\frac{9}{10}$ of a dollar: at that rate how much will 30 books cost?

59. If a tract of land containing 15 acres is divided into lots containing $\frac{5}{8}$ of an acre each, how many lots are there in all?

60. How many times $\frac{3}{4}$ is $\frac{5}{6}$?

Solution. $\frac{3}{4} = \frac{9}{12}$ and $\frac{5}{6} = \frac{10}{12}$; $\frac{1}{12}$ is $\frac{1}{9}$ of $\frac{9}{12}$, and $\frac{10}{12}$ is 10 times $\frac{1}{9}$, or $\frac{10}{9}$ of $\frac{9}{12}$, equal to $1\frac{1}{9}$ times $\frac{3}{4}$.

How many times:

61. $\frac{2}{3}$ is $\frac{3}{4}$?	**65.** $\frac{1}{8}$ is $\frac{5}{7}$?	**69.** $\frac{3}{8}$ is $\frac{3}{7}$?
62. $\frac{4}{5}$ is $\frac{5}{6}$?	**66.** $\frac{1}{10}$ is $\frac{1}{6}$?	**70.** $\frac{4}{11}$ is $\frac{7}{15}$?
63. $\frac{4}{7}$ is $\frac{5}{8}$?	**67.** $\frac{3}{7}$ is $\frac{5}{8}$?	**71.** $\frac{5}{11}$ is $\frac{6}{13}$?
64. $\frac{3}{4}$ is $\frac{6}{7}$?	**68.** $\frac{4}{7}$ is $\frac{7}{9}$?	**72.** $\frac{5}{12}$ is $\frac{5}{6}$?

73. A man owning $\frac{5}{6}$ of a ship sold $\frac{3}{4}$ of his share: what part of the ship did he sell?

74. If $3\frac{1}{4}$ yards of silk cost \$13, how much will $13\frac{1}{2}$ yards cost?

75. Thomas had $\frac{4}{5}$ of a dollar and gave his sister $\frac{3}{10}$ of a dollar: what part of his money did he give his sister?

76. William having $\frac{3}{4}$ of a dollar lost $\frac{1}{3}$ of a dollar: what part of $\frac{3}{4}$ of a dollar remained?

77. Mary had $\frac{9}{10}$ of a dollar and spent $\frac{1}{6}$ of a dollar: what part of $\frac{9}{10}$ of a dollar was left?

78. A man bought $\frac{5}{8}$ of an acre of land and sold $\frac{1}{4}$ of an acre: what part of $\frac{5}{8}$ of an acre had he left?

79. The owner of $\frac{2}{3}$ of a mill sold $\frac{1}{5}$ of his share: what part of the mill did he then own?

80. A house insured for $\frac{7}{8}$ of its value was injured by fire to the extent of $\frac{2}{3}$ of its insurance: what was the loss of the insurance company?

81. At the rate of 5 oranges for $12\frac{1}{2}$ cents, what will 6 oranges cost?

82. If 6 men earn $\$16\frac{4}{5}$ in one day, how much will 5 men earn in the same time?

83. What will 3 pounds of tea cost at the rate of 7 pounds for $\$2\frac{4}{5}$?

84. How much did Jennie pay for 10 apples at the rate of $\frac{4}{5}$ of an apple for a cent?

85. If $\frac{5}{6}$ of a yard of cloth is worth 25 dimes, what is 5 yards worth?

86. Two thirds of the cost of a watch-chain was \$18, which was $\frac{1}{3}$ of the cost of a watch: what was the cost of the watch?

87. If a boy can earn $\$3\frac{3}{5}$ in 6 days, how much can he earn in $1\frac{1}{4}$ days?

88. If 3 books cost $\$1\frac{4}{5}$, how much will 5 books cost?

89. A man walked 30 miles in 6 hours: at that rate how far can he walk in $3\frac{1}{6}$ hours?

90. Mary having 40 pins lost $\frac{3}{4}$ of them, and then found $\frac{4}{5}$ as many remained: how many had she then?

91. A boy having 24 cents, spent $\frac{3}{4}$ of them, and his father gave him $\frac{2}{3}$ as many as remained: how many had he then?

92. John's age diminished by its $\frac{1}{3}$ and $\frac{1}{4}$ is 5 years: what is his age?

93. Mary's age increased by its $\frac{1}{2}$ and $\frac{1}{3}$ is 11 years: what is her age?

94. Thomas's age diminished by the difference between its $\frac{1}{2}$ and $\frac{1}{5}$ is 14 years: how old is he?

95. William's coat cost \$20, and $\frac{3}{4}$ of this is 3 times the cost of his trousers, and his vest cost $\frac{4}{5}$ as much as his trousers: required the cost of his trousers and vest.

Complex Fractions.

ART. 161. A **Complex Fraction** is one whose numerator, or denominator, or both, are fractional (Art. 129).

Thus, $\frac{2\frac{1}{2}}{5}$; $\frac{5}{2\frac{1}{2}}$; $\frac{3\frac{1}{3}}{4\frac{1}{4}}$; $\frac{\frac{3}{5}}{\frac{5}{6}}$, are complex fractions.

Complex fractions express unexecuted divisions.

1. Find the values of $\frac{2\frac{1}{2}}{5}$, $\frac{5}{2\frac{1}{2}}$, $\frac{3\frac{1}{3}}{4\frac{1}{4}}$, $\frac{\frac{3}{5}}{\frac{5}{6}}$.

Processes.

$$\frac{2\frac{1}{2}}{5} = \frac{5}{2} \div \frac{5}{1} = \frac{\cancel{5}}{2} \times \frac{1}{\cancel{5}} = \frac{1}{2}.$$

$$\frac{5}{2\frac{1}{2}} = \frac{5}{1} \div \frac{5}{2} = \frac{\cancel{5}}{1} \times \frac{2}{\cancel{5}} = 2.$$

$$\frac{3\frac{1}{3}}{4\frac{1}{4}} = \frac{10}{3} \div \frac{17}{4} = \frac{10}{3} \times \frac{4}{17} = \frac{40}{51}.$$

$$\frac{\frac{3}{5}}{\frac{5}{6}} = \frac{3}{5} \div \frac{5}{6} = \frac{3}{5} \times \frac{6}{5} = \frac{18}{25}.$$

Analysis.—Since, as in simple fractions, the numerator represents the dividend and the denominator the divisor, we complete the division according to the methods already explained.

Change to simple fractions:

2. $\frac{3\frac{1}{3}}{4}$.	6. $\frac{5}{3\frac{1}{6}}$.	10. $\frac{2\frac{1}{5}}{3\frac{2}{3}}$.	14. $\frac{\frac{5}{8}}{\frac{7}{9}}$.
3. $\frac{5\frac{1}{3}}{6}$.	7. $\frac{4}{7\frac{1}{2}}$.	11. $\frac{5\frac{1}{5}}{4\frac{4}{5}}$.	15. $\frac{\frac{5}{7}}{\frac{4}{5}}$.
4. $\frac{3}{2\frac{1}{3}}$.	8. $\frac{7}{6\frac{1}{2}}$.	12. $\frac{\frac{3}{8}}{\frac{1}{11}}$.	16. $\frac{3\frac{5}{8}}{2\frac{3}{4}}$.
5. $\frac{4}{3\frac{1}{4}}$.	9. $\frac{5\frac{4}{5}}{7\frac{3}{4}}$.	13. $\frac{\frac{2}{9}}{3\frac{5}{6}}$.	17. $\frac{21\frac{2}{3}}{\frac{1}{3} \text{ of } 2\frac{1}{3}}$.

Review Problems

ORAL EXERCISES.

ART. 162.—1. John had $\frac{3}{4}$ of a pie and gave Thomas $\frac{1}{6}$ of it: how much did Thomas receive?

2. Charles had $\frac{1}{5}$ of a dollar and gave Joseph $\frac{1}{5}$ of it: how much did Joseph receive?

3. Ralph found $\frac{3}{4}$ of a dollar and lost $\frac{1}{3}$ of it: how much did he lose?

4. Anthony divided $\frac{4}{5}$ of a loaf of bread equally among 5 beggars: how much did each receive?

5. Susan divided $\frac{4}{5}$ of a dozen candies equally among 4 girls: how much did each receive?

6. If 3 pounds of sugar cost 18 cents, what will $\frac{2}{3}$ of a pound cost?

7. Four times $4\frac{1}{4}$ years is $\frac{1}{2}$ of Mr. Bamford's age: how old is he?

8. Six times $\$5\frac{5}{6}$ is $\frac{7}{8}$ of Warner's money: how much money has Warner?

9. If 1 yard of ribbon cost $\frac{3}{4}$ of a dollar, how many yards can be bought for \$8?

10. If Thomas walks $\frac{4}{5}$ of a mile in an hour, how long will it take him to walk 8 miles?

11. Mr. A. bought $2\frac{1}{2}$ bushels of potatoes at the rate of 2 bushels for \$6: what did they cost?

12. A dairyman exchanged 25 quarts of milk worth 6 cents a quart for cheese at $12\frac{1}{2}$ cents a pound: how many pounds did he obtain?

13. If 1 quart of maple syrup costs $\frac{4}{5}$ of a dollar, how many quarts can be bought for \$12?

14. If it takes 4 men 7 days to build a wall, how long will it take 10 men?

15. Jenkins bought 8 tons of coal at $\$4\frac{1}{4}$ a ton and

paid for it with wood worth \$4 a cord: how many cords did it take?

16. Peter distributed $8\frac{3}{4}$ pears among a certain number of boys, giving to each $1\frac{3}{4}$ pears: how many boys were there?

17. Eighteen is $\frac{2}{3}$ of how many times 9?

18. Twenty is $\frac{5}{6}$ of how many times 12?

19. Forty-five is $\frac{3}{4}$ of how many times 20?

20. Three fourths of 48 is how many times $\frac{1}{6}$ of 36?

21. Anderson is 35 years old and $\frac{5}{7}$ of his age is $\frac{5}{4}$ of the age of Henry: how old is Henry?

22. A coat cost \$20 and $\frac{3}{4}$ of the cost of the coat was 3 times the cost of the vest: how much did the vest cost?

23. If $\frac{1}{3}$ of a peach costs $\frac{1}{4}$ of a cent, what is $\frac{1}{6}$ of a peach worth?

WRITTEN EXERCISES.

ART. 163. -**1.** What number multiplied by $\frac{5}{12}$ will produce $19\frac{5}{6}$?

2. If Walter earns $7\frac{3}{4}$ dollars in one week and spends \$$4\frac{1}{8}$, how much will he save in 6 weeks?

3. If Frank walks $7\frac{3}{5}$ miles in 3 hours, how far will he walk in 5 hours?

4. How many yards of cloth at $\frac{3}{8}$ of a dollar a yard can be bought for \$$7\frac{1}{4}$?

5. Seba spent $\frac{5}{6}$ of \$$20\frac{1}{4}$ for apples at $\frac{5}{8}$ of a dollar a bushel: how many bushels did he buy?

6. Three fifths of \$$7\frac{1}{2}$ is $\frac{1}{3}$ of $\frac{1}{4}$ of Robert's money: how much money has Robert?

7. If $\frac{5}{7}$ of a yard of velvet cost \$$4\frac{1}{7}$, how many yards can be bought for \$$21\frac{3}{4}$?

8. What number divided by 18_{3} gives $14\frac{5}{7}$?

9. If 1 man eats $\frac{3}{2}$ pounds of meat a day how long will $5\frac{1}{4}$ pounds last 7 men?

10. At $\frac{2}{3}$ of a dollar for a score of eggs, how many score can be bought for $\frac{3}{5}$ of $\$3\frac{1}{3}$?

11. A farmer exchanged $17\frac{1}{2}$ cords of wood at $\$2\frac{3}{4}$ a cord for coal at $\$4\frac{3}{8}$ a ton: how many tons did he buy?

12. Randolph rode 110 miles on his bicycle in $3\frac{1}{3}$ days: how many miles did he average a day?

13. Thirty-seven-and-one-half dollars will buy how many pounds of tea at $\frac{1}{3}$ of $\frac{3}{4}$ of a dollar a pound?

14. To how many boys can Joseph give $\$3\frac{1}{3}$, if $18 is $\frac{3}{5}$ of his money?

15. Sophie had $\frac{5}{8}$ of 40 cents; she spent $\frac{3}{5}$ of it for oranges at $\frac{1}{2}$ of 6 cents apiece: how many did she buy?

16. What will $2\frac{1}{4}$ yards of cloth cost, if $6\frac{2}{3}$ yards cost $\$33\frac{1}{3}$?

17. When land is $\$74\frac{2}{3}$ an acre, how many acres can be bought for $\$746\frac{2}{3}$?

18. The difference between $\frac{5}{7}$ and $\frac{3}{8}$ of the value of an estate is $1,691: what is the value of the estate?

19. The difference between two sums is $\$3\frac{3}{4}$; the less is $\$5\frac{4}{5}$: what is the greater?

20. If $13\frac{1}{2}$ tons of coal cost $\$64\frac{4}{5}$, how many tons will $\$19\frac{1}{5}$ buy?

21. Thaddeus owning $\frac{5}{6}$ of 1,200 acres of land, sold $\frac{3}{8}$ of his share: how many acres did he retain?

22. If 9 men build a wall in $\frac{1}{3}$ of $\frac{3}{5}$ of $16\frac{1}{3}$ days, how long will it take 8 men to build the wall?

23. A man owning $\frac{4}{5}$ of a store, sold $\frac{1}{6}$ of his share for $1,200: what was the store worth?

24. Mr. H owning $\frac{3}{7}$ of a ship, sold $\frac{1}{3}$ of his share. If what he still owned was worth $3,300, what was the value of the ship?

25. A man bequeathed $\frac{3}{5}$ of his money to his son, $\frac{1}{2}$ of the remainder to his daughter, and the remaining \$5,000 to his wife: what was the value of his estate?

26. A, B and C entered into partnership. A furnished $\frac{3}{8}$ of the capital; B $\frac{1}{3}$, and C the remainder, which was \$7,000: what was the aggregate capital?

27. A grocer bought 56 pounds of sugar at $6\frac{1}{4}$ cents a pound; it dried out $3\frac{1}{2}$ pounds in weight and he sold the remainder at $7\frac{1}{3}$ cents a pound: what did he gain?

28. If $\frac{2}{3}$ of 9 acres of land cost \$1,140, what is the cost of $\frac{5}{7}$ of $2\frac{1}{3}$ acres?

29. If peaches are worth $\frac{4}{5}$ of a dollar a basket, and a man sells them at $\frac{5}{4}$ of a dollar a basket, how many baskets will it take to yield a profit of \$81?

30. Quintin bought a farm for \$7,[illegible]00, and sold $\frac{2}{5}$ of it to Moses, and $\frac{1}{2}$ the remainder to John; what was the value of the remainder?

31. If at 3 P. M., a pole $37\frac{1}{2}$ feet high casts a shadow $58\frac{1}{4}$ feet long, how tall a pole will it take to cast at the same hour, a shadow of 67 feet?

32. How many jars holding $\frac{3}{4}$ of a gallon can be filled from a cask containing $15\frac{3}{4}$ gallons?

33. Illustrate by an original problem, addition of fractions.

34. Illustrate by an original problem, subtraction of fractions.

35. Illustrate by an original problem, multiplication of fractions.

36. Illustrate by an original problem, division of fractions.

37. Illustrate by an original problem, addition and subtraction of fractions.

38. Illustrate by an original problem, multiplication and division of fractions.

REVIEW QUESTIONS.

What is an integer? An even number? An odd number? A composite number? A prime number? A prime factor? A common divisor? The greatest common divisor? A multiple? A common multiple? The least common multiple?

Give the rule for finding the prime factors of a number; for finding the greatest common divisor; for finding the least common multiple.

What is cancellation? Give the rule. From what are all fractions and numbers derived? Of what is the unit 1 the measure? How do you analyze a fraction?

How is the value of a fraction affected by multiplying its numerator? Why? By dividing its denominator? Why? By multiplying its denominator? Why? By dividing its numerator? Why? By multiplying both numerator and denominator by the same number? Why? By dividing both numerator and denominator by the same number? Why?

What is a fraction? How is it expressed? What does the denominator of a fraction show? The numerator? What are the terms of a fraction? What is a proper fraction? An improper fraction? A simple fraction? A complex fraction? A mixed number? The reciprocal of a number? Of a fraction?

How do you change a fraction to an equivalent fraction having a given denominator? When is a fraction in its lowest terms? Give the rule. How are mixed numbers changed to improper fractions? How is an improper fraction changed to a whole or mixed number? Give the rule for changing fractions to equivalent fractions having a common denominator; for changing them to fractions having the least common denominator.

Give the rule for addition of fractions; for subtraction of fractions; for multiplication of fractions. To what is the word "of" between fractions equivalent? How many cases are usually given in multiplication of fractions? Show how they can be reduced to a single case. Give the rule for division of fractions? Give the reasons for inverting the terms of a fraction in order to divide by it.

What are complex fractions? What do they express? Illustrate. What is meant by the relations of numbers? The analysis of numbers? What is the first step in solving a problem in analysis of numbers? Upon what principle is their solution based?

Decimal Fractions.

If a unit be divided into 10 equal parts, what is one of the parts called?

If a tenth be divided into 10 equal parts, what is one of the parts called?

If a hundredth be divided into 10 equal parts, what is one of the parts called?

If a thousandth be divided into 10 equal parts, what is one of the parts called?

How many thousandths make a hundredth? How many hundredths make a tenth? How many tenths make a unit, or one?

ART. 164.—A Decimal Fraction is one whose denominator is 10, or 10 multiplied by itself one or more times.

A decimal fraction, for convenience, is written without the denominator and with a point after the units' place. Thus, $\frac{8}{10}$, $\frac{8}{100}$, $\frac{8}{1000}$, and $\frac{8}{10000}$, are written decimally .8, .08, .008, .0008.

The point is called the *decimal point*, and the fraction thus written is called a *decimal*.

The decimal point separates the decimals on the right from whatever whole numbers may be written on the left of it.

ART. 165.—A Mixed Decimal is an integer and decimal written together; as, 8.08; 245.32.

ART. 166.—A Complex Decimal is a decimal fraction and common fraction written together; as, $.48\frac{1}{3}$; $.212\frac{5}{8}$; $45.42\frac{1}{8}$.

For brevity, mixed decimals and complex decimals are called *decimals*.

ART. 167. As we proceed from left to right in whole numbers, their value decreases in a tenfold ratio. The same law governs the value of decimal fractions.

Beginning at the decimal or unit point, the first place to the right is *tenths;* the second, *hundredths;* the third, *thousandths;* the fourth, *ten-thousandths;* the fifth, *hundred-thousandths;* the sixth, *millionths,* and so on.

3.	units.
.3	tenths.
.03	hundredths.
.003	thousandths.
.0003	ten-thousandths.
.00003	hundred-thousandths.
.000003	millionths.

ART. 168. In reading decimals, first read the figures representing them as whole numbers; then give the decimal order of the last figure as a name for the entire decimal. Thus, .315 is first read as 315; then as the 5 or last figure stands in *thousandths* place, we read the decimal as 315 *thousandths.* Use the word *and* only between the whole number and the decimal. Thus, 24.056 is read twenty-four *and* fifty-six thousandths.

Read the following:

1.	.7	**13.**	10.07	**25.**	9.09
2.	.67	**14.**	30.094	**26.**	.400
3.	4.8	**15.**	214.214	**27.**	.004
4.	5.32	**16.**	.0361	**28.**	50.0005
5.	6.07	**17.**	.0178	**29.**	701.0701
6.	31.015	**18.**	741.09	**30.**	3400.043
7.	21.0048	**19.**	31.9019	**31.**	6000.006
8.	315.326	**20.**	.00007	**32.**	4000.0007
9.	43.5178	**21.**	500.05	**33.**	70.009
10.	.00864	**22.**	7000.007	**34.**	61010.101
11.	.101	**23**	900.0009	**35.**	41720.0019
12.	.9019	**24.**	4100.0195	**36.**	10000.00001

Express decimally:

1. 4 tenths.
2. 1 tenth.
3. 12 hundredths.
4. 9 hundredths.
5. 25 hundredths.
6. 75 hundredths.
7. 146 thousandths.
8. 65 thousandths.
9. 2364 ten-thousandths.
10. 769 ten-thousandths.
11. 11 ten-thousandths.
12. 87 hundred-thousandths.
13. 2002 hundred-thousandths.
14. 5007 millionths.
15. 1001 ten-millionths.
16. Sixty-five thousandths.
17. Seven hundred seven hundred thousandths.
18. Eight ten-thousandths.
19. One hundred five ten-millionths.
20. Three thousand and three hundred thousandths.
21. Four hundred four and ten thousand five millionths.
22. Five million and five millionths.
23. Seventy thousand seven and three thousand seven hundred-thousandths.
24. One thousand one and one thousand one millionths.
25. One and one hundred one ten-thousandths.
26. Nine thousand and nine tenths.
27. Seventeen and seventeen hundred-thousandths.
28. Seven and seventeen thousandths.
29. One hundred thousand and one hundred-thousandth.
30. Two hundred thousand two and two hundred two ten-thousandths.
31. Seven thousand and seven millionths.
32. Eighty thousand and eight hundred-thousandths.
33. One million and one ten-thousandth.
34. One hundred million ten and ten thousand one hundred-millionths.

Express decimally:

35. $\frac{63}{100}$.	**39.** $13\frac{13}{100}$.
36. $\frac{19}{1000}$.	**40.** $770\frac{513}{1000}$.
37. $\frac{312}{10000}$.	**41.** $\frac{307}{1000}$.
38. $\frac{2001}{100000}$.	**42.** $7001\frac{505}{10000}$.

ART. 169. Changing Decimals to Common Fractions and Common Fractions to Decimals.

1. Change .84 to a common fraction of equivalent value.

Process.

$.84 = \frac{84}{100} = \frac{21}{25}$.

Analysis. — .84 written as a common fraction is $\frac{84}{100}$, which changed to its lowest terms is $\frac{21}{25}$.

ART. 170. Rule for changing a decimal to a common fraction of equivalent value.— *Write the denominator under the decimal, omit the decimal point, and change the fraction to its lowest terms.*

Change the following to equivalent common fractions:

2. .8	**8.** 18.0205	**14.** 13.305
3. .06	**9.** 5.0025	**15.** 201.08
4. .25	**10.** 17.17	**16.** 93.003
5. .0125	**11.** 32.064	**17.** 11.9
6. 4.28	**12.** .185	**18.** 30.06
7. 13.075	**13.** 78.16	**19.** 109.008

1. Change $\frac{3}{8}$ to an equivalent decimal.

Process.

$\frac{3}{8} = \frac{3000}{8000}$; $\frac{3000}{1000} \div 8 = \frac{375}{1000} = .375$

Or, thus, $8)\underline{3.000}$
$\quad .375$

Analysis. — $\frac{3}{8} = \frac{3000}{8000}$, which changed to thousandths is $\frac{375}{1000}$. This expressed decimally is .375.

ART. 171. -Rule for changing a common fraction to a decimal of equivalent value.—*Annex ciphers to the numerator, divide by the denominator, and point off as many decimal places as there are ciphers annexed.*

Change the following to equivalent decimals:

2. $\frac{2}{3}$.

NOTE.—If, after annexing ciphers, the numerator is not exactly divisible by the denominator the decimal may be completed by writing the remainder as a common fraction, or the sign + may be added to the quotient to show that the division is not exact. Thus, $\frac{2}{3}$ = .666+

3. $\frac{4}{5}$.	**9.** $\frac{1}{111}$.	**15.** .33$\frac{1}{4}$.
4. $\frac{25}{600}$.	**10.** 74.3$\frac{1}{3}$.	**16.** 61.2$\frac{3}{5}$.
5. $\frac{45}{100}$.	**11.** 101.1$\frac{5}{8}$.	**17.** .71$\frac{9}{10}$.
6. 16$\frac{2}{3}$.	**12.** 412$\frac{5}{8}$.	**18.** 9.03$\frac{4}{5}$.
7. $\frac{13}{50}$.	**13.** .42$\frac{1}{2}$.	**19.** 6.6$\frac{3}{4}$.
8. $\frac{23}{1000}$.	**14.** 101 $\frac{1}{1000}$.	**20.** 503.05$\frac{1}{25}$.

21. Write a proper fraction and change it to a decimal of equivalent value.

22. Write a mixed number and change to a mixed decimal.

Addition of Decimals.

ART. 172.—**1.** What is the sum of $\frac{5}{10}$ and $\frac{3}{10}$? Of .5 and .3?

2. What is the sum of $\frac{13}{100}$ and $\frac{24}{100}$? Of .18 and .22?

3. What is the sum of $\frac{3}{10}$ and $\frac{6}{100}$? Of .4 and .32?

4. What is the sum of $\frac{7}{10}$ and $\frac{3}{100}$? Of .8 and .016?

5. What is the sum of .3, .15 and .004?

WRITTEN EXERCISES.

1. What is the sum of .12, 34.8, 7.004?

Process.

.12	**Analysis.**—We write the numbers so that units of the same order stand in the same column, and add as in whole numbers.
34.8	
7.004	
41.924	

ART. 173.—Rule for Addition of Decimals.—*Write the numbers so that units of the same order shall stand in the same column, and add as in whole numbers.*

What is the sum of:

2. 765.789, 34.908, 65.8, 1001.0897?

3. 9.675, 35.6789, 80.6784, .80907?

4. .235, 27.6542, .0007, 6000.0872, 754.0001?

5. 1001.987, 10001.432, 5478, 9009, 5437, 9004?

6. Seventeen and seventeen thousandths, one thousand ten and one hundred eleven ten thousandths, fifteen and seventy five hundred-thousandths, one million and one millionth?

7. Ten thousand ten and one hundred one hundred-thousandths, five hundred fifty five and eight hundred eighteen millionths, three hundred eighty-four thousand and six hundred-thousandths.

8. Twenty seven and four thousand four hundred-thousandths, ten thousand ten and nineteen millionths, sixty-four thousand and seventeen ten-thousandths?

9. Seven hundred seven and four hundred thirteen thousandths, nine hundred nine and nineteen hundredths, four tenths, three thousand three and eleven thousandths.

10. What is the sum of 10 dollars 4 cents, 28 dollars

27 cents, 47 dollars 5 mills, 17 dollars 7 cents, 101 dollars 1 mill, $3\frac{1}{2}$ dollars, and $\frac{3}{5}$ of a dollar?

11. Mr. A paid the following bills: groceries, \$17.80; shoes, \$23.42; milk, \$3.75; clothing, \$37.48; fuel, \$19.50: how much did he pay?

12. A man paid \$154.75 for a horse, \$19.48 for a cow, \$195.40 for a pair of mules, and \$3.75 for a dog: what did he pay for all?

13. A farmer sold some land for \$524.50, hay for \$21.38, oats for \$17.75, wheat for \$21.16, and corn for \$19.20: how much did he receive?

14. Find the sum of 1,010 dollars 20 cents, 9 dollars 9 mills, 1 dollar 40 cents, 30 dollars 8 cents, 75 cents, and 17 dollars 1 mill.

15. A clerk's expenses for one month were: board, \$24.40; washing, \$3.25; gifts, \$7.10; incidentals, \$3.21: what was the total?

16. Illustrate by an original problem addition of decimals.

Subtraction of Decimals

ART. 174.—1. From $\frac{7}{10}$ take $\frac{3}{10}$. From .8 take .7.

2. From $\frac{6}{100}$ take $\frac{3}{100}$. From .17 take .01.

3. From $\frac{23}{1000}$ take $\frac{17}{1000}$. From .043 take .023.

4. From 8 take $\frac{6}{10}$. From 9 take .9.

5. From .24 take .18. From .35 take .02.

WRITTEN EXERCISES.

1. From 21.304 take 17.017.

Process.

$$\begin{array}{r} 21.304 \\ \underline{17.017} \\ 4.287 \end{array}$$

Analysis.—We write the numbers so that units of the same order stand in the same column, and subtract as in whole numbers.

ART. 175.—Rule for Subtraction of Decimals.—*Write the subtrahend under the minuend, so that units of the same order shall stand in the same column, and subtract as in whole numbers. If there are more decimal places in the subtrahend than in the minuend, fill the vacant orders of the minuend with ciphers.*

Find the difference between

2. 203.456 and 199.009.

3. 89.076 and 64.0009.

4. 3003.765 and 2101.1001.

5. 90.678954 and 1001.0077.

6. 78786.0008 and 9999.090909.

7. .7 and 7 ten-millionths.

8. 4 and 14 hundred-thousandths and 2 and 22 millionths.

9. 5 hundredths and 5 millionths.

10. From 33 dollars 5 cents take 21 dollars 6 mills.

11. From 113 dollars 7 mills take 101 dollars 8 cents.

12. From 19 dollars 20 cents take 11 dollars 18 cents 1 mill.

13. A clerk's income was $1,235.60 and his expenses $750.23: how much did he save?

14. A man handed his grocer a twenty-dollar bill in payment for a debt of $17.60: how much change did he receive?

15. Illustrate by an original problem subtraction of decimals.

Multiplication of Decimals.

ART. 176.—**1.** What is the product of $\frac{7}{10} \times 5$? Of $\frac{4}{10} \times 5$? Of $.2 \times 4$?

2. What is the product of $\frac{4}{100} \times 7$? Of $.03 \times .6$?

3. What is the product of $\frac{7}{10} \times \frac{9}{100}$? Of $.5 \times .11$?

4. What is the product of $\frac{4}{100} \times \frac{7}{100}$? Of $.03 \times .12$?

5. What is the product of $\frac{13}{100} \times \frac{6}{1000}$? Of $.15 \times .006$?

WRITTEN EXERCISES.

1. Multiply .248 by .36.

Process.

```
 .248
  .36
 1488
 744
.08928
```

Analysis.—Since multiplying thousandths by hundredths gives hundred-thousandths, the product is 8928 hundred-thousandths, or .08928. It is seen that the decimal places in the product equal the number in the multiplicand and multiplier. The same result may be shown thus:

$$\frac{248}{1000} \times \frac{36}{100} = \frac{8928}{100000} = .08928.$$

ART. 177. Rule for Multiplication of Decimals.—*Multiply as in whole numbers, and point off as many decimal places in the product as there are in the multiplicand and multiplier. In case there are not the required number of decimal places in the product, prefix as many ciphers as are necessary to make the required number.*

Multiply:

2. 25.05 by .25.
3. 4.003 by .36.
4. 74.74 by .0009.
5. 384.31 by .64.
6. 78.78 by 78.7.
7. 320.32 by .3001.
8. .584 by 1.
9. 38.62 by 10.
10. 380.37 by 101.
11. 6361.01 by .3.
12. 10000 by .001.
13. 742.303 by 6.
14. .0009 by 125.
15. .0101 by 101.
16. 3103 by .25.
17. 600.006 by 1.006.
18. 301.09 by 13.04.
19. 690.008 by 6.003.
20. 31.01 by 31.01.
21. 117.11 by 17.013.
22. 3214 by .0001.
23. 1480 by .0702.

24. .0325 by 25.	**27.** 17.20 by .20.
25. 1.017 by 1.2.	**28.** 2400 by 1.1.
26. 23.07 by 3.08.	**29.** .3 by .0008.

30. What is the value of 123 acres of land at $113.75 an acre?

31. What is the cost of 29.5 pounds of sugar at $.0625 a pound?

32. What is the value of 38 barrels of apples at $4.50 a barrel?

33. The purchases made by a lady were: 23 yards of calico at $.08, 12 yards of gingham at $.10, 11 yards of muslin at $.12, 24 yards of drilling at $.115: what was the total cost of her purchases?

34. Illustrate by an original problem multiplication of decimals.

Division of Decimals.

ART. 178.—1. What is the product of $.4 \times .6$?

2. What is the quotient of $2.4 \div 6$?

3. What is the product of $.5 \times .7$?

4. What is the quotient of $.35 \div .7$?

5. What is the product of $.06 \times .06$?

6. What is the quotient of $.0036 \div .06$?

WRITTEN EXERCISES.

1. Divide 15.424 by 3.2.

Process.

```
3.2)15.424(4.82
    12 8
    262
    256
    ---
      64
      64
```

Analysis. Since the dividend is the product of the divisor by the quotient, the number of decimal places in the quotient must equal the excess of those in the dividend over those in the divisor. The same result may be shown thus:

$$\tfrac{15424}{1000} \div \tfrac{32}{10} = \tfrac{482}{100} = 4.82.$$

ART. 179. -Rule for Division of Decimals.—*Divide as in whole numbers, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.*

NOTE When the dividend has fewer decimal places than the divisor, annex enough ciphers to the dividend to make them equal. When the number of quotient figures is less than the excess of decimal places in the dividend over those in the divisor, prefix *as many ciphers as there are places lacking.*

Divide:

2. 324.547 by 7.32.

3. 789.67 by 394.835.

4. 70.087543 by 6.876.

5. 6068.05 by 6.876.

6. 32.657 by 4.8702.

7. 2.76453 by 24.72.

8. .0656 by .042.

9. 54.785 by .002.

10. 765 by .765.

11. 3.53 by .0642.

12. .0456 by 9.8.

13. .03246 by 124.

14. .005 by 500.

15. .75 by 600.

16. $\$18\frac{3}{4}$ by 45.

17. 1 by .0001.

18 At $6.35 a ton, how many tons of coal can be bought for $76.20?

19. Illustrate by an original problem division of decimals.

ART. 180.—To find the cost when the quantity and the price of 100 or 1000 are given.

1. If the cost of 100 articles is given, how can you find the cost of 200? Of 500? Of 800? Of 900?

2. If the cost of 100 articles is given, how can you find the cost of 250? 350? 375? 550? 575? 600? 625?

3. How many times 100 is 200? 225? 250? 275?

4. How many times 1000 is 5000? 5500? 5800? 5900? 5950?

WRITTEN EXERCISES.

1. What is the cost of 550 pounds of plaster at $4.25 per 100 pounds.

Process.

```
 $4.25
   5.50
  21250
 2125
$23.3750
```

Analysis.—Since 100 pounds cost $4.25, 550 pounds, or 5.50 times 100 pounds cost 5.50 times $4.25 or $23.375.

NOTE.—The letters C and M respectively represent the words *hundred* and *thousand*.

ART. 181.—Rule to find the cost, when the quantity and the price of 100 or 1000 are given.—*Point off two places for price per hundred, or three places for price per thousand, and multiply the price by the quantity.*

2. What is the cost of 15,500 shingles @ $5.75 per M.?

3. What is the cost of 8,548 feet of scantling @ $4.25 per M.?

4. I paid $6.25 per M. for 16,850 bricks: what was the cost?

5. A compositor was paid 48 cents per thousand ems for setting a book of 250 pages of 1,040 ems each: how much was he paid?

6. What is the cost of 1,275 bananas @ $3.25 per C.?

7. What must I pay for 780 envelopes @ $2.04 per C.?

8. What are 1,333 pine-apples worth @ $68.50 per M.?

9. A paid $1.75 per C. for 1849 feet of boards: what was the cost?

10. What is the cost of 1,960 pounds of hay @ $16.50 per ton (2,000 pounds)?

United States Money.

ART. 182.—United States Money is based on the decimal system. Calculations involving United States Money are performed in the same manner as in whole numbers and decimals.

ART. 183.—The dollar is the unit; the dime is one tenth of a dollar; the cent one tenth of a dime, and the mill one tenth of a cent.

There is no coin known as the *mill;* it is used only in making calculations.

10 mills (m.)	= 1 cent, ct.	10 dimes	= 1 dollar, $.
10 cents	= 1 dime, d.	10 dollars	= 1 eagle, E.

The demands of business have compelled the subdivision of coins, so that those now used are:

Bronze, 1 ct. and 2 ct. pieces; *Nickel*, 3 ct. and 5 ct. pieces; *Silver*, 5 ct., 10 ct., 25 ct., 50 ct., and $1 pieces; *Gold*, $1. $2½, $5, $10, $20, and $50. Some of the branch mints have coined 50 ct. and 25 ct. pieces of gold, but such coinage is illegal. The $10 piece is the eagle, the $5 piece the half eagle, and the $2½ piece the quarter eagle.

Paper money consists of bills of the denominations of $1, $2, $5, $10, $20, $50, $100, $500, and $1,000.

Write:

1. Fifty dollars fifty cents.
2. Fifty-five dollars fifty five cents five mills.
3. Seventy dollars eight cents six mills.
4. One hundred dollars three cents.

5. One thousand one dollars six-and-a-half cents.

6. Twenty-four dollars one mill.

7. Eight dollars one cent one mill.

8. Ten dollars three-and-a-half cents.

9. One hundred dollars two-and-a-half mills.

10. One hundred four dollars four mills.

Read the following:

1. $75.25.	5. $98.005.	9. $100.015.
2. $200.09.	6. $76.008.	10. $7.1875.
3. $100.07.	7. $200.002.	11. $20.0073.
4. $27.027.	8. $700.007.	12. $1001.001.

WRITTEN EXERCISES.

1. What is the sum of $63.05 and $75.125?

2. What is the sum of $270.005 and $.295?

3. From the sum of $20.28 and $1001.018 take the sum of 1 half eagle, 12 dimes, and 6 mills.

4. From $3.50 take 62½ cents.

5. A farmer sold 27½ bushels of apples at $.75 a bushel: how much did he receive?

6. If 7½ yards of broadcloth cost $16.35, how much will 45 yards cost?

7. A stock dealer bought a horse for $125.50, another for $160, and a third for $200. He sold them for $500: how much did he gain?

8. Samuel bought a coat for $20.25, a vest for $7.50, and a pair of trousers for $9.75. He handed the merchant a twenty-dollar bill: how much did he still owe?

9. If 8½ cords of wood cost $38.25, how much will 42½ cords cost?

10. If a man smoke 5 cigars a day, and the cost of

each is $6\frac{1}{4}$ cents, how much will he spend in 15 years of 365 days, and 5 years of 366 days each?

11. A merchant sold 4 pieces of muslin, each containing $30\frac{1}{4}$ yards, at $37\frac{1}{2}$ cents a yard, and took his pay in wheat at $1.25 a bushel: how many bushels did he receive?

12. A grocer bought 1,250 sacks of coffee at $18.50 a sack; he sold 624 sacks at $24, and the remainder at $19.125: how much did he gain?

13. If board is $8.75 a week, how long can a man board for $206.25?

14. A contractor paid $763.75 to 25 men for doing a piece of work that took them 23.5 days: what was the daily wages of each man?

15. The salary of the President of the United States is $50,000 a year. Reckoning 365 days to a year, how much does he receive per day?

16. The charge for sending a telegram from St. Louis to Chicago being 25 cents for 10 words, and 2 cents for each additional word, what is the cost of a dispatch of 29 words?

17. A gentleman bought a watch for $75.75, a chain for $18.20, and a gold key for $5.90: he sold them so as to gain $11.80: what did he receive for them?

18. A lady bought 4 sets of chairs at $9.50 a set, 3 tables at $4.75 apiece, 3 rocking chairs, one at $5.35, and the others at $6.25 apiece, and 48 yards of carpet at $1.375 a yard: what was the amount of her bill?

19. A dealer bought a boat-load of coal for $300, and by retailing it at $6.25 a ton, gained $75: how many tons were in the load?

20. A man bought 12 barrels of cider, each containing 28.5 gallons, at $.25 a gallon, and sold it at 33 cents a gallon: how much did he gain?

21. I bought 111 barrels of apples at $2.75 a barrel,

and sold 32 barrels at \$2.50 a barrel, and the remainder at \$3.20 a barrel: did I gain or lose, and how much?

22. A man bought 120 acres of land at \$75 an acre, and 216 acres at \$88 an acre. He sold it all at \$84 an acre: did he gain or lose, and how much?

23. How many days must a laborer work at \$1.62½ a day, to pay for 5 tons of coal at \$5.25 per ton, and 3 cords of wood at \$4.25 per cord?

24. At \$35.60 a ton, how many tons of railroad iron can be bought for \$5,119.28?

25. A man bought 24 boxes of lemons at \$4.75 a box and sold them at \$3.98 a box: how much did he lose?

26. A man used \$1,310.40 in paying 90 workmen. To 30 he gave \$12 apiece: how much did he give to each of the others?

27. A boy spent .5 of his money, gave away .25 of what remained, and then had \$29.91 left: how much had he at first?

28. If 17 yards of cloth cost .5 of \$289, how much will 63 yards cost?

29. What will $\frac{1}{2}$ of $\frac{2}{3}$ of 30 bushels of apples cost at $\frac{3}{4}$ of $\frac{4}{5}$ of a dollar a bushel?

30. A newsboy paid \$2.75 for a hundred papers. He sold $\frac{1}{2}$ at 3 cents apiece, $\frac{1}{4}$ at 2 cents apiece, and the remainder at 4 cents apiece: how much did he gain?

31. Illustrate by an original problem addition of United States Money.

32. Illustrate by an original problem subtraction of United States Money.

33. Illustrate by an original problem multiplication of United States Money.

34. Illustrate by an original problem division of United States Money.

Bills and Accounts.

ART. 184.—A **Bill of Goods** is a written statement of articles sold, the quantity and price of each, the date of each sale, the date upon which the bill of goods is made out, the total indebtedness, and the name of the debtor and the creditor.

ART. 185.—The **Debtor** is the one that owes the debt, and the **Creditor** is the one to whom the debt is due.

ART. 186.—An **Account** is a record of the debit and credit transactions between parties.

ART. 187.—A **Statement of Account** is a list of items in an account.

ART. 188.—An **Invoice** is a full statement of merchandise sold, including all expenses incurred.

ART. 189.—A **Bill is Receipted** by writing the words "Received Payment" or "Paid," at the bottom of the bill or across the face, with the signature of the creditor attached. If this is done by an authorized agent, he adds the word "Per" and his own name or initials.

ART. 190.—To **Balance an Account,** the difference between the sum of the credit and debit accounts is added to the less side. This shows which party owes the other and how much.

The following are the principal abbreviations used in bills and accounts:

@	at.	Mdse	merchandise.
Acct. or %/c	account.	Mo	month.
Bo't	bought.	No. or #	number.
¢ or cts.	cents.	Pay't	payment.
Co.	company.	Pd	paid.
Cr.	credit.	Per.	by.
Dr	debtor.	Pr.	pair.
Do or ditto	the same.	Prox.	next month.
Fr't	freight.	Rec'd	received.
Fol.	folio.	S.S.	steamship.
Inst	the present month.	Sunds.	sundries.
Int	interest.	Ult	last month.

1. St. Louis, Mo., Jan. 8, 1886.

Mr. Jonas Jones,

Washington, D. C.

Bought of Whitman Agricultural Co.

3 Rebound Plunger Hay Presses	@ $275.00		
2 Two Horse Railway Powers	" 125.00		
4 Magic Feed Mills for Belt Power	" 37.00		
3 Swing Saw Tables, 24 inch saw	" 40.00		
2 Monarch Corn and Cob Mills	" 25.00		
4 No. 7 St. Louis Feed Cutters	" 37.50		
6 Pacific Broadcast Seed Sowers	" 18.50		
2 Americus Senior Cider Mills	" 25.00		
4 Garden Barrows, No. 3	" 4.50		
5 Steel Bottom Road Scrapers	" 5.00		

Rec'd payment,

Whitman Agricultural Co.

Per C. E. Whitman.

2. St. Louis, Jan. 14, 1886.

Mr. Geo. C. Townsend,
Ninth St. and Washington Ave.

Bought of Simmons Hardware Co.

2 Cook Stoves and Furniture .	@	$20.50
3 Wash Tubs	"	.85
3 " Boards	"	.10
2 Clothes Wringers . . .	"	2.25
2 Refrigerators	"	10.50
2 Pr. Carvers	"	1.75
2 Doz. Knives and Forks . .	"	4.50
3 " Plated Tea Spoons. .	"	2.10
2 " Casters . . .	"	3.25
3 Feather Dusters . . .	"	.90
4 Scrubbing Brushes . . .	"	.30
2 Student Lamps . . .	"	3.25

3. St. Louis, Jan. 4, 1886.

Messrs. J. R. Scott & Co.,
Sedalia, Mo.

Bought of Robt. D. Patterson Stationery Co.

10 Rms. 6 lb. Com'l Note . .	@	$ 1.50
5 " 10 lb. Letter . .	"	2.50
5 " 12 lb. Cap . . .	"	3.00
2 " 14 lb. Legal . .	"	3.50
1 Patterson's Paper Case . .	"	10.00
1 Gross Patterson's Best Pencils	"	5.00
1 " Irving Nickel " .	"	3.50
1 lb. Irving Rubbers . .	"	1.00
5 C. Slate Pencils . . .	"	.15
1 Gross Carter's Ink . .	"	3.50
1 " Esterbrook Pens . .	"	.60
1 " Irving Bookkeeper's Pens	"	1.25
2 M. 215–4 Envelopes . .	"	1.35
½ Doz. Gem Inkstands . .	"	1.50

4. ST. LOUIS, Jan. 14, 1886.

MR. THOS. STEVENSON,

Vineland, Mo.

Bought of D. CRAWFORD & Co.

40 yds. $\frac{10}{4}$ Bleached Pepperell Muslin	@ $ .20			
55 " Semper Idem Muslin	" .07$\frac{1}{2}$			
16 " Jeans	" .15			
20 " "	" .20			
10 " "	" .22$\frac{1}{2}$			
10 " Cassimere	" .32$\frac{1}{2}$			
12 " "	" .65			
12 Linen Towels	" .12$\frac{1}{2}$			
20 " "	" .15			
7 yds. Linen Damask	" .35			
3 Window Shades	" .50			
1 Honey-comb Spread	" .65			
3 Bed Comfortables	" .60			
2 prs. White Blankets	" 3.25			
10 yds. Black Cashmere	" .50			

Received payment,

D. CRAWFORD & Co

Per J. M. SMITH.

5. ST. LOUIS, Jan. 16, 1886.

MR. JOHN RANDOLPH,

Bought of T. B. BOYD & Co.

12 pr. Socks	@ $ .75		
6 Collars	" .25		
6 Hdkfs	" .80		
6 Neckties	" 1.25		
6 Shirts	" 3.00		
12 pr. Cuffs	" .30		

Received payment,

T. B. BOYD & Co.

6. St. Louis, Jan. 4, 1886.

Mr. John Smith,

Washington, D. C.

Terms: 60 days. Bought of

Via Baltimore & Ohio R. R. Richardson Drug Co.

25 ℔ Pure Cream Tartar	@ $ .37½		
5 ℔ Powd. Arsenic	" .08		
2 ℔ Carb. Magnesia	" .20		
1 ℔ Powd. Rhubarb	" 1.00		
2 ℔ Soccotrine Aloes Powd	" .25		
5 ℥ Quinine, ozs.	" .85		
1 ℥ Sulph. Morphia	" 3.20		
10 ℥ " Cinchonidia	" .25		
3 ℔ Carb. Iron	" .15		
2 ℔ American Calomel	" .80		
2 ℔ " Blue Mass	" .45		
2 ℔ Chloroform	" .60		
2 ℔ Citric Acid	" .54		
2 ℔ Cryst. Carbolic Acid	" .30		
50 ℔ Refined Borax	" .08		
10 ℔ Gum Camphor	" .27		
1 ℔ Subnit. Bismuth	" 2.25		
1 ℔ Gum Opium	" 3.60		
5 ℔ Calabria Licorice	" .26		
Box, 25; Cartage, 35	.60		
		$41	91

7. St. Louis, May 27, 1886.

Mrs. George L. Duncan,

Bought of The Wm. Barr Dry Goods Co.

May 27,	11 yds. Muslin	@ $ .12	$1	32
	4 yds. Silk	" 1.50	6	00
	Carried forward		$7	32

	Brought forward . . .			$7 32
May 27,	3 yds. Lining . .	@	$ .12½	38
	4 Spools Silk . .	"	.15	60
	½ Doz. Braid . .	"	.75	37
	11 yds. Lawn . . .	"	.17½	1 93
	15 yds. Satteen . .	"	.32½	4 87
	4 yds. Farmers' Satin .	"	.65	2 60
	14 yds. Cambric . .	"	.06¼	88
	7 yds. Ribbon . .	"	.15	1 05
	9 yds. Dimity . .	"	.35	3 15
	1½ yds. Veiling . .	"	.35	52
	6 Pair Hose . .	"	.37½	2 25
				$25 92

Received Payment.

WM. BARR DRY GOODS CO.

May 27, 1886. J. S. CONCANNON.

REVIEW QUESTIONS.

What is a decimal fraction? How, for convenience, is it written? Explain the use of the decimal point. What is a mixed decimal? A complex decimal? What law of decrease in value from left to right governs decimal fractions?

How do you change decimals to common fractions? Common fractions to decimals? What should be done when the numerator is not exactly divisible by the denominator?

Give the rule for addition of decimals; for subtraction of decimals; for multiplication of decimals; for division of decimals. What should be done when the dividend has fewer decimal places than the divisor?

Give the rule for finding the cost when the quantity and the price of 100 or 1,000 are given.

On what is United States Money based? How are calculations involving it performed? What is the unit? The dime? What of the mill? Give the table. Name the coins made of bronze; of nickel; of silver; of gold. Of what does paper money consist?

What is a bill of goods? Who is meant by the debtor? By the creditor? What is an account? A statement of account? An invoice? How is a bill receipted? How do you balance an account?

Denominate Numbers.

ART. 191.—A **Denominate Number** is a concrete number composed of one or more denominations: as 12 pounds, 7 ounces.

ART. 192.—**Denominate Numbers** are either Simple or Compound.

ART. 193. A **Simple Denominate Number** is composed of units of the same kind or denomination: as 7 days, 6 cents, 3 inches, 11 books.

ART. 194. A **Compound Denominate Number** consists of two or more denominate numbers of the same nature: as 7 feet 3 inches; 4 pounds 11 ounces; 21 hours 18 minutes.

ART. 195.—A **Denominate Fraction** is one or more of the equal parts of a denominate unit: as $\frac{1}{4}$ of a year, $\frac{5}{8}$ of an acre, .75 of a ton.

Operations in denominate numbers embrace:

1. Changing Denominate Numbers to lower denominations, called *Reduction Descending.*

2. Changing Denominate Numbers to higher denominations, called *Reduction Ascending.*

Measures of Weight.

Weight is the measure of gravity, or the force by which all bodies are attracted toward the center of the earth.

AVOIRDUPOIS WEIGHT.

ART. 196. Avoirdupois Weight is used in weighing everything in common use, and the metals, except gold and silver.

TABLE.

16 ounces (oz.)	=	1 pound, lb.
100 pounds	=	1 hundred weight, cwt.
20 hundred weight	=	1 ton, T.

The unit is the pound, equal in weight to 7,000 grains Troy, or the weight of 27.7015 cubic inches of distilled water at 39.2° Fahrenheit, the barometer being at 30 inches.

In Great Britain 28 pounds make a quarter, 112 pounds a hundred weight, and 2,240 pounds a ton, known in this country as the *gross* or *long ton*. It is used in weighing coal at the mines, chalk in ballast, and in calculating duties on goods brought from England.

1. Change 7 T. 11 cwt. 15 lb. 10 oz. to oz.

Process.

```
T.  cwt.  lb.  oz.
 7   11    15   10
20
---
151
  100
15115
   16
-----
 90700
15115
241850
```

Analysis.—Since in 1 ton there are 20 cwt., in 7 tons there are 7 times 20 cwt. = 140 cwt., to which the 11 cwt. given are added = 151 cwt.

Since in 1 cwt. there are 100 lb., in 151 cwt. there are 151 times 100 lb. = 15100 lb., to which the 15 lb. given are added = 15115 lb.

Since in 1 lb. there are 16 oz., in 15115 lb. there are 15115 times 16 oz. = 241840 oz., to which the ten oz. given are added = 241850 oz.

ART. 197. Rule for Reduction Descending. *Multiply the highest denomination by the number of units of the next lower denomination required to make a unit of the higher; add to the product the given number, if any, of the lower denomination.*

Proceed in like manner with each successive denomination until the required denomination is reached.

2. Change 241850 oz. to higher denominations.

Process.

16)241850
100)15115 + 10 oz.
20)151 + 15 lb.
7 T. + 11 cwt.

Analysis.—Since there are 16 oz. in 1 lb., in 241850 oz. there are as many lb. as 16 oz. is contained times in 241850 oz., or 15115 lb., with 10 oz. remaining.

Since there are 100 lb. in 1 cwt., in 15115 lb. there are as many cwt. as 100 lb. is contained times in 15115 lb., or 151 cwt., with 15 lb. remaining.

Since there are 20 cwt. in 1 T., in 151 cwt. there are as many tons as 20 cwt. is contained times in 151 cwt., or 7 T., with 11 cwt. remaining.

Therefore, 241850 oz. = 7 T. 11 cwt. 15 lb. 10 oz.

ART. 198. Rule for Reduction Ascending.—*Divide the given denominate number by the number of units required to make one of the next higher denomination. Divide the quotient in like manner, and continue the operation until the required denomination is reached. The last quotient, with the several remainders annexed in their proper order, is the answer.*

ART. 199.—The following denominations are now in use, or were formerly:

56 lb. of butter	= 1 firkin.
84 lb. of butter	= 1 tub.
100 lb. of dried fish	= 1 quintal.
100 lb. of grain or flour	= 1 cental.
100 lb. of nails	= 1 keg.
100 lb. of raisins	= 1 cask.
196 lb. of flour	= 1 barrel.
200 lb. of pork, fish, or beef	= 1 barrel.
280 lb. of salt	= 1 barrel.
240 lb. of lime	= 1 cask.

NOTE.—It is more convenient to estimate many articles by weight than by measure, and the tendency is in that direction. The following standard has been adopted by the Pennsylvania and other railroad companies.

WEIGHT OF OIL PER GALLON

Lard oil and engine-oil	7.5 lb.
Well-oil and passenger-car oil	7.4 lb.
Signal oil	7.1 lb.
Head-light oil	6.6 lb.

ORAL EXERCISES.

How many:

1. Ounces in 6 lb.?
2. Ounces in 100 lb.?
3. Pounds in 25 cwt.?
4. Cwt. in 10 tons?
5. Cwt. in 2,400 lb.?
6. Cwt. in $\frac{1}{4}$ of a ton?

WRITTEN EXERCISES.

Change:

1. 4 T. 15 cwt. 50 lb. to lb.
2. 6 T. 20 lb. to oz.
3. 18,462 lb. to higher denominations.
4. 26,848 oz. to higher denominations.
5. What will $\frac{1}{4}$ of a firkin of butter cost at $.24 a pound?
6. When flour is worth $8 a barrel, how many pounds can be bought for $2?
7. How many barrels of salt are there in 4,060 lb.?
8. Show by an original problem in avoirdupois weight how to change higher to lower denominations.

9. Show by an original problem in the miscellaneous table how to change lower to higher denominations.

TROY WEIGHT.

ART. 200.—Troy Weight is used in weighing gold, silver, jewels, and fluids in philosophical experiments.

TABLE.

24 grains (gr.)	= 1 pennyweight, pwt.
20 pennyweights	= 1 ounce, oz.
12 ounces	= 1 pound, lb.

The unit of weight is the Troy pound, which is equal in weight to 22.794377 cubic inches of distilled water, at the temperature of 39.2° Fahrenheit, the barometer standing at 30 inches. It is the same as the Imperial Troy pound of Great Britain.

ORAL EXERCISES.

How many—

1. Grains in 4 pwt.?
2. Grains in 1 ounce?
3. Pwts. in 96 grains?
4. Pwts. in 5 ounces?
5. Pwts. in 1 pound?
6. Ounces in 120 pwt.?
7. Ounces in 1 pound?
8. Pounds in 144 ounces?
9. Pounds in 240 ounces?
10. Grains in 1 pound?

WRITTEN EXERCISES.

Change:

1. 8 pwt. 12 grains to grains.
2. 41 lb. 4 oz. 11 pwt. 10 gr. to grains.
3. 1,240 gr. to higher denominations.
4. 1,000 pwt. to higher denominations.
5. 9 lb. to pwt.
6. 7,777 gr. to higher denominations.
7. Show by an original problem in Troy weight how to change higher to lower denominations.

8. Show by an original problem in Troy weight how to change lower to higher denominations.

APOTHECARIES' WEIGHT.

ART. 201.—Apothecaries' Weight is used in prescribing and compounding medicines. Medicines are bought and sold by avoirdupois weight.

TABLE.

20 grains (gr.)	= 1 scruple, ℈.	8 drams	= 1 ounce, ℥.
3 scruples	= 1 dram, ʒ.	12 ounces	= 1 pound, ℔.

The unit is the pound, and is the same as the Troy pound. The only difference between the two weights is in the subdivision of the ounce.

APOTHECARIES' FLUID MEASURE.

60 minims or drops (♏ or gtt.)	= 1 fluid dram (f ʒ).
8 fluid drams	= 1 fluid ounce (f ℥).
16 fluid ounces	= 1 pint (O.).
8 pints	= 1 gallon (Cong.).

ORAL EXERCISES.

How many—

1. ℔ in 96 ℥?
2. ℔ in 108 ℥?
3. ℥ in 48 ʒ?
4. ʒ in 6 ℥?
5. gr. in 6 ℈?
6. ʒ in 1 ℔?

WRITTEN EXERCISES.

Change:

1. 6 ℔ 5 ℥ to scruples.
2. 1 ℔ 4 ʒ to grains.
3. 5760 gr. to pounds.
4. 8420 ℈ to ℔.
5. 1 ℔ 1 ℥ 1 ʒ 1 ℈ 1 gr. to grains.
6. 7642 gr. to ℔.
7. Illustrate by an original problem in apothecaries' weight how to change lower to higher denominations.
8. Illustrate by an original problem in apothecaries' weight how to change higher to lower denominations.

Measures of Value.

ART. 202. Money is the measure of value. (For the currency used in this country see Arts. 182, 183.)

ENGLISH CURRENCY.

ART. 203. English Currency or Sterling Money is the legal currency of Great Britain and Ireland.

The sovereign or pound sterling is the standard unit of English money. Its value in United States money is $4.8665.

The coins of Great Britain in general use are: *Gold*, the sovereign and half sovereign; *Silver*, the crown, half crown, florin, shilling, six-penny, and three-penny pieces; *Copper*, the penny, half-penny, and farthing.

TABLE.

4 farthings (far.)	=	1 penny, d.
12 pence	=	1 shilling, s.
2 shillings	=	1 florin, fl.
5 shillings	=	1 crown, cr.
20 shillings	=	1 sovereign, sov. 1 pound, £.

1. Change £7 0s. 6d. 3 far. to farthings?
2. Change £5 16s. 11d. to far.
3. Change 8 cr. 1 fl. 11d. to pence.
4. Change 6850 farthings to higher denominations.
5. Change £50 to farthings.
6. Change 6242d. to florins, crowns and sovereigns.
7. Change $389.32 to pounds.
8. Change $3,163.225 to pounds.
9. Change £35 10s. to farthings.

10. Change £1,124 to dollars.

11. Illustrate reduction descending by an original problem.

12. Illustrate reduction ascending by an original problem.

FRENCH MONEY.

ART. 204.—French money is the legal currency of France. The unit is the franc, equal to $.193 of United States currency.

TABLE.

10 millimes (m)	=	1 centime, ct.
10 centimes	=	1 decime, dc.
10 decimes	=	1 franc, fr.

The bronze coins of France are the 1, 2, 5 and 10 centime pieces.

The silver coins are 1, 2 and 5 franc pieces.

The gold coins are the 5, 10, 20, 40 and 100 franc pieces.

The French currency is founded on the decimal notation. All the operations, therefore, are similar to those in United States money.

The decime, like our dime, is not used in business calculations, but is expressed by centimes. Thus, 6 decimes are called 60 centimes, 85 francs 4 decimes and 5 centimes are written 85.45 francs.

CANADA MONEY.

ART. 205.—Canada money is the legal currency of the Dominion of Canada. In 1858, the currency was made the same as that of the United States, previous to which time it was sterling money.

GERMAN MONEY.

ART. 206.—The New Empire of Germany has adopted a uniform system of coinage.

The unit is the mark (Reichsmark), whose value is 23.85 cents.

The mark is divided into 100 pennies (Pfennige).

The nickel coins are the 5 and the 10 penny pieces, and those of smaller value.

The silver coins are the 20 penny, 1 mark, and 2 mark pieces.

The gold coins are the 5, 10 and 20 mark pieces.

The silver groschen and silver thaler are in general use. The silver groschen is worth $2\frac{1}{2}$ cents, and the silver thaler is equal to $.746.

Measures of Extension.

LONG MEASURE.

ART. 207.—**Long Measure** is used in measuring lengths and distances.

TABLE.

12 inches (in.)	= 1 foot, ft.
3 feet	= 1 yard, yd.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod, rd.
320 rods	= 1 mile, mi.
60 geographical miles	= 1 degree.
69.16 statute miles	= 1 degree (deg. or °) of longitude at the equator.
360°	= circumference of the earth.

The unit of Long Measure is the yard, which is formed from the scale of Troughton, at the temperature of 62° Fahrenheit. It is identical with the imperial yard of Great Britain.

ORAL EXERCISES.

How many—

1. Yards in 48 inches?
2. Feet in 64 miles?
3. Inches in 5 feet?
4. Feet in 2 rods?
5. Rods in 9 miles?
6. Miles in 640 rods?

WRITTEN EXERCISES.

Change:

1. 1 mile to feet; 2 mi. 39 rd. to ft.
2. 15 rods to yards; [illegible] m. 87 rd. 7 yd. to ft.
3. 1,280 inches to yards; 8679687 in. to ft.
4. 620,560 inches to higher denominations.
5. 5 miles 6 rods 4 yards 2 feet 10 inches to inches.
6. Show by an original example in long measure how to change higher to lower denominations.
7. Show by an original example in long measure how to change lower to higher denominations.

CLOTH AND MARINERS' MEASURES.

ART. 208. Cloth Measure is used in measuring cloth, laces, ribbons, etc. Its principal unit is the yard, which is divided into halves, quarters, eighths, and sixteenths.

ART. 209.—Mariners' Measure is used in measuring the depth of the sea.

TABLE.

6 feet	=	1 fathom.
120 fathoms	=	1 cable length.
880 fathoms	=	1 mile, mi.

SURVEYORS' MEASURE.

ART. 210.—Surveyors' Measure is used by surveyors and engineers in measuring land and distances.

ART. 211.—The **Surveyors' Chain**, called Gunter's chain, is 4 rods or 66 feet long, and is subdivided as follows:

7.92 inches (in.)	= 1 link, l.
25 links	= 1 rod, rd.
4 rods or 100 links	= 1 chain, ch.
80 chains	= 1 mile, mi.

ORAL EXERCISES.

How many:

1. Inches in $\frac{1}{4}$ of a yard?
2. Feet in 8 fathoms?
3. Fathoms in 2 miles?
4. Fathoms in 144 feet?
5. Links in 5 chains?
6. Miles in 240 chains?
7. Inches in 2 links?
8. Rods in 200 links?

WRITTEN EXERCISES.

1. Reduce 8 miles to links.
2. Reduce $5\frac{1}{2}$ miles to fathoms.
3. Reduce 1 mile to inches.
4. Reduce 2,432 sixteenths to yards.
5. How many eighths in $63\frac{1}{2}$ yards?
6. How many fathoms in $16\frac{3}{4}$ miles?
7. Show by an original problem in cloth measure how to change higher to lower denominations.
8. Show by an original problem in mariners' measure how to change lower to higher denominations.

Measures of Surface.

SQUARE MEASURE.

ART. 212.—Square Measure is used in measuring surfaces, or that which has length and breadth only; as plastering, papering, flooring, land, etc.

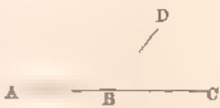
TABLE.

144 square inches (sq. in.)	= 1 square foot, sq. ft.
9 square feet	= 1 square yard, sq. yd.
$30\frac{1}{4}$ square yards or $272\frac{1}{4}$ square feet	= 1 square rod, sq. rd.
160 square rods	= 1 acre, A.
640 acres	= 1 square mile, sq. mi.

The unit of Land Measure is the acre, subdivided as follows:

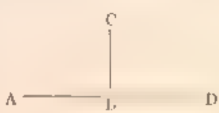
625 square links	= 1 square rod, sq. rd.
16 square rods	= 1 square chain, sq. ch.
10 square chains or 160 square rods	= 1 acre.

ART. 213.—An **Angle** is the opening between two straight lines that meet. Thus, A B D and D B C are angles.



ART. 214.—The **Vertex** is the point where the lines meet. Thus, B is the vertex of the angles A B D and D B C.

ART. 215.—A **Right Angle** is formed when one of the lines which meet is perpendicular to the other. Thus, A B C and C B D are right angles.



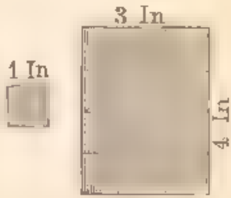
ART. 216.—Right Angles are always equal to each other.

ART. 217. A **Parallelogram** is a four-sided figure, whose opposite sides are parallel.

ART. 218. A **Rectangle** is a parallelogram whose angles are all right angles.

ART. 219. A **Square** is a rectangle whose sides are equal. Thus, a square foot is a square, each side of which is a foot in length.

The **Area** of a Figure is its amount of surface.



The larger figure represents a surface 4 inches long and 3 inches wide. The dividing lines of the figure show that its surface contains as many times 3 sq. inches as there are inches in the length; that is, 4 × 3 sq. in. or 12 sq. in. *The length and breadth of a rectangular surface being given in the same denomination, their product is the number of superficial units in the area.*

ORAL EXERCISES.

How many:

1. Square inches in 2 square feet? in 4 sq. ft.?

2. Square yards in 81 square feet? in 108 sq. ft.?

3. Square feet in 11 square yards? in 76 sq. yd.?

4. Square rods in $3\frac{1}{2}$ acres? in 17 acres?

5. Acres in 480 square rods? in 640 sq. rd.?

6. Square yards in 4 square rods? in 6 sq. rd.?

WRITTEN EXERCISES.

Change:

1. 1 acre to square feet; 7 A; 9 A.

2. 6840 square rods to acres; 8694 sq. rd.

3. 1 acre 126 square rods to square feet.

4. 3 acres 1 foot to square inches.

5. 87,120 square feet to acres.

6. 1 acre 1 square rod 1 foot to inches.

7. Show by an original problem in square measure how to change lower to higher denominations.

8. Show by an original problem in square measure how to change higher to lower denominations.

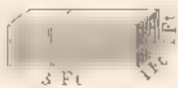
Measures of Volume.

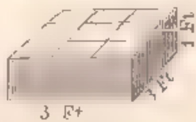
CUBIC MEASURE.

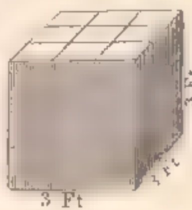
ART. 220.—Cubic Measure is used in measuring solids.

A cubic foot is a cube or solid, one foot wide, one foot deep, and one foot long.

Suppose we take 3 blocks, each of which is a foot long, a foot wide, and a foot deep. Now, if they are laid on the table beside each other, you see that it is 3 feet from one side to the other. We will lay 3 similar blocks beside the first row, and 3 more beside the second row. We now have a layer consisting of 3 rows. Above this layer let us put 2 other layers exactly like it. How many feet is it across the front of the pile? How wide, then, is the pile? How high? How deep?







The pile thus formed being 3 feet each way, there are 9 sq. ft. on the top, but each layer is a foot deep, so that each must contain 9 cu. ft., or 27 cu. ft. in the three layers. Hence, *the product of the number of linear units in length, breadth and thickness is the number of cubic units in volume.*

TABLE.

1728 cubic inches (cu. in.)	= 1 cubic foot, cu. ft.
27 cubic feet	= 1 cubic yard, cu. yd.
16 cubic feet	= 1 cord foot, cd. ft.
8 cord feet 128 cubic feet	= 1 cord, cd.

NOTE.—The cord foot is rarely used.

ORAL EXERCISES.

1. How many cubic feet in a block 2 feet wide, 3 feet long and 1 foot thick?

2. How many cubic feet in a block 3 feet wide, 3 feet long and 2 feet thick?

3. How many cubic inches in a block 3 feet wide, 3 feet long and 3 feet thick?

4. How many cubic inches in a brick 8 inches long, 4 inches wide and 2 inches thick?

5. How many cubic inches in a block 6 inches long, 6 inches wide and 6 inches thick?

6. How many cords in a pile of wood 24 feet in length, 4 feet in width and 6 feet in height?

7. In 108 cubic feet how many cubic yards?

8. What part of a solid foot is a block 4 inches high, 6 inches wide and 6 inches long?

WRITTEN EXERCISES.

1. In 10 cubic feet, how many cubic inches?

2. In $337\frac{1}{2}$ cubic feet, how many cubic yards?

3. A pile of wood 12 feet long, 4 feet wide and 1 foot high contains what part of a cord?

4. A pile of wood 20 feet long, 8 feet high and 4 feet wide contains how many cords?

5. In 10,896 cubic feet, how many cords?

6. Illustrate by an original problem the measurement of wood.

Measures of Capacity.

ART. 221.—**Capacity** means amount of room or space.

ART. 222. **Measures of Capacity** are divided into two classes, Dry Measure and Liquid Measure.

DRY MEASURE.

ART. 223.—**Dry Measure** is used in measuring grain, fruit, salt, and like articles.

TABLE.

2 pints (pt.)	=	1 quart, qt.
8 quarts	=	1 peck, pk.
4 pecks	=	1 bushel, bush.

The standard bushel is $18\frac{1}{2}$ inches in diameter and 8 inches deep, and contains 2,150.42 cubic inches.

In measuring grain, seeds and small fruits, the measure must be *even full;* but in measuring corn in the ear, apples and coarse vegetables, the measure must be *heaped.* A heaped bushel contains $\frac{5}{4}$ of an even or stricken bushel. One pint dry measure equals $1\frac{1}{6}$ pints liquid measure.

ORAL EXERCISES.

How many—

1. Pints in 3 quarts?
2. Pints in 2 pecks?
3. Quarts in 16 pints?
4. Pecks in 72 quarts?
5. Pecks in 128 pints?
6. Bushels in 256 pints?
7. Pints in 1 bushel?
8. Pints in 5 bushels?
9. Quarts in $3\frac{3}{4}$ bushels?
10. Pints in $4\frac{1}{2}$ pecks?

WRITTEN EXERCISES.

How many—

1. Quarts in $3\frac{1}{4}$ pecks?
2. Pints in $2\frac{1}{2}$ bushels?
3. Pints in 17 bush. 2 pks. 5 qts. 1 pt.?
4. Bushels in 4,862 pints?

5. How many quarts in $3\frac{3}{4}$ bushels?

6. How many pints in 1 bush. 1 pk. 1 qt.?

7. Show by an original problem in dry measure how to change lower to higher denominations.

8. Show by an original problem in dry measure how to change higher to lower denominations.

LIQUID MEASURE.

ART. 224.—Liquid Measure is used in measuring liquids.

TABLE.

4 gills (gi.)	=	1 pint, pt.
2 pints	=	1 quart, qt.
4 quarts	=	1 gallon, gal.
$31\frac{1}{2}$ gallons	=	1 barrel, bbl.
63 gallons	=	1 hogshead, hhd.

The unit is the Winchester gallon, containing 231 cubic inches. The barrel and hogshead vary in capacity.

ORAL EXERCISES.

How many—

1. Gills in 5 pints?
2. Quarts in 1 hogshead?
3. Gills in 3 quarts?
4. Gills in 3 gallons?
5. Gallons in 128 pints?
6. Hhd. in 378 quarts?

WRITTEN EXERCISES.

How many—

1. Gills in 1 hhd. 2 qts. 1 pt. 3 gi.?
2. In 3 hhd. 1 pt. 1 gi.?
3. Hhd. in 252 quarts?
4. Barrels in 1,295 pints?
5. Hhd. in 1,800 gi.?
6. Hhd. in 10,000 pints?

7. Show by an original problem in liquid measure how to change lower to higher denominations.

8. Show by an original problem in liquid measure how to change higher to lower denominations.

CIRCULAR MEASURE.

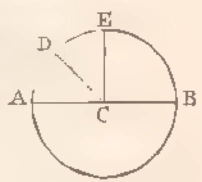
ART. 225.—**Circular Measure** is used in measuring angles, latitude and longitude, the motions of the heavenly bodies, etc. It is sometimes called Angular Measure.

TABLE.

60 seconds (″)	= 1 minute, ′
60 minutes	= 1 degree, ° or deg.
30 degrees	= 1 sign, S.
12 signs or 360 degrees	= 1 circumference.

The unit of Circular Measure is the degree.

The figure in the margin represents a circle. The curved line is the circumference. The distance from one side to the other, through the center, is the diameter.



ART. 226.—An **Arc** is any part of a circumference. Half a circumference is a semi-circumference, $\frac{1}{4}$ is a quadrant, $\frac{1}{6}$ a sextant, and $\frac{1}{12}$ a sign.

There are 360 degrees in every circumference, hence, the length of a degree depends upon the size of the circle. A degree of longitude at the equator, and the average length of a degree of latitude adopted by the United States Coast Survey is 69.16 statute miles. At latitude 30° a degree of longitude is equal to 59.89 miles; at 60° to 34.58 miles.

ART. 227. **Signs** are used in astronomy as measures of the Zodiac.

ART. 228. A **Minute** of the earth's circumference is called a geographical or nautical mile, and equals 1.152$\frac{2}{3}$ common miles.

ORAL EXERCISES.

How many:

1. Seconds in a degree? **2.** Minutes in a sign?

3. Degrees in 3,600 seconds?

4. Degrees in a semi-circumference? In a quadrant? In a sign?

WRITTEN EXERCISES.

How many:

1. Degrees in 12,000 seconds? **3.** Degrees in 28,464″?

2. Seconds in 18′ 18″? **4.** Seconds in 5 S. 13° 15′?

5. Seconds in 1 S. 11° 11′ 11″?

6. Show by an original problem in circular measure how to change lower to higher denominations.

7. Show by an original problem in the same measure how to change higher to lower denominations.

TIME.

ART. 229.—Time is a portion of duration.

TABLE.

60 seconds (sec.)	= 1 minute, min.
60 minutes	= 1 hour, h.
24 hours	= 1 day, d.
7 days	= 1 week, w.
365 days or 12 calendar months	= 1 common year, yr.
366 days	= 1 leap year.
100 years	= 1 century.

The unit of time is the day. One rotation of the earth on its axis makes a day. This rotation causes sunrise, day, sunset and night. The revolution of the earth around the sun produces the year and causes the change of seasons.

ART. 230.—The **Sidereal Day** is the exact time of the rotation of the earth on its axis.

ART. 231.—The **Solar Day** is the time of the apparent revolution of the sun around the earth.

ART. 232.—The **Astronomical Day** is the solar day, beginning and ending at noon.

ART. 233.—The **Civil Day** is the average or mean solar day. It is the day adopted by the government for business purposes. In nearly all countries it begins and ends at midnight, and the halves are divided into 12 hours each.

All the hours between midnight and noon are designated as A.M. (*ante meridiem*, before meridian, or noon); noon by M. (meridian); afternoon by P.M. (*post meridiem*, after meridian, or afternoon).

Occasional attempts have been made to name the hours without regard to the division of the day into two equal parts. Thus 7 o'clock P.M. would be called 19 o'clock. The attempt, however, has met with little success, and is not likely to prevail.

The months in the year, and the number of days in each month, are as follows:

January,	31 days.	July,	31 days.
February,	28 days or 29 days during leap year.	August,	31 days.
March,	31 days.	September,	30 days.
April,	30 days.	October,	31 days.
May,	31 days.	November,	30 days.
June,	30 days.	December,	31 days.

A true year, or the time of one revolution of the earth around the sun, is 365 days 5 hours 48 minutes 46.15 seconds. It will be noticed, therefore, that the common year, or 365 days, is very nearly a quarter of a day shorter than the true year. To correct this, every 4 years 1 day is added to February, the shortest month, such year being known as leap year.

This, however, is slightly in excess of the time needed, and in 105 years it amounts to more than three-fourths of a day. To equalize matters, the extra day is added only to every fourth centennial year. (A centennial year is one whose expression in figures ends with 2 ciphers.)

Every year exactly divisible by 4, except the centennial years, is a leap year, and every centennial year exactly divisible by 400 is a leap year: the others are common years. Thus, A.D. 1600 and A.D. 2000 are leap years, but A.D. 1700, A.D. 1800, and A.D. 1900 are common years.

ART. 234.—To Reduce Denominate Fractions or Decimals to Whole Numbers; and to Reduce Denominate Numbers to Fractions or Decimals.

1. Reduce .07964 sq. mi. to integers of lower denominations.

Process.

```
         .07964 sq. mi.
            640
         318560
        47784
   A. 50.96960
          160
       5817600
       96960
sq. rd. 155.13600
           30¼
           34
          408
   sq. yd. 4.114
             9
   sq. ft. 1.026
           144
           104
          104
          26
  sq. in. 3.744
```

Analysis.—Since there are 640 A. in 1 sq. mi., in .07964 sq. mi., there are .07964 times 640 A. = 50.9696 A. Since there are 160 sq. rd. in 1 A., in .9696 A. there are .9696 times 160 sq. rd. = 155.136 sq. rd. Since there are 30¼ sq. yd. in 1 sq. rd., in .136 sq. rd. there are .136 times 30¼ sq. yd. = 4.114 sq. yd. Since there are 9 sq. ft. in 1 sq. yd., in .114 sq. yd. there are .114 times 9 sq. ft. = 1.026 sq. ft. Since there are 144 sq. in. in 1 sq. ft., in .026 sq. ft. there are .026 times 144 sq. in. = 3.744 sq. in. Therefore, .07964 of a sq. mi. = 150 A. 155 sq. rd. 4 sq. yd. 1 sq. ft. 3.744 sq. in.

Reduce to integers of lower denominations:

2. .076438 of a T.

3. .468754 of a bush.

4. .87468 of a mi.

5. .61478 of a gal.

6. .46865 of an A.

7. .59846 of a lb. Troy.

8. Reduce $\frac{7}{11}$ of a mile to integers of lower denominations.

Process.

$\frac{7}{11}$ mi. $= \frac{7}{11}$ of 320 rd. $= \frac{2240}{11}$ rd. $= 203\frac{7}{11}$ rd.
$\frac{7}{11}$ rd. $= \frac{7}{11}$ of $5\frac{1}{2}$ yd. $= \frac{7}{2}$ yd. $= 3\frac{1}{2}$ yd.
$\frac{1}{2}$ yd. $= \frac{1}{2}$ of 3 ft. $= \frac{3}{2}$ ft. $= 1\frac{1}{2}$ ft.
$\frac{1}{2}$ ft. $= \frac{1}{2}$ of 12 in. $= \frac{12}{2}$ in. $=$ 6 in.

Analysis.—Since there are 320 rd. in 1 mi., in $\frac{7}{11}$ mi. there are $\frac{7}{11}$ of 320 rd. $= 203\frac{7}{11}$ rd. Since there are $5\frac{1}{2}$ yd. in 1 rd., in $\frac{7}{11}$ rd. there are $\frac{7}{11}$ of $5\frac{1}{2}$ yd. $= 3\frac{1}{2}$ yd. Since there are 3 ft. in 1 yd., in $\frac{1}{2}$ yd. there is $\frac{1}{2}$ of 3 ft. $= 1\frac{1}{2}$ ft. Since there are 12 in. in 1 ft., in $\frac{1}{2}$ of a ft. there is $\frac{1}{2}$ of 12 in. $=$ 6 in.

Therefore, $\frac{7}{11}$ of a mile $=$ 203 rd. 3 yd. 1 ft. 6 in.

Reduce to integers of lower denominations:

9. $\frac{7}{9}$ of a ton.
10. $\frac{8}{13}$ of a hhd.
11. $\frac{5}{6}$ of a yr.
12. $\frac{9}{13}$ of lb. Troy.
13. $\frac{8}{15}$ of a sovereign.
14. $\frac{4}{7}$ of an acre.

15. Reduce 264 rd. 3 yd. 1 ft. 6 in. to the decimal of a mile.

Process.

```
        12)6.0 in.
          3)1.5 ft.
  5½ |    3.5 yd.
   2 |      2
  11 |    7.0
 320 |264.63 7/11
  11 |     11
3520)2911.00(.826 87/88 mi.
     28160
       9500
       7040
      24600
      21120
Rem.   3480
```

Analysis.—Since 1 in. is $\frac{1}{12}$ of a ft., 6 in. are $\frac{6}{12}$ ft. $=$ .5 ft., to which the 1 ft. given is prefixed $=$ 1.5 ft. Since 1 ft. is $\frac{1}{3}$ of a yd., 1.5 ft. is $\frac{1}{2}$ of a yd. $=$ .5 yd., to which the 3 yd. given are prefixed $=$ 3.5 yd. Since 1 yd. is $\frac{2}{11}$ of a rd., 3.5 yd. are $\frac{7}{11}$ rd. $= .63\frac{7}{11}$ rd., to which the 264 rd. given are prefixed $= 264.63\frac{7}{11}$ rd. Since 1 rd. is $\frac{1}{320}$ of a mile, $264.63\frac{7}{11}$ rd. are $\frac{2911}{3520}$ of a mile $= .826\frac{87}{88}$ mi. Therefore, 264 rd. 3 yd. 1 ft. 6 in. $= .826\frac{87}{88}$ mi.

Reduce to a decimal of the highest denomination:

16. 14 cwt. 9 lb. 12 oz.
17. 3 pk. 7 qts. 1 pt.
18. 89 sq. rd. 8 sq. yd. 8 sq. ft.
19. 124 rd. 4 yd. 2 ft. 8 in.
20. 56 gal. 3 qts. 1 pt. 2 gi.
21. 87° 29′ 52″.

22. What part of a ton is 14 cwt. 72 lb. 12 oz.?

Process.

12 oz. = $12 \times \frac{1}{16}$ lb. = $\frac{3}{4}$ lb.
72 lb. + $\frac{3}{4}$ lb. = $72\frac{3}{4}$ lb.
$72\frac{3}{4}$ lb. = $72\frac{3}{4} \times \frac{1}{100}$ cwt. = $\frac{291}{400}$ cwt.
14 cwt. + $\frac{291}{400}$ cwt. = $14\frac{291}{400}$ cwt.
$14\frac{291}{400}$ cwt. = $\frac{5891}{400} \times \frac{1}{20}$ T. = $\frac{5891}{8000}$ T.

Analysis.—Since 1 oz. is $\frac{1}{16}$ of a lb., 12 oz. are $\frac{12}{16}$ lb. = $\frac{3}{4}$ lb., to which the 72 lb. given are added. Since 1 lb. is $\frac{1}{100}$ of a cwt., $72\frac{3}{4}$ lb. are $\frac{291}{400}$ cwt., to which the 14 cwt. given are added = $14\frac{291}{400}$ cwt. Since 1 cwt. is $\frac{1}{20}$ of a ton, $14\frac{291}{400}$ cwt. is $\frac{5891}{8000}$ of a ton. Therefore, 14 cwt. 72 lb. 12 oz. = $\frac{5891}{8000}$ T.

Reduce to a fraction of the highest denomination:

23. 3 pk. 7 qts. 1 pt.
24. 80° 14′ 18″.
25. 36 sq. rd. 15 sq. yd.
26. 185 rd. 4 yd. 2 ft.
27. 16 gal. 3 qts. 1 pt. 3 gi.
28. 15s. 6d. 3 far.

Addition of Denominate Numbers.

1. What is the sum of 12 lb. 8 oz. 11 pwt. 9 gr.; 8 lb. 7 oz. 8 pwt. 8 gr.; 21 lb. 6 oz. 10 pwt.; 27 lb. 5 oz. 21 gr.?

Process.

lb.	oz.	pwts.	gr.
12	8	11	9
8	7	8	8
21	6	10	0
27	5	0	21
70	3	19	14

Analysis.—We write like denominations in the same column, and begin with the lowest denomination to add.

21 gr. 8 gr. and 9 gr. are 38 gr., equal to 1 pwt. and 14 gr. We write 14 under the column of gr. and add 1 pwt. to the column of pwt.

20 pwt. 8 pwt. 11 pwt. are 39 pwt., equal to 1 oz. and 19 pwt. We write 19 under the column of pwt., and add 1 oz. to

the column of oz. 6 oz. 6 oz. 7 oz. and 8 oz. are 27 oz., equal to 2 lb. and 3 oz. We write 3 under the column of oz., and add 2 to the column of lb. 29 lb. 21 lb. 8 lb. and 12 lb. are 70 lb., which we write under the column of pounds.

2. Find the sum of £18 11s. 7d.; £110 18s. 9d.; £188 13s. 1d.; £99 2s. 5d. 3 far.; £90 8d. 2 far.

3. What is the sum of 18 mi. 88 rds. 5 yds. 1 ft. 9 in.; 19 mi. 5 yds. 2 ft. 11 in.; 74 mi. 200 rds. 4 yds. 2 ft. 6 in.; 144 rds. 5 yds. 7 in.?

4. What is the sum of 18 bush. 3 pks. 6 qts.; 23 bush. 4 qts.; 11 bush. 1 pk. 7 qts. 1 pt.; 3 pks. 1 pt.; 40 bush. 4 qts. 1 pt.; 29 bush. 6 qts. $1\frac{1}{2}$ pts.; 77 bush. 2 pks. 4 qts.?

5. Add 15° 28′ 47″; 24° 36′ 52″; 5 S. 20° 51′ 11″; 9 S. 15° 34′ 54″.

6. Add 3 A. 110 sq. rds. 10 sq. yds.; 4 A. 3 sq. rds. 11 sq. yds.; 120 sq. rds. 20 sq. yds.

7. Add $\frac{1}{7}$ of a mile and $\frac{5}{6}$ of a rod.

Process.

$\frac{1}{7}$ mi. = $\frac{1}{7}$ of 320 rd. = $45\frac{5}{7}$ rd.
$\frac{5}{7}$ rd. = $\frac{5}{7}$ of $\frac{11}{2}$ yd. = $3\frac{13}{14}$ yd.
$\frac{13}{14}$ yd. = $\frac{13}{14}$ of 3 ft. = $2\frac{11}{14}$ ft.
$\frac{11}{14}$ ft. = $\frac{11}{14}$ of 12 in. = $9\frac{3}{7}$ in.

$\frac{5}{6}$ rd. = $\frac{5}{6}$ of $\frac{11}{2}$ yd. = $4\frac{7}{12}$ yd.
$\frac{7}{12}$ yd. = $\frac{7}{12}$ of 3 ft. = $1\frac{3}{4}$ ft.
$\frac{3}{4}$ ft. = $\frac{3}{4}$ of 12 in. = 9 in.

	rd.	yd.	ft.	in.
	45	3	2	$9\frac{3}{7}$
		4	1	9
	46	2($\frac{1}{2}$)	1	$6\frac{3}{7}$
$\frac{1}{2}$ yd. =			1	6
$\frac{1}{2}$ yd. =	46	3	0	$\frac{3}{7}$

Analysis.—Reduce the denominate fractions to integers of lower denominations, and add. If a fraction of a higher denomination occurs, as the $\frac{1}{2}$ yd. in this example, it must be reduced to integers of lower denominations and the result must be changed accordingly.

8. Add $\frac{1}{8}$ of a ton to $\frac{1}{4}$ of a pound.

9. Add $\frac{1}{6}$ A., $\frac{1}{4}$ sq. rd., $\frac{3}{4}$ sq. yd.

10. Illustrate addition of denominate numbers by a problem in long measure.

11. Illustrate addition of denominate numbers by a problem in cubic measure.

Subtraction of Denominate Numbers.

ART. 235.—1. From 33 gal. 2 qts. 1 pt. 3 gi. take 27 gal. 3 qts. 1 pt. and 2 gi.

Process.

gal.	qts.	pts.	gi.
33	2	1	3
27	3	1	2
5	3	0	1

Analysis.—We write like denominations in the same column and begin with the lowest to subtract. 2 gi. from 3 gi. leaves 1 gi., which we write under the column of gills. 1 pt. from 1 pt. leaves 0 pts., which we write under the column of pts. 3 qts. cannot be taken from 2 qts., so we increase 2 qts. by 1 gal., making 6 qts. 3 qts. from 6 qts. leaves 3 qts., which we write under the column of qts. Since we increased the minuend by 1 gal., we increase the subtrahend by 1 gal. 28 gal. from 33 gal. leaves 5 gal., which we write under the column of gallons.

Subtract:

2. 121 lb. 1 oz. 11 pwt. 17 gr. from 144 lb. 2 oz. 10 pwt. 18 gr.

3. 99 hhd. 24 gal. 3 qts. from 100 hhd. 20 gal. 1 pt.

4. 180 mi. 220 rds. 4 yds. 1 ft. 9 in. from 202 mi. 218 rds. 3 yds. 1 ft. 8 in.

5. A jeweler having 11 lb. of gold, used 1 lb. 2 oz. 5 pwt. 5 gr. in making watch chains, and 7 lb. 9 oz. 9 pwt. 20 gr. in other articles: how much gold was left?

6. General U. S. Grant died July 23, 1885, and General G. B. McClellan, October 29 of the same year: what is the difference between the two dates?

7. Sir Moses Montefiore was born October 24, 1784, and died July 28, 1885: what was his age when he died?

8. The steamship *Alarm* left New York at 4.15 P.M. December 19, 1885, and reached the Galveston bar at midnight December 25: what was the time occupied in the passage?

9. Alaska was ceded by Russia to the United States, June 20, 1867: how long was that after the discovery of America by Columbus, October 12, 1492?

10. How many years between the breaking out of the American Revolution, April 19, 1775, and the declaration of war against Great Britain, June 19, 1812?

11. War was declared against Mexico, May 13, 1846, and closed February 2, 1848: how long did the war last?

12. The war for the Union began at Fort Sumter, April 14, 1861, and General Lee surrendered April 12, 1865: how long did the war last?

13. The Military Academy at West Point was founded by Congress, March 16, 1802, and the Naval Academy at Annapolis was opened October 10, 1845: how much time between the dates?

14. The first passenger railroad was opened in England, September 27, 1825: how long was that after the arrival of the *Savannah* (the first steamer which crossed the Atlantic) at Liverpool, May 24, 1819?

15. The first public telegram that ever passed over a wire was sent from Baltimore to Washington, May 27, 1844: how long ago was it?

16. Washington was inaugurated first President April 30, 1789: how long was that previous to the first success of the telephone, May 14, 1877?

17. How long after the surrender of Cornwallis at Yorktown, October 19, 1781, was fought the battle of Waterloo, June 18, 1815?

18. Illustrate by an original problem subtraction of denominate numbers.

Multiplication of Denominate Numbers.

ART. 236.—1. Multiply 15 gal. 2 qts. 1 pt. by 5.

Process.

gal.	qts.	pts
15	2	1
		5
78	0	1

Analysis.—5 times 1 pt. is 5 pts., equal to 2 qts. and 1 pt. We write 1 in the place of pts. and add 2 to the product of qts. 5 times 2 qts. are 10 qts., and 2 qts. added are 12 qts., equal to 3 gal. and 0 qts. We write 0 in the place of qts., and add 3 to the product of gal. 5 times 15 gal. are 75 gal., and 3 gal. added are 78 gal., which we write in the place of gal.

Multiply:

2. 17 cwt. 75 lb. 11 oz. by 5.

3. 15 lb. 9 oz. 14 pwts. 18 gr. by 7.

4. 18 hhd. 33 gal. 3 qts. 1 pt. by 8.

5. 16 mo. 8 d. 18 h. 32 min. 30 sec. by 10.

6. 11 ℔ 8 ℥ 6 ʒ 2 ℈ 16 gr. by 12.

7. How much wood is there in 9 piles, each containing 11 C. 80 cu. ft. 23 cu. in.?

8. If 30 gal. 3 qts. 1 pt. of water runs into a hogshead every hour and 10 gal. 2 qts. 1 pt. leaks out every hour, how much will the hogshead lack of being full at the end of three hours?

9. If Lemuel walked 19 mi. 128 rds. 4 yds. each day for 11 days, and Humphrey averaged 19 mi. 200 rds. 3 yds. for 12 days, how much farther did Humphrey walk than Lemuel?

10. Illustrate multiplication of denominate numbers by an original problem in square measure.

11. Illustrate the same subject by an original problem in time measure.

Division of Denominate Numbers.

ART. 237.—1. Divide 42 bush. 3 pks. 4 qts. 1 pt. by 5.

Process.

	bu.	pks.	qts.	pt.
5)	42	3	4	1
	8	2	2	1

Analysis.—$\frac{1}{5}$ of 42 bush. is 8 bush., and 2 bush. remaining = 8 pks. 8 pks. and 3 pks. = 11 pks. $\frac{1}{5}$ of 11 pks. is 2 pks. and 1 pk. remaining = 8 qts. 8 qts. and 4 qts. = 12 qts. $\frac{1}{5}$ of 12 qts. is 2 qts. and 2 qts. remaining = 4 pts. 4 pts. and 1 pt. = 5 pts. $\frac{1}{5}$ of 5 pts. = 1 pt.

Divide:

2. £45 16s. 7d. by 4.

3. 54 lb. 11 oz. 12 pwts. 10 grs. by 5.

4. 118 T. 16 cwt. 81 lb. by 8.

5. 94 hhd. 38 gal. 3 qts. 1 pt. 3 gi. by 9.

6. If a pile of wood containing 12 cords 96 cu. ft. be equally divided among 5 persons, how much will each receive?

7. If George picks 3 bush. 3 pks. 7 qts. of apples a day, how many days will it take him to pick 43 bush. 2 pks. 5 qts.?

8. If 28 cwt. 16 lb. of sugar is put up in packages, each containing 2 lb. 8 oz., how many packages will there be?

9. If three boys gather daily 2 gal. 3 qts. 1 pt. of sap from each of 4 trees, how long will it take them to gather $2\frac{1}{3}$ hogsheads of sap?

10. Illustrate division of denominate numbers by an original problem in cubic measure.

11. Illustrate the same subject by a problem in which the dividend and divisor are denominate numbers.

LATITUDE AND LONGITUDE.

ART. 238. The **Equator** is an imaginary line, passing around the earth midway between the north and south poles.

ART. 239. The **Latitude** of a place is its distance north or south of the equator. Northward 90° from the equator is the north pole, and southward 90° from the equator is the south pole: consequently, no place on the globe can have more than 90° of latitude.

ART. 240. A **Meridian** is a semi-circumference of a great circle passing around the earth through the poles.

ART. 241. The **Longitude** of a place is its distance east or west of a certain meridian.

ART. 242. Latitude and longitude are reckoned in degrees, minutes and seconds.

Since 360° comprise the circumference of the earth, and since longitude is numbered both east and west from a given meridian, the greatest longitude a place can have is 180°.

The system of reckoning longitude in use in most parts of the world, is attended with great inconvenience. Our school atlases, as a rule, give the longitude from Washington at the top of the page, and that from Greenwich, in England, at the bottom. Germany, besides Greenwich, uses for her topographical maps, the longitude of Berlin; Russia uses Greenwich, Paris, and St. Petersburg; the Netherlands employ Greenwich and Amsterdam; France reckons from Paris only; Denmark from Copenhagen; Spain uses San Fernando; while Italy uses San Fernando, Turin, and Milan.

So urgent has become the need for a single prime

meridian for the world, that an International Conference was convened in Washington in October, 1884, for the purpose of fixing upon a meridian. There were forty delegates present, representing Austria, Hungary, Brazil, Colombia, Costa Rica, Denmark, France, Germany, Great Britain, Guatemala, Hawaii, Italy, Japan, Mexico, Netherlands, Paraguay, Russia, San Domingo, San Salvador, Spain, Sweden and Norway, Switzerland, Turkey, Venezuela, and the United States. A resolution was adopted providing that longitude shall be counted from the meridian of Greenwich in two directions up to 180° east longitude plus, and west longitude minus.

The Conference also adopted a universal day for the purposes for which it may be found convenient. This day does not interfere with the use of local or other standard times where desirable.

This universal day is to be a mean solar day, and is to begin for all the world at the moment of mean midnight at the initial meridian. It coincides with the beginning of the civil day at that meridian, and is to be counted from zero up to twenty four hours.

The earth turns entirely around every 24 hours, causing day and night. Since each circle contains 360 degrees, every part of the earth passes through 360 degrees in 24 hours. Hence, in 1 hour it passes through $\frac{1}{24}$ of 360°, or 15°.

It follows, therefore, that if the position of the sun is noted at any moment it will be 15° farther west an hour later, appearing to descend the sky at that rate until it disappears below the horizon, to reappear later in the east and continue its ceaseless round.

Hence, for every 15° we move eastward, the time will be found 1 hour later, and the reverse prevails as we move westward.

From these facts we can readily ascertain the difference of time between two places whose longitude is given, and

can calculate the difference in longitude when the difference in time is known.

All know the inconvenience of computing time as indicated. A person traveling eastward or westward finds his watch continually too slow or too fast, even though he sets it every few miles of his journey. This is especially felt since the increased facilities in railroad traveling of the last few years, the variation in a single day amounting almost to an hour.

The Pacific steamers sailing westward are obliged to drop a day, which is "picked up" on the return. Thus, in going from San Francisco to China, one of the weeks, of necessity, contains only 6 days, while in coming back, a certain week must contain 8 days.

STANDARD TIME.

ART. 243.—The great inconvenience attending the ordinary division of time led to the adoption throughout the United States in 1884 of what is known as *Standard Time.*

By general agreement and partial legislation, four standard meridians were selected, by which railway trains are run and local time regulated. These meridians are 15 degrees apart. On the equator they are 69.16 × 15 miles apart. There is a difference of just one hour in time from one meridian to another, since in the earth's circumference there are 360°, which divided by 24 hours gives 15 degrees to an hour.

The Eastern meridian, 75° W. from Greenwich, passes 4' west of New York City. The Central meridian, 90° W. longitude, passes near New Orleans and St. Louis. The Mountain standard meridian, 105° W. longitude, passes near Pike's Peak, Rocky Mountains. The Pacific meridian, 120° W. longitude, is near the coast, San Francisco being $9\frac{1}{2}$ miles west of it.

Standard time has been only partially adopted, and in

many cases the boundaries have been changed. Thus, the Central and Mountain divisions vary in width from 8° to 20°.

In all places in the Eastern Division the time is the same. Passing westward, the time throughout the Central Division is just one hour earlier; the Mountain Division time is two hours earlier, and that of the Pacific Division three hours earlier.

ORAL EXERCISES.

1. Where do we begin to reckon latitude?

2. Where do we begin to reckon longitude?

3. What is the latitude of the south pole? Of the north pole?

4. Find out from your maps, the latitude of Quito, South America.

5. Find out what place on the globe has neither latitude nor longitude. How many such places are there?

6. There is a point on the earth from which if you travel southward in a direct line for 1,000 miles, then eastward 1,000 miles, and finally in a direct line northward 1,000 miles, you will arrive at the point from which you started: name the point.

7. What is the latitude of a place lying $\frac{1}{2}$ of the way between the equator and the north pole? $\frac{1}{3}$ of the way? $\frac{1}{4}$ of the way? $\frac{2}{3}$ of the way? $\frac{1}{5}$ of the way? $\frac{3}{5}$ of the way?

8. Of what use are latitude and longitude?

9. If Mr. Brown travels 180° east, and Mr. Jones, starting from the same point, journeys 180° west, how many degrees will their positions differ in longitude?

10. If you travel 200° east from Greenwich, in what longitude will you be? If you travel 195° west, in what longitude will you be?

11. What difference of time is caused by 60° longitude? By 75°? By 90°? By 180°? By 15°? By 30°? By 45°?

12. What difference in longitude produces a difference of 2 hours in time? 5 hours? 3 hours? 7 hours? 1 hour? 30 minutes? 15 minutes? 10 minutes? 5 minutes?

13. When it is noon in Boston, ascertain from your geography what is the hour in Philadelphia, reckoning both by Standard Time? What is the hour in Harrisburg? In Cincinnati? In Chicago? In Denver? In St. Louis? In San Francisco?

WRITTEN EXERCISES.

TABLE.

15°	in longitude	makes	1 hour	difference	in time.
15′	"	"	1 minute	"	"
15″	"	"	1 second	"	"
1°	"	"	4 minutes	"	"
1′	"	"	4 seconds	"	"

1. The difference in longitude between two places is 28° 40′ 30″: what is the difference in time?

Process.

$$15)\underline{28°\ 40'\ 30''}$$
$$1°\ 54'\ 42''$$

Analysis.—Since a difference of 15° longitude makes a difference of 1 hour in time, 15′ longitude 1 minute, and 15″ longitude 1 second, then 28° 40′ 30″ will make a difference of as many hours, minutes, and seconds, as 15 is contained times in 28° 40′ 30″. Dividing as in denominate numbers we obtain 1 h. 54 min. 42 sec. Therefore, etc.

ART. 244.—Rule for finding the difference in time between two places whose longitude is given.—*Divide the difference of longitude in degrees, minutes and seconds, by 15, and the quotient will express the difference in hours, minutes, and seconds.*

2. Berlin lies in 13° 23′ 44″ E. longitude and New Orleans 90° W. longitude: what is the difference in time?

3. The longitude of Chicago is 87° 36′ 42″ W. and that of London 5′ W.: required their difference in time.

4. San Francisco is 122° 24′ 40″ W. and St. Petersburg 30° 18′ 23″ E.: what is their difference in time?

5. Paris is 2° 20′ 9″ E.; Omaha 96° W. When it is midnight in Paris, what is the time in Omaha?

6. The longitude of Rome is 12° 27′ 14″ E. and that of Washington 77° 2′ 48″ W. At noon in Washington what is the time in Rome?

7. Mr. Smith's watch was right when he left Boston, but when he reached Cincinnati it was 53 minutes 25 seconds fast: what is the difference in longitude between Boston and Cincinnati?

Process.

	min.	sec.
	53	25
		15
13 h.	21 min.	15 sec.

representing 13° 21′ 15″

Analysis.—Since a difference of 1 hour of time is caused by a difference of 15° of longitude, and 1 minute's difference of time is caused by 15′ difference of longitude, and 1 second's difference of time by 15″ difference of longitude, then 15 times the number of hours, minutes, and seconds of difference in time is equal to the number of degrees, minutes and seconds difference in longitude. 53 minutes and 25 seconds multiplied by 15 equals 13 h. 21 min. 15 seconds, which represents 13° 21′ 15″, the difference in longitude between Cincinnati and Boston.

ART. 245. Rule for finding the difference of longitude between two places when the difference of time is known.—*Multiply the difference in time by* 15, *and the product will represent the difference in longitude.*

8. When it is 5 o'clock A. M. in Philadelphia, it lacks 2 hours of noon in London: what is the difference in longitude?

9. The difference between the time in New York city and St. Louis is 1 hour $2\frac{1}{3}$ minutes: what is the difference in longitude?

10. On reaching San Francisco from New York, I found my watch 3 hours 14 minutes 8 seconds too fast: what is the longitude of New York, that of San Francisco being 122° 24′ 40″?

11. Illustrate by an original example the method of finding the difference of time between two places.

12. Illustrate by an original example the method of finding the difference of longitude between two places.

Practical Applications of Measurements.

SQUARE MEASURE.

ART. 246. The **Area of a Rectangle** equals the length multiplied by the breadth.

It follows, therefore, that the

Area ÷ breadth = length. | Area ÷ length = breadth.

WRITTEN EXERCISES.

Find the area of the rectangles:

1. 16 in. by 18 in. **3.** $12\frac{1}{2}$ ft. by $10\frac{1}{3}$ ft.

2. 25 ft. by 16 ft. **4.** 24 ft. 8 in. by 18 ft. 4 in.

SUGGESTION.—24 ft. 8 in. = $24\frac{2}{3}$ ft.
18 ft. 4 in. = $18\frac{1}{3}$ ft.

5. 8 yds. 2 ft. 6 in. by 4 yds. 1 ft. 8 in.

SUGGESTION.—8 yds. 2 ft. 6 in. = 318 in.
4 yds. 1 ft. 8 in. = 164 in.

6. A certain school had 5 blackboards, each 9 feet 4 inches long, and 3 feet wide: how many square feet of blackboard were in the room?

7. What will it cost to slate a roof 36 feet long and 24 feet wide, at $4.50 a square yard?

8. How many square yards of carpeting will cover the floor of a room 20 feet long and 15 feet wide?

9. At $1.10 per square yard, what will it cost to carpet a room 24 feet 3 inches long and 12 feet 6 inches wide?

10. How many planks 14 feet long and 8 inches wide will be required to floor a room 32 feet long and 28 feet wide?

11. What will it cost to glaze 8 windows, each 8 feet 4 inches by 5 feet 3 inches, at $.80 a square foot?

12. Find the cost of slating a roof 48 feet 9 inches long and 40 feet wide, at $12.50 per square yard.

13. How much will it cost to sod a yard 10 rods long and 35 feet wide, at 7 cents a square yard?

14. What will it cost to cement a cellar floor 37 ft. 3 in. long by 26 ft. 8 in. wide, at $.40 a square yard?

15. A certain pavement is 300 feet long, and contains 250 square yards: what is its width?

16. What part of an acre is a plot of ground 118 yards long and 80 feet wide?

17. Mr. Jones's parlor is 34 feet long and 16 feet 8 inches wide: what will it cost to cover the floor with Brussels carpet $\frac{3}{4}$ yard wide, at $2.62½ per yard?

18. A dining room is 15 feet 8 inches long and 12 feet 4 inches wide. The mantel projects 1 foot into the room for a distance of 5 feet; the register occupies a space on the floor $2\frac{1}{2}$ feet long, and 1 foot 3 inches wide: how much will it cost to oil and polish the floor at 18 cents a square yard?

19. I papered my sitting room, 18 feet long 12 feet wide and 9 feet 6 inches high. The room had two windows, 3 feet wide and 8 feet high; two doors, 3 feet wide and 9 feet high; and the wash-board extended 6 inches from the floor: what was the expense, if each piece of paper was 14 feet long 18 inches wide and cost 12 cents a piece?

20. The staging erected at a certain school exhibition was 24 feet 6 inches long, and it required 477¾ square feet of pine boards: how deep was the stage?

21. How many acres in a field 140 rods square?

22. What will it cost to cover with Lincrusta-Walton the walls of a room 18 ft. 6 in. long, 9 ft. high and 14 ft. 9 in. wide, at $3.75 a square yard, allowing 72 square feet for doors and windows?

23. How many yards of chintz $\frac{7}{8}$ of a yard wide will it take to line $28\frac{1}{2}$ yards of cloth $\frac{3}{4}$ of a yard wide?

24. What will it cost to cover the floor of an office with linoleum at $1.25 a square yard, if the office is 24 feet long, and for half its length 8 feet wide, and for the rest 6 feet wide?

25. A rectangular piece of ground containing an acre is 264 feet long: how wide is it?

26. A rectangular field of 40 acres is 140 rods long: what is its width?

27. Illustrate by an original problem how to find the cost of carpeting a room.

28. Illustrate by an original example how to find the cost of plastering the four sides and ceiling of a room.

CUBIC MEASURE.

ART. 247.—A **Solid** has length, breadth and thickness.

ART. 248. A **Rectangular Solid** has 6 rectangular surfaces.

ART. 249.—The **Volume** of a cube or rectangular solid equals the product of the length, breadth and thickness or height.

It follows, therefore, that the

Volume ÷ (breadth × height) = length.

Volume ÷ (length × height) = breadth.

Volume ÷ (length × breadth) = thickness or height.

Masonry is generally estimated by the perch or cubic foot and sometimes by the square foot or yard.

A perch of masonry is $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide and 1 foot high, and contains $24\frac{3}{4}$ cubic feet. When stone is laid in the form of a wall, 22 cubic feet make a perch, an allowance of $2\frac{3}{4}$ cubic feet being made for mortar and filling.

NOTE 1.—Since a perch of masonry contains $24\frac{3}{4}$ cubic feet, the number of perches in any piece of masonry is found by dividing the contents in feet by $24\frac{3}{4}$.

NOTE 2.—The common size of a brick is 8 inches long, 4 inches wide, and 2 inches thick.

Excavations and embankments are estimated by the cubic yard; brick work by the thousand, and an allowance of $\frac{1}{10}$ to $\frac{1}{8}$ of the solid contents of a wall is made for mortar.

WRITTEN EXERCISES.

1. How many cubic feet in a block of marble 12 feet long, 6 feet wide and 5 feet thick?

2. What is the volume of a solid 6 feet 6 inches long, 4 feet 4 inches wide, and 6 feet 6 inches thick?

3. How many cubic yards can be taken from a cellar 30 feet long, 28 feet wide and $6\frac{3}{4}$ feet deep?

4. A certain pile of wood is 24 feet long, 4 feet wide and $8\frac{1}{2}$ feet high; another is 30 feet long, 4 feet wide and $6\frac{1}{2}$ feet high. If the owner sells the first pile at $\$4\frac{3}{4}$ a cord, and the second at $\$5\frac{1}{2}$ a cord, how much money will he receive?

5. What is the cost of digging and walling a cellar, whose length is 42 feet, width 36 feet and depth 8 feet, if the walls are $1\frac{1}{2}$ feet thick, the cost of excavating \$.60 a cubic yard, and the stone and mason work \$3.60 a perch?

6. How many perches of stone in a wall 40 rods long, 2 feet wide and 6 feet high, no allowance being made for mortar and filling?

7. How many cubic yards in a rectangular body 144 feet long, 9 feet wide and 4 feet thick?

8. How many cubic inches in a rectangular mass of stone 360 feet long, $12\frac{1}{2}$ feet wide and $10\frac{3}{4}$ feet thick?

9. With an average of 4.5 feet below grade, what will be the cost of filling in a street 480 feet long and 80 feet wide, if the cost of filling in is \$.50 a cubic yard?

10. How many cubic feet in a stick of timber 48 feet long, 10 inches thick, 14 inches wide at one end and 7 inches wide at the other end?

11. How many perches in a wall 12 feet high, 2 feet thick, 8 rods long?

12. How much will it cost at \$8.50 per thousand to furnish the bricks for a wall 120 feet long, 20 feet high and 18 inches thick, if $\frac{1}{11}$ is deducted for mortar?

13. How many board feet are in a board 18 feet long and 9 inches wide?

NOTE.—In the lumber business boards are assumed to be 1 inch thick, hence, the volume of such boards is represented by their surface measurement. Where the thickness exceeds 1 inch, $\frac{1}{4}$ is added to the cost for each increase of $\frac{1}{4}$ of an inch in the thickness. When the thickness is less than 1 inch, usually no deduction is made.

14. How many board feet in a plank 12 feet long, 10 inches wide and $2\frac{1}{2}$ inches thick?

15. How many board feet in a board 16 feet long, 1 inch thick, 18 inches wide at one end and 10 inches wide at the other?

NOTE.—When a board tapers uniformly its average width is one half the sum of the widths of the two ends.

16. A lumber dealer bought 18 three-inch planks, 24 feet long, and 18 inches wide, at $16.50 per M.: how much did they cost?

17. What will be the expense of flooring a room 18 feet by 24, with two-inch flooring at $20 per M., allowing $\frac{1}{8}$ for matching?

18. Illustrate the measurement of lumber by an original problem.

19. Illustrate by an original problem how to find the contents of a rectangular solid.

CAPACITY OF BINS, TANKS AND CISTERNS.

ART. 250. The capacity of bins for grain is commonly measured in bushels, and that of tanks, cisterns, etc., in gallons or barrels.

The standard bushel of the United States contains 2,150.42 cubic inches. The capacity of a bin, therefore, is found by dividing its contents in cubic inches by 2,150.42.

A liquid gallon contains 231 cubic inches, and a gallon dry measure 268.8 cubic inches. 6 dry gallons are nearly equal to 7 liquid gallons ($231 \times 7 = 1617$, and $268.8 \times 6 = 1612.8$).

Since a bushel contains about $1\frac{1}{4}$ cubic feet, the capacity of a bin in bushels may be considered as $\frac{4}{5}$ of the number of cubic feet it contains.

1. What is the capacity in gallons of a cistern 6 feet square and 5 feet deep?

2. How many bushels are contained in a bin 8 feet long, $4\frac{1}{2}$ feet wide and 4 feet deep?

3. What is the capacity in barrels ($31\frac{1}{2}$ gal.), gallons and quarts, of a tank $8\frac{1}{2}$ feet square and 8 feet high?

4. A bin holds 296 bushels of wheat: what is its volume in cubic feet?

5. How many bushels of corn will a bin hold having a capacity of 2,000 cubic feet?

6. A tank containing 150 cubic feet will hold how many gallons of water?

7. A cellar 30 feet long and 25 feet wide contains 2¼ feet of water: what will be the cost of pumping it out at 5 cents a hogshead?

8. A man bought 6 bushels of hickory nuts at $2.50 a bushel dry measure, and retailed them at 14 cents a quart liquid measure: what was his gain?

9. Illustrate by an original problem the method of determining the capacity of bins, tanks and cisterns.

THE MEASUREMENT OF HAY.

The following method, while not strictly accurate, is convenient in estimating the measurement of hay in bulk. There must of necessity be considerable variation in the weight of grass, according to its quality, degree of dryness and the method of packing.

Average meadow hay is estimated as follows:

One ton equals: 10 cubic yards in bales for shipping.
15 cubic yards in stacks.
20 cubic yards in barns and on wagons.
25 cubic yards in windrows.

1. How many tons in a pile of baled hay 20 yards long, 30 feet wide and 20 feet high?

2. What are the contents of a stack of hay that measures 16 feet by 9 feet and is 12 feet high?

3. What are the contents of a mow of clover hay 44 feet by 24 feet and 16 feet deep, allowing 580 feet to a ton?

NOTE. A ton of clover hay in the mow is estimated to measure from 550 to 640 cubic feet. A cube whose sides are 8 feet (512 cu. ft.), taken from a mow of ordinary hay is estimated to contain a ton.

4. A shipped 16 bales of hay, each 4 feet long, 3 feet wide and 2 feet thick: what was its value at $13.25 a ton?

5. Illustrate the measurement of hay by an original problem.

REVIEW QUESTIONS.

What is a denominate number? What is reduction descending? Reduction ascending?

What is English currency? What is the unit? What is its value in United States money?

What is French money? What is the unit? What is its value? Repeat the table.

What is said of the money of Canada?

What has the Empire of Germany done? How is the mark divided?

For what is avoirdupois weight used? Repeat the table.

For what is Troy weight used? Repeat the table.

How are medicines bought and sold?

Repeat the table of long measure.

Repeat the table of surveyors' measure.

Repeat the table of square measure.

What is an angle? The vertex? A right angle? What is a square? A rectangle? What is the area of a figure?

How is the volume of a rectangular solid found?

What is meant by capacity? Repeat the table of dry measure. What is the standard bushel? How does a heaped bushel compare with a stricken bushel?

Repeat the table of liquid measure.

Repeat the table of circular measure.

What is the average length of a degree of latitude?

What is time? Repeat the table.

How are the hours previous to noon designated? How is noon designated? Afternoon? Repeat the months in the year, and give their length.

What is the true length of a year? What centennial years are leap years?

How many units make a dozen? A gross? A great gross? How many sheets make a quire? A ream? A bundle? A bale?

What is meant by folio? A quarto or 4to? An octavo or 8vo? A duodecimo or 12mo? A 16mo? An 18mo? A 24mo?

What is latitude? What is longitude?

Knowing the difference in longitude between two places, how do you find the difference in time? Knowing the difference in time, how do you find the difference in longitude?

Explain Standard Time.

Percentage.

ART. 251.—Per centum or per cent. means *by the hundred.* One per cent. means one hundredth, expressed also $\frac{1}{100}$ or .01; five per cent., or five hundredths, is expressed $\frac{5}{100}$ or .05; ten per cent. is expressed $\frac{10}{100}$ or .10 or .1. Per cent. is usually written %.

Per cent. is expressed in several ways, thus:

5 per cent., or	5 %	= $\frac{5}{100}$	=	.05.
4 per cent., or	4 %	= $\frac{4}{100}$	=	.04.
15 per cent., or	15 %	= $\frac{15}{100}$	=	.15.
$12\frac{1}{2}$ per cent., or	$12\frac{1}{2}$%	= $\frac{25}{200}$	=	.125.
$6\frac{1}{4}$ per cent., or	$6\frac{1}{4}$%	= $\frac{25}{400}$	=	.0625.
$\frac{1}{2}$ per cent., or	$\frac{1}{2}$%	= $\frac{1}{200}$	=	.005.
$\frac{1}{4}$ per cent., or	$\frac{1}{4}$%	= $\frac{1}{400}$	=	.0025.
125 per cent., or	125 %	= $\frac{125}{100}$	=	1.25.

ART. 252.—Every number is $\frac{2}{2}$ or $\frac{3}{3}$ or $\frac{5}{5}$ or $\frac{10}{10}$ or $\frac{50}{50}$ or $\frac{100}{100}$ of itself, since each of these fractions is equal in value to 1. It follows, therefore, that when we speak of 100 per cent. of any amount, we mean the whole amount.

ORAL EXERCISES.

1. Tell what part of any number is meant by 50% of it; by 25%; by 75%; by 20%; by 40%; by 60%; by 80%; by 90%; 125%; 150%; 175%; 200%.

2. What per cent. of a number is $\frac{1}{100}$ of it? $\frac{5}{100}$? $\frac{10}{100}$? $\frac{20}{100}$? $\frac{25}{100}$? $\frac{30}{100}$? $\frac{65}{100}$? $\frac{70}{100}$? $\frac{85}{100}$?

3. What is 5% of 100? 200? 500? 1000?

4. What is 10% of 50? 80? 90? 190?

5. What is 25% of 200? 300? 400? 600?

6. What is 50% of 300? 180? 250? 300?

7. What is 100% of 500? 400? 100? 750?

8. Express decimally 1%; 5%; 6%; 8%; 10%; 12½%; 20%; 25%; 33⅓%; 50%; 62½%; 75%; 80%; 100%; 125%; 150%; 200%.

DEFINITIONS.

ART. 253. **Percentage** is the result obtained by taking a certain per cent. of the base.

ART. 254.—The **Base** is the number on which the percentage is computed.

ART. 255. The **Rate** per cent. is the number of hundredths of the base to be taken.

ART. 256. The **Amount** is the sum of the base and percentage.

ART. 257. The **Difference** is the remainder after subtracting the percentage from the base.

Thus, if we take 12% of $500, the result is $60. The *percentage* is $60; the *base* $500; the *rate* .12; the *amount* $560, and the *difference* $440.

ART. 258. The percentage, base and rate bear such relations to each other that if two of them are given, the third can be found.

ART. 259.—To find the percentage, the base and rate being given.

ORAL EXERCISES.

1. What is 5% of $100?

Solution.—5% of any number is $\frac{5}{100}$ of that number; $\frac{1}{100}$ of $100 is $1, and $\frac{5}{100}$ is 5 times $1, which is $5.

What is —

2. 8% of 100 bushels?
3. 10% of 150 gallons?
4. 12% of 200 men?
5. 12½% of 120 horses?
6. 25% of 120 acres?
7. 30% of 180 miles?
8. 50% of $75?
9. 60% of 320 feet?
10. 90% of 500 yards?
11. 100% of 1 cent?

12. I bought 12 bushels of chestnuts and sold 25% of them: how many bushels did I sell?

13. Harmon receives a monthly salary of $40, and saves 40%: how many dollars does he save each month?

14. Susan had 45 pins, and gave Ellen 60% of them: how many pins did Ellen receive?

15. In a certain election the successful candidate received 600 votes, and his opponent 90% as many: how many votes did the latter receive?

16. Thomas bought a horse for $240, and sold it at a gain of 30%: what did he receive?

17. Walter bought a cow for $90, and sold it at a loss of 20%: how much did he receive?

18. James paid $12.50 for a railroad ticket, and his son paid 50% as much: how much did his son's ticket cost?

19. A certain school has 120 pupils, of whom 40% are boys: how many are girls?

20. A freight train ran 20 miles an hour. If the speed of a passenger train was 150% of that, how many miles an hour did it run?

21. John rode 40 miles on his bicycle on Monday; 20% farther on Tuesday, and 25% farther on Wednesday than on Tuesday: how many miles did he ride on Wednesday?

WRITTEN EXERCISES.

1. What is 15% of $275?

Process.

```
 $275
   .15
 1375
 275
$41.25
```

Analysis.—15% of any number is .15 of it, and .15 of $275 is $41.25.

ART. 260.—Rule for finding the percentage, the base and rate being given.—*Multiply the base by the rate per cent. expressed decimally.*

Formula.—*Percentage = base × rate per cent.*

What is—

2. 3% of $268?

3. 4% of $305.50?

4. $4\frac{1}{2}$% of 362 feet?

5. 6% of 250 bbl.?

6. 11% of 75.5 pounds?

7. 15% of 140 bushels?

8. $\frac{1}{2}$% of $60?

9. $\frac{3}{4}$% of 200 miles?

10. What is the difference between 15% and $37\frac{1}{2}$% of 160 bushels?

11. A tract of land bought for $225 increased 125% in value: what was its value after the increase?

12. The weekly edition of a newspaper is 28,500 copies, and the daily edition is 70% of the weekly edition: what is the daily edition?

13. From a bill of goods amounting to $1,238.50, a deduction of 12% was made for cash: how much cash was paid?

Solution.—100% − 12% = 88%; $1238.50 × .88 = $1089.88.

14. Franklin having 600 bushels of grain, sold $12\frac{1}{2}$% of it to one man, and $37\frac{1}{2}$% of it to another: how many bushels did he keep?

15. The yearly production of quinine throughout the world is 4,500,000 avoirdupois ounces, of which America consumes 40%: how many tons are consumed in America?

16. A bookkeeper receives an annual salary of $3,500. He expends 10% for board, 6% for clothing, 4% for charity, and 35% for other expenses: how much does he save each year?

17. A merchant bought $\frac{3}{4}$ of a vessel for $15,000, and sold $\frac{3}{4}$ of his share at a gain of 8%: how much did he receive?

18. Illustrate by an original problem the method of finding the percentage when the base and rate per cent. are given.

ART. 261. —To find the rate, when the base and percentage are given.

ORAL EXERCISES.

1. What per cent. of 20 is 5?

Solution.—20 is 100% of 20; and since 5 is $\frac{1}{4}$ of 20, it is $\frac{1}{4}$ of 100%, or 25% of 20.

What per cent. of—

2. 25 is 10? 15?	**5.** 75 is 60? $37\frac{1}{2}$?
3. 40 is 20? 25?	**6.** 80 is 20? 25?
4. 50 is 30? 40?	**7.** 90 is 30? 50?

8. A house rents for $600, which is 5% of its value: what is its value?

9. John is 12 years old, and his brother's age is 16 years: what per cent. of his brother's age is John's age?

10. Mr. Hall had $550 in bank, and drew out $50: what per cent. did he draw out?

WRITTEN EXERCISES.

1. What per cent. of 60 is 36?

Process.

$36.00 \div 60 = .60.$

Analysis. 60 is 100% of 60; and since 36 is $\frac{3}{5}$ of 60, it is $\frac{3}{5}$ of 100%, or 60% of 60.

REMARK. To obtain hundredths in the quotient, the dividend must contain two more decimal places than the divisor.

ART. 262. Rule for finding the rate when the base and percentage are given. *Divide the percentage by the base, and the quotient is the rate per cent. required.*

Formula. *Rate per cent. = percentage ÷ base.*

What per cent. of—

2. 125 bush. is 100 bush.? **4.** $62\frac{1}{2}$ yds. is $12\frac{1}{2}$ yds.?

3. 81 qts. is 27 qts.? **5.** $3\frac{1}{2}$ in. is 7 in.?

6. $\frac{4}{5}$ of 25 is what % of $\frac{5}{6}$ of 60?

7. A drover had 1,254 cattle, and sold 418 of them: what % of his drove did he sell?

8. Horace, while fishing, brought 12 fishes to the surface, but 3 got away: what per cent. did he land?

9. Newbold, having 24 apples, gave $\frac{1}{3}$ of them to his sister and 25% of the remainder to his brother: what % of them had he left?

10. From a hogshead of molasses containing 63 gallons, 10 gallons 2 quarts leaked away: what % of the whole remained?

11. Timothy set out to walk 68 miles. He walked 22 miles the first day, 24 the second, and the remainder on the third day: what % of the distance did he walk on the third day?

12. A pound Troy is what % of a pound avoirdupois?

13. If a miller keeps 3 quarts of every bushel he grinds, what % does he keep as pay for grinding?

14. A grocer having 1 ton $2\frac{1}{2}$ cwt. of sugar, sold 5 cwt. 25 lb.: what % of the whole remained?

15. A company numbering 98 men went into battle; 17 were killed and 23 wounded: what % were unhurt?

16. Certain mining stock bought for $53.25 a share was sold for $129.75 a share: what % was the increase in the value?

17. Illustrate by an original problem the method of finding the rate when the base and percentage are given.

ART. 263.—To find the base, when the percentage and rate per cent. are given.

ORAL EXERCISES.

1. 90 is 15% of what number?

Solution.—15% of any number is $\frac{15}{100}$ or $\frac{3}{20}$ of it; and since $\frac{3}{20}$ of a certain number = 90, $\frac{20}{20}$, or that number, is 20 times $\frac{1}{3}$ of 90 = 600.

2. 15 pounds is 20% of how many pounds?

3. 18 days is 6% of how many days?

4. 32 cows is 25% of how many cows?

5. 2.5 weeks is 10% of how many weeks?

6. 360 degrees is 120% of how many degrees?

7. 120 miles is 150% of how many miles?

8. 75 yards is 100% of how many yards?

9. 39 feet is $33\frac{1}{3}$% of how many feet?

WRITTEN EXERCISES.

1. What number increased by 25% of itself equals 555?

Process.

$100\% + 25\% = 1.25$

$555 \div 1.25 = 444$

Analysis.—A number increased by 25% or $\frac{1}{4}$ of itself is 125% or $\frac{5}{4}$ of itself; and since $\frac{5}{4}$ of a certain number = 555, $\frac{4}{4}$, or that number = 4 times $\frac{1}{5}$ of 555 = 444.

REMARK.—Percentage is a certain number of hundredths of any number, and hence includes the sum or difference of 100% and any given rate per cent.

ART. 264.—Rule for finding the base, when the percentage and rate are given. *Divide the percentage by the rate per cent expressed decimally, and the quotient is the required number.*

Formula.—*Base = percentage ÷ rate per cent.*

2. 360 is 45% of what number?

3. 25.50 is $37\frac{1}{2}$% of what number?

4. $62\frac{1}{2}$ is 14% of what number?

5. 35.4 is 14% of what number?

6. $250\frac{3}{4}$ is 60% of what number?

7. 600.75 is 70% of what number?

8. What fraction increased by 20% of itself equals $1\frac{5}{17}$?

9. I sold 148 bushels of wheat which was 37% of what I raised: how much did I raise?

10. A man owning 35% of a ship, sold 40 per cent. of his share for $907.55: what was the value of the ship?

11. A sold 25% of his apples to B and 50% of the remainder to C. If C bought 189 bushels, how many bushels had A at first?

12. B loaned $33\frac{1}{3}$% of his money to his brother, who paid a debt of $550 with $16\frac{2}{3}$% of what he borrowed: how much money had B at first?

13. Sold two horses for $360 each. On one I gained 25%, and on the other I lost 25%: did I gain or lose, and how much?

14. A merchant failed owing $18,500, with $7,200 assets: what per cent. of his debts could he pay?

15. 124 cords of wood was sold for $589.79, which was 11 per cent. more than it cost: what did it cost a cord?

16. Illustrate Art. 259 by an original problem.

17. Illustrate Art. 261 by an original problem.

18. Illustrate Art. 263 by an original problem.

Applications of Percentage.

Profit and Loss.

ORAL EXERCISES.

1. At a gain of 50% what part of the value equals the gain?

Solution.—50% gain on anything is $\frac{50}{100}$ or $\frac{1}{2}$ of the value.

2. What part of the value equals the gain, when the gain is 4%? 5%? 8%? 15%?

3. What part of the value equals the loss, when the loss is 10%? 20%? 25%? 35%? 60%?

4. What part of the cost equals a gain of $8\frac{1}{3}$%? $16\frac{2}{3}$%? $62\frac{1}{2}$%?

5. A farmer bought a horse for $120 and sold it at a gain of 10%: for what sum did he sell the horse?

6. A merchant paid $60 for a piece of cloth and sold it at a gain of 20%: what sum did he receive?

7. Coats worth $40 apiece were sold at a loss of 25%: how much money was lost on each coat?

8. Harvey paid $150 for a carriage and sold it at a gain of 20% to his neighbor, who sold it at a loss of 25%: how much did the neighbor receive?

9. 10% of $50 is $\frac{1}{2}$ of what Horace paid for a dictionary: how much did the dictionary cost?

10. Susan bought 8 yards of muslin for 64 cents and sold it at a gain of 25%: how much did she receive per yard?

11. Edgar gave $20 for a colt and sold it for $30: what per cent. did he gain?

Solution.—He gained the difference between $30 and $20, or $10. $20 is 100% of $20; and since $10 is $\frac{1}{2}$ of $20, it is $\frac{1}{2}$ of 100%, or 50% of $20.

12. A boy bought a knife for 50 cents and sold it for 40 cents: what per cent. did he lose?

13. Silas bought a pair of shoes for $2.50, for which the dealer paid $2: what per cent. did the dealer gain?

14. William bought 8 bushels of chestnuts for $24, and sold them for $32: what per cent. did he gain?

15. $\frac{3}{4}$ of 16 is twice what per cent. of $\frac{3}{5}$ of 50?

16. $50 is $\frac{5}{6}$ of what Honora paid for 12 books: she sold them for $6.25 apiece: what per cent. did she gain?

17. Thomas bought 1 dozen shawls at $8 apiece, and sold $\frac{2}{3}$ of them at $10 apiece, and the remainder for cost: what per cent. did he gain?

18. Edwin sold a cow for $60 and thereby cleared $\frac{1}{4}$ of the money: what per cent. would he have gained, if he had sold it for $63?

19. Ira sold his bicycle for $110 and thereby gained 10%: what was the cost of the bicycle?

Solution.—A gain of 10% is $\frac{1}{10}$ of the cost; $\frac{1}{10}$ of the cost plus the cost equals $\frac{11}{10}$ of the cost, or $110; $\frac{1}{10}$ of the cost equals $\frac{1}{11}$ of $110, or $10, and $\frac{10}{10}$ of the cost equals 10 times $10, or $100.

20. Jennie sold a hat for $7 and thereby lost 30%: what did she pay for the hat?

21. By selling his dog for $9 Charles gained 80%: what did he pay for the dog?

22. Sarah sold some silk for $1.80 a yard, and thereby gained 20%: how much did she pay for the silk?

23. Albert gained 30% by selling his horse for \$260 : for what should he have sold it to lose 8 per cent.?

24. Thomas sold his watch for \$81, thereby losing 10% : for what sum should he have sold it in order to gain 10 per cent.?

25. A drover sold a lot of horses so that $\frac{2}{3}$ of what he received equaled what he paid : what was his gain per cent.?

26. 40 is 20% less and 25% more than what numbers?

27. A merchant sold some goods for \$60 and thereby lost 20%; he then bought more goods for \$60 and sold them at a gain of 20% : what was his entire gain or loss?

28. A farmer sold a horse and carriage for \$300, receiving twice as much for the horse as for the carriage. On the horse he gained 25% and on the carriage he lost 20% : did he gain or lose, and how much?

DEFINITIONS.

ART. 265.—Profit and **Loss** are terms used to denote the gain or loss in business transactions. They are generally estimated at a rate per cent. on the cost, or money invested.

When an article is sold for more than cost, it is sold at a *premium* or *profit* or *gain*; when sold for less than cost, it is sold at a *loss* or *discount*.

1. The **Base** is the cost or sum invested.
2. The **Rate** is the rate per cent. of profit or loss.
3. The **Percentage** is the gain or loss.

Formulas.—*Gain or loss = cost × rate %*

Rate % = gain or loss ÷ cost.

Cost = gain or loss ÷ rate %.

$$\textit{Cost} = \textit{selling price} \div \begin{cases} 1 + \textit{rate \% of gain.} \\ 1 - \textit{rate \% of loss.} \end{cases}$$

WRITTEN EXERCISES.

1. A merchant bought $650 worth of cloth and sold it at an advance of 20%: how much did he gain?

2. A farm that cost $18,500 was sold at a loss of 15%: what was received for it?

3. Potatoes bought at $1.20 a bushel were sold at a discount of 12½%: for what sum per bushel were they sold?

4. A drover bought cattle at $48.50 a head and sold them at 8% advance: what did he get for each animal?

5. A merchant sold a lot of goods at 16⅔% advance, and thereby gained $375.60: what did he pay for them?

6. A horse was sold for $240, which was at a loss of 4%: how much did it cost?

7. A tailor marked a coat 25% above cost, but fell 20% of his asking price: did he gain or lose?

8. A merchant marked his goods 25% above cost, but a rise in the market caused him to add 20% to the asking price: what was his gain per cent.?

9. Bought 72 cords of wood at $3.20 a cord, and sold it at a gain of 25 per cent.: what was the gain, and what the selling price per cord?

10. A merchant's stock of goods cost $8,400: if he sold at an advance of 15%, what were his profits after deducting $364.75 for expenses?

11. Bought a hogshead of sugar containing 8 cwt. 48 lb. for $63.60: at what price per pound must it be sold to gain 20%?

12. A drover bought 160 head of cattle at $38 per head, and sold them for $5,472: what was his loss per cent?

13. At what price must cloth which cost $2.40 per yard be sold to gain 12½%?

14. A dealer sold two pianos for $460 each: on one he

made 20%, and on the other 15%: what was the total gain?

15. A stationer bought goods at 20% off the list price: if he retails them at the list price, what will be his per cent. of profit?

16. A horse and carriage were sold for $400, the horse bringing 3 times the price of the carriage. If there was a profit of 20% on the horse and a loss of 20% on the carriage, what was the gain on the two?

17. I bought two donkeys at $75 each, and sold one at an advance of $33\frac{1}{3}$%, and the other at a loss of $33\frac{1}{3}$%: did I gain or lose?

18. If a house is sold for $\frac{2}{3}$ of its cost, what is the loss per cent.?

19. A house and lot were sold for $8,500, the gain being $1,500: if $1,500 more had been received, what would have been the gain per cent.?

20. How shall goods costing $600 be marked, so that the seller may fall 10% of the price asked, and still make 5%?

SUGGESTION—After a fall of 10% on the asking price, there must be left a profit of 5% of $600, hence, $630 must be the price received, but this is a fall of 10% or $\frac{1}{10}$, so that $630 is $\frac{9}{10}$ of the price that should be asked

21. If I buy wood at $4.80 per cord, what price shall I ask so that, after a deduction of 15% is made, the gain shall be $6\frac{1}{4}$%?

22. By selling an interest in a ship for $13,370, Captain Burton gained 18%: what price would he have received had he lost 8%?

23. I bought pottery ware at 25%, 20% and 10% off: what per cent. did I pay for my goods?

NOTE.—The foregoing is an example of "Trade Discount," or continuous discounts. It means that 25% is deducted from the list price, 20% from the remainder, and 10% from what is left. Thus, the list price or 100% − 25% is 75%, 20% of 75% is 15%, and 75% − 15% = 60%, 10% of 60% is 6%, and 60% − 6% = 54%, which is the actual cost.

24. If I buy a book marked \$4.80 at 25% and 20% off, and sell at a gain of 25%, what will I gain?

25. Walker bought a Newfoundland dog for \$25, and set a price, so that after falling \$15 he gained 20%: what per cent. of the asking price did he deduct?

26. If the population of New York is 50% more than that of Philadelphia, what per cent. less is the population of Philadelphia than that of New York?

27. Harry sold Philip a pony, and gained 25%; Philip sold it to Peter for \$48, and gained 20%: what did Harry pay for the pony?

28. If a house is bought for $\frac{4}{5}$ of its value, and sold for 20% more than its value, what is the gain per cent.?

29. A grocer bought 4 barrels of sugar, each containing 240 pounds, at 8 cents a pound, and sold it at a profit of 25 per cent.: what was the selling price per pound?

30. A farmer sold two loads of produce for \$24.36 each. On one he lost 20 per cent., and on the other gained 20 per cent.: was there a gain or loss on the sale of both, and how much?

31. A man bought flour for \$1,000, but 40% proved worthless, and he sold the remainder at a gain of 50%: did he gain or lose, and how much?

32. A ship lost $33\frac{1}{3}\%$ of its cargo in a storm; $33\frac{1}{3}\%$ of what was left was sold at cost, and on the remainder a gain of $33\frac{1}{3}\%$ was made: what was the result of the transaction?

33. Illustrate Profit and Loss by an original problem.

Commission and Brokerage.

ORAL EXERCISES.

1. If I sell goods for \$1,200, and receive 3% of the sales, how much do I get?

2. If I get $2\frac{1}{2}$% for buying \$1,000 worth of goods, how much do I receive?

3. Mr. A paid an agent 6% for collecting a debt of \$250: how much did he pay the agent?

4. Mr. B received 2% for selling cotton for \$8,000: how much did he receive?

5. Mr. C charged 3% for selling goods for \$2,000: how much did he receive? How much did he return to his employer?

DEFINITIONS.

ART. 266.—An **Agent** or **Commission Merchant** is a person that transacts business for another. He is known also as a *Factor*, *Broker*, *Collector*, etc.

ART. 267.—The **Consignor** or **Shipper** is the one that sends the merchandise to be sold

ART. 268.—The **Consignment** is the goods shipped.

ART. 269.—The **Consignee** or **Correspondent** is the one to whom the merchandise is sent.

ART. 270.—**Commission** is the compensation allowed the agent who sells the goods, and is a percentage of the money expended or collected.

ART. 271.—**Brokerage** is the name generally given to the compensation of a broker.

ART. 272.—The **Net Proceeds** of a sale or collection is the money due the consignor after the commission and all other expenses are deducted.

The processes in Commission and Brokerage are based on the principles of Percentage, already explained. The terms are:

1. The Base is the sales or sum invested.

2. The Rate is the rate per cent. of commission.

3. The Percentage is the commission.

4. The Amount is the purchase price plus the commission.

5. The Difference is the net proceeds.

Formulas.—*Commission or Brokerage = base × rate %.*
Rate % = commission or brokerage ÷ base.
Base = commission or brokerage ÷ rate %.
Base = base + commission or brokerage ÷ (1 + rate %).

WRITTEN EXERCISES.

1. A commission merchant sold 650 barrels of flour at $5.87½ a barrel: what was his commission at 2½%?

2. Mr. A sold $4,850 worth of goods at auction, and charged 3½% commission and 1% for insuring payment: what were the net proceeds?

3. A collector having a debt of $1,258 to collect compromised for 90%; he charged 3¼% commission: how much money did he remit to his employer?

4. An agent sold a consignment of wheat for $4,328.60, and 1,200 barrels of apples at $2.25 per barrel: his commission being 2¼%, what sum did he remit to the consignor?

5. An attorney's commission at $2\frac{1}{2}\%$ for collecting debts was \$400 : what was the amount of his collections ?

6. I received \$8,346.24 with which to buy horses, after deducting my commission of $3\frac{1}{2}\%$: how many horses did I buy at \$192 apiece ?

SUGGESTION.—The sum, \$8,346.24, includes the $3\frac{1}{2}\%$ commission and 100% the money to be invested ; hence, it is $103\frac{1}{2}\%$ of the money expended in the purchase of horses.

7. A correspondent bought goods at $1\frac{1}{4}\%$ commission ; the goods and commission together amounted to \$1,800 : what was his commission ?

8. A broker sold to a merchant funds for \$4,825, at a brokerage of $4\frac{1}{5}\%$: how much was the brokerage ?

9. A broker receives 2% commission and $2\frac{1}{2}\%$ for guaranteeing payment : how much does he receive on sales amounting to \$18,250 ?

10. My agent compromised with a debtor owing me \$3,828 at 75%, and his commission was 5% : how much did he receive ?

11. A cotton broker in New Orleans received \$12,300 with which to buy cotton, after deducting his commission of $2\frac{1}{2}\%$: if he paid \$.25 a pound, how many pounds of cotton did he buy ?

12. How much wool at 40 cents a pound can be bought for \$4,160, after deducting a commission of 4% ?

13. A speculator sent his broker \$6,828 to invest in stocks at \$60 a share, after deducting $\frac{1}{8}\%$ brokerage : how many shares were purchased ?

14. I sent my agent in Trenton, N. J., the money with which to buy \$824 worth of pottery and pay a commission of $2\frac{3}{4}\%$: how much did I send him ?

15. B, of Fort Worth, bought 800 Texas cattle at a commission of $3\frac{1}{2}\%$; he paid \$40 for keeping and \$60 for

transportation, which, with his commission, amounted to $296: how much did the cattle cost apiece?

16. Smith & Bro.'s agent in Omaha sent them $7,296.06 as the proceeds of the sale of some land, the agent retaining $167.94: what rate of commission did he charge?

17. A consignee charged $240 for selling goods to the amount of $9,600: what was his rate of commission?

18. An architect charged ½% for his plans and specifications, and 1¼% for superintending the erection of some buildings costing $240,000: how much was his fee?

19. An agent charged 1½% commission and $58.60 expenses for selling my house, and sent me $11,761.40: for how much did he sell the house?

20. What is the total cost of 620 yards of cloth at $1.50 a yard, if I pay 2½% commission for purchasing, ¼% for a draft covering cost and agent's commission, and $15.75 for freight and cartage?

21. Sent $360 to an agent to be invested in prints at 15 cents a yard, after deducting his commission of 3½%: how many yards did he purchase?

22. Sold sugar at 2½% commission, and after deducting my commission, I invested the remainder in flour at 1% commission. What did I get for the sugar, if I bought flour to the amount of $126.75?

23. Illustrate by an original problem the method of finding the commission, when the amount of sales and the rate % of commission are given.

24. Illustrate by an original problem the method of finding the rate of commission or brokerage, when the commission and amount of sales are given.

25. Illustrate by an original problem the method of finding the amount to be invested, when the rate % of commission and the net proceeds are given.

Insurance.

ART. 273.—Insurance is an agreement by one party to pay another for damage or loss. It is divided into two classes: *Property Insurance* and *Life Insurance*.

ART. 274. Property Insurance includes *Fire Insurance*, or indemnity against loss of property by fire; *Marine Insurance*, or indemnity against loss by accidents of navigation; and *Live Stock Insurance*, or indemnity against loss of horses, cattle, etc.

ART. 275. Life Insurance is indemnity for loss of life.

ART. 276. Accident Insurance is indemnity for sickness or accident.

ART. 277. The **Underwriter** is the party who writes the insurance.

ART. 278. The **Policy** is the written contract between the insured and the company insuring.

ART. 279. The **Premium** is the sum paid for insurance.

In property insurance the premium is computed at a certain rate per cent. on the value of the property insured.

Life Insurance is computed at a certain sum or rate ¢ on \$100 or \$1,000 insured, the rate varying with age and occupation.

The processes in Insurance are based on the principles of Percentage.

1. The Base is the amount insured.

2. The Rate is the rate of premium.

3. The Percentage is the premium.

Formulas.—*Premium* = *amount insured* × *rate %.*
Rate % = *premium* ÷ *amount insured.*
Amount insured = *premium* ÷ *rate %.*
Property and premium = *property* ÷ (1 − *rate %*).

WRITTEN EXERCISES.

1. A insured his store for \$7,500 at $2\frac{3}{4}$ %: what was the premium?

2. If I insure my mill for \$12,000 at .0265, what amount of premium do I pay?

3. If the premium on an insurance of \$20,000 is \$350, what is the rate?

4. A steamer and cargo worth \$450,000 are insured for $\frac{3}{5}$ of their value. The premium is \$6,750: what is the rate?

5. A drover paid \$75 premium at \$.30 per \$100 for insurance on the value of a drove of cattle: what was the value of the herd?

6. What will it cost to insure at $2\frac{1}{2}$%, property valued at \$6,000, to cover $\frac{2}{3}$ of the property?

7. A man obtained a life policy for \$5,000 at the rate of \$10.70 per \$1,000: what was his annual premium?

8. A man at the age of 40 years was insured for \$3,000 at the rate of \$17.60 per \$1,000, payable semi-annually. At the age of $45\frac{1}{2}$ years, he was lost at sea. How much did the sum due his family exceed the amount of the premiums paid?

9 Illustrate by an original problem the method of finding the premium, when the rate and amount insured are given.

10. Illustrate by an original problem the method of

finding the rate of insurance, when the premium and amount of insurance are given.

11. Illustrate by an original problem the method of finding the amount of insurance, when the premium and rate per cent. are given.

Taxes.

ART. 280.—**Taxes** are sums of money levied on persons, property or business for public purposes.

ART. 281.—**Poll** or **Capitation Tax** is a tax on every citizen without regard to his property.

ART. 282.—**Property Tax** is a tax upon property, and is assessed at a given rate per cent. of the valuation, or at the rate of a given number of mills on a dollar, or a number of cents on each $100.

ART. 283.—**Real Estate** is such property as houses and lands.

ART. 284.—**Personal Property** is movable property, such as merchandise, cattle, stocks, etc.

ART. 285.—An **Assessor** is an officer who appraises property and apportions taxes.

The processes in Taxes are governed by the principles of Percentage.

Formulas.—*Tax* = *property* × *rate %.*
Rate % = *tax* ÷ *property.*
Amt. of taxes collected = *net proceeds* ÷ (1 − *rate % charged for collection*).

WRITTEN EXERCISES.

1. The taxable property of a town was $850,000 and the rate 8 mills on a dollar: what amount was raised?

2. A tax of $6,450 was levied upon the taxable property of a county, valued at $1,935,000: what was the rate, and what was the tax upon a farm worth $5,600?

3. The real estate in Bayville was assessed at $1,250,000; personal property at $1,050,000; and there were 320 polls assessed at $1.50 each: what was the total amount of tax raised, the rate being 6 mills on a dollar?

4. How much did A pay, whose real estate was worth $20,000, personal property $15,000?

5. How much did B pay, whose real estate was worth $16,500, personal property, $14,000, and who paid for 2 polls?

6. A tax of $7,200 was levied upon property valued at $1,800,000: what was the rate?

7. If property valued at $36,000 is taxed $120, what is the value at the same rate, of property taxed $75?

8. A's entire tax, including three polls at $1.50, is $204.50: if the rate is 8 mills on $1, what is the value of his property?

9. A certain town is taxed $16,500; the real estate amounts to $850,000 and personal property to $350,000; there are 500 polls, each taxed at $1: what is the rate?

10. How much is C's tax, who pays for 3 polls and whose taxable property is $8,500?

11. In a certain school, the salary of the teacher was $800; the cost of fuel $36, and of repairs $30. The school fund, received from the State, was $386, and the rest of the expense was paid by the patrons of the school. There were 60 pupils enrolled, and their average attendance during the school year, was 40 days apiece: what did Mr. Glenn pay, who sent two children 150 days each and one child 124 days?

12. Illustrate by an original problem the method of assessing taxes.

Duties and Customs.

ART. 286. –**Duties** and **Customs** are taxes assessed by the Government on imported goods, for revenue or for the protection of home industry.

ART. 287. –**Ports of Entry** are established by the Government for the collection of duties. At every port of entry there is a custom house, in charge of a *Collector of Customs.*

ART. 288. –Duties are of two kinds—*Ad Valorem* and *Specific.*

ART. 289. –**Ad Valorem Duty** is a certain percentage assessed on the actual cost of the goods in the country from which they are imported.

ART. 290. –**Specific Duty** is a tax assessed on the quantity of the goods.

ART. 291. **Tare** is a specified allowance for the weight of the box, bag, or cask containing the goods.

ART. 292. –**Leakage** and **Breakage** are allowances for the loss of liquids.

ART. 293. –An **Invoice** or **Manifest** is a written statement of a ship's cargo, showing where the goods were shipped, their quantity, quality, to whom consigned, etc.

ART. 294. –**Gross Weight** or **Value** is the weight or value of the goods before any allowances are made.

ART. 295. **Net Weight** or **Value** is the weight or value of the goods, after all allowances have been made.

The processes under Duties and Customs are based on the principles of Percentage.

WRITTEN EXERCISES.

Formulas.—*Ad valorem duty = net inv. price × rate %.*
Net inv. price = ad val. duty ÷ rate %.

1. What is the duty on 500 yards of broadcloth, valued at $1.75 per yard, at 25% ad valorem?

2. What is the duty on 2,800 pounds of ginger-root at 2 cents a pound, tare being 4%?

3. What is the duty at 4 cents a pound on 3,250 pounds of coffee, allowing 5% for tare?

4. Coudray & Company imported from Havana 50 hogsheads of molasses, each containing 63 gallons, duty 3 cents a gallon; 50 boxes of sugar weighing 450 pounds each, duty $2\frac{1}{4}$ cents a pound, and 200 boxes of cigars, invoiced @ $18 per box, duty $33\frac{1}{3}$% ad valorem: how much was the total duty paid?

5. What is the ad valorem duty at 25%, on 140 chests of tea, each weighing 50 pounds, and invoiced at 44 cents a pound, the tare being 4 pounds per chest?

6. The duty on 400 drums of figs, containing 15 pounds each, invoiced at $4\frac{1}{2}$ cents a pound, was $27: what was the rate?

7. A merchant imported 40 dozen bottles of champagne costing $15 per dozen: what was the duty at $5.50 per dozen, breakage being 5%?

8. A wine merchant imported 48 casks of Madeira wine valued at $55 a cask, and 50 casks of sherry wine valued at $64 a cask: if an allowance of 3% is made for leakage, what will be the duty at 42% ad valorem?

9. The duty on 1,800 yards of silk at $33\frac{1}{3}$% ad valorem was $1,200: for what price per yard must it be sold that the importer may clear $16\frac{2}{3}$%?

10. Illustrate Duties and Customs by an original problem.

Stocks and Investments.

ART. 296.—**Capital** is property or money invested in business.

ART. 297.—A **Company** is an association of persons whose object is the transaction of business.

ART. 298. When a company is organized and governed by law, it is said to be *incorporated.* Such a company is also called a *Corporation,* and the law governing it is its *Charter.*

ART. 299. **Stock** is the capital of an incorporated company. It is divided into equal parts called *Shares.*

Shares are generally $100 each.

ART. 300.—**Certificates of Shares** are called *Stock Certificates,* or *Scrip;* their owners are *Stockholders,* and the stock may be bought and sold like any other property.

ART. 301. The **Par Value** of stock is the amount named in the certificate.

ART. 302.—The **Market Value** is the price for which it can be sold.

When stock sells for its original or face value, it is *at par,* or worth 100%; when it brings more, it is *above par,* or *at a premium;* when it brings less than its face value, it is *below par,* or *at a discount.*

ART. 303.—A **Dividend** is a sum paid to stockholders out of the profits of the business of the company.

ART. 304. An **Assessment** is a sum assessed upon stockholders to meet the losses or expenses of business.

ART. 305. A **Stock Broker** is one that buys and sells stocks for a commission, which is called *Brokerage.*

ART. 306.—A Bond is an instrument, properly signed and authenticated, guaranteeing the payment of a sum of money on a specified date.

The processes in Stocks and Investments are based on the same principles as those under Percentage.

WRITTEN EXERCISES.

Formulas.—*Assessment or dividend* = *stock* × *rate* %.
Rate % = *assessment or dividend* ÷ *stock*.
Discount or premium = *par value* × *rate* %.
Market value = *par value* × (1 ± *rate* %).
Par value = *market value* ÷ (1 ± *rate* %).

1. What is the cost of 50 shares in the West Creek Bridge Company, at 1.07¾, brokerage ⅛%?

Solution.—1.07¾ + ⅛ = 1.07⅞, and $5,000 × 1.07⅞ = $5,393.75.

2. What is the cost of 5 U. S. bonds of 1891 at 110¼, brokerage ⅛%?

3. How many Trenton Watch Company bonds at 112½, brokerage ¼%, can be bought for $13,304.50?

SUGGESTION.—$13,304.50 ÷ $112.75 = 118.

4. How many shares of Western Union can be bought for $2,775 at 110⅞, brokerage ⅛%?

5. How many shares of the Blue Stone Company, bought at 50¼ and sold at 50⅞, brokerage ⅛% each way, must be bought and sold to yield a profit of $600?

SUGGESTION.—($50⅞ − $⅛) − ($50¼ + $⅛) = $⅜ gain on each share.

6. What sum must be invested in stock at 108½, which pays a semi-annual dividend of 4%, to yield an income of $1,200?

SUGGESTION.—1 share yields $8 annually; $1,200 ÷ $8 = 150 shares; $108½ × 150 = $16,275.

7. What sum must be invested in Camden and Amboy Railroad bonds at 108⅞, yielding an annual interest of

8%, brokerage being $\frac{1}{8}$%, to secure an annual income of $800?

8. What annual income is secured by an investment of $18,050 in railroad stock at 90, brokerage $\frac{1}{4}$%, if 5% semi-annual dividends are declared?

9. What rate per cent. is realized from bonds bought at 140, and paying a semi-annual interest of 6%?

SUGGESTION.—Annual income, $12 ÷ investment, $140 = .08$\frac{4}{7}$ = 8$\frac{4}{7}$%.

10. What price should be paid for stock paying an annual dividend of 8%, to yield an income of 7%?

SUGGESTION.—Since the annual income is $8, this is 7% of the price that must be paid. $8 ÷ .07 = 114\frac{2}{7}$.

11. What price must be paid for bonds paying 8% annual interest, to yield an income of 10%?

12. For what price must 4% bonds be bought that they may yield an income of 5%?

13. What will 200 shares of bank stock cost at 104$\frac{1}{4}$?

14. How many shares of railroad stock at 95 can be bought for $11,780?

15. How much money invested in 7% bonds will yield an annual income of $1,190?

16. Mrs. H. owns enough 5% stock to give her a semi-annual income of $1,200: how much stock does she own?

17. A guardian invested $20,700 in 4% bonds at 103: what income did it produce?

18. If 4$\frac{1}{2}$% bonds are bought at 90, what % do they pay on the investment?

19. What price must be paid for 6% bonds to yield 10% on the money invested?

20. What price must be paid for 6% bonds to yield 4% on the money invested?

21. A person invested \$3,480 in bank stock at 116, and received a semi-annual dividend of \$120: what was the annual rate of dividend?

22. A medical college has its endowment fund invested in 4% state bonds, bought at par, from which a quarterly income of \$1,120 is realized: how much money is thus invested?

23. If a United States bond, bought at 108, pays \$4.50 interest annually, what rate is received on the money invested?

24. How much income will I receive annually by investing \$3,088 in 5% bonds purchased at 48, allowing $\frac{1}{4}$% brokerage?

25. Which is more profitable, and how much, to invest \$2,000 in 6% bonds purchased at 80, or in 5% bonds purchased at 66$\frac{2}{3}$%?

26. How much must be invested in 5% bonds purchased at 80 to secure the purchaser an income of \$1,200 annually?

27. The net earnings of a company whose capital stock is \$1,500,000 was \$80,000: if \$5,000 is reserved as a surplus fund, what per cent. dividend can be declared?

28. I invested \$13,500 in stocks at 60, brokerage 2$\frac{1}{2}$% on the par value: what was the par value of the stocks?

29. A broker bought stock at 8% discount, and sold it at 10% premium, thereby gaining \$540: how many shares did he purchase?

30. The net earnings of a water company are \$35,000 and the capital stock is \$650,000: if a surplus of \$2,500 is reserved, what rate of dividend can be declared?

31. Illustrate Stock Investments by an original problem.

Interest.

ART. 307.—Interest is money paid for the use of money.

ART. 308.—The **Principal** is the money for which interest is paid.

ART. 309.—The **Rate of Interest** is the per cent. of the principal paid for its use for a year, or month.

ART. 310. The **Time** is the period during which the money is on interest.

ART. 311. The **Amount** is the sum of the principal and interest.

ART. 312. **Simple Interest** is interest on the principal only.

ART. 313. **Compound Interest** is interest on the principal and the interest.

ART. 314. -**Legal Interest** is interest at a rate fixed by law.

ART. 315.—Usury is interest at a rate higher than that fixed by law.

ART. 316. Interest differs from the applications of Percentage already explained only in the use of *time* as an element in connection with the *rate per cent.*

ART. 317.—The *Principal* is the **Base**; the *Per cent. per annum*, the **Rate**; the *Interest*, the **Percentage**; the *Sum* of principal and interest, the **Amount.**

The legal rate of interest varies in the different States as shown by the following table:

INTEREST LAWS IN THE UNITED STATES.

Laws of each State and Territory regarding Rates of Interest and Penalties for Usury, with the Law or Custom as to Days of Grace on Notes and Drafts.

STATES AND TERRITORIES.	Legal Rate of Interest.	Rate allowed by contract.	PENALTIES FOR USURY.	Grace or No grace.
	per cent.	per cent.		
Alabama	8	8	Forfeiture of entire interest	Grace
Arizona	10	Any rate	None	Grace
Arkansas	6	10	Forfeiture of principal and interest	No stat.
California	7	Any rate	None	No grace.
Colorado	10	Any rate	None	Grace.
Connecticut	6	6	None	Grace
Dakota	7	12	Forfeiture of excess	Grace
Delaware	6	6	Forfeiture of principal	Grace
Dist. of Columbia	6	10	Forfeiture of entire interest	Grace
Florida	8	Any rate	None	No stat.
Georgia	7	8	Forfeiture of excess	Grace
Idaho	10	18	Forfeiture of 3 times excess of int.	No grace.
Illinois	6	8	Forfeiture of entire interest	Grace
Indiana	6	8	Forfeiture of excess of interest	Grace.
Iowa	6	10	Forfeit. of 10% per year on amt.	Grace
Kansas	7	12	Forfeiture of excess of interest	Grace
Kentucky	6	10	Forfeiture of excess over 10%	Grace.
Louisiana	5	8	Forfeiture of entire interest.	Grace.
Maine	6	Any rate	None	Grace.
Maryland	6	6	Forfeiture of excess of interest	Grace.
Massachusetts	6	Any rate	None	Grace.
Michigan	7	10	Forfeiture of excess of interest	Grace.
Minnesota	7	10	Forfeiture of excess over [illegible]%	Grace.
Mississippi	6	10	Forfeiture of excess of interest	Grace
Missouri	6	10	Forfeiture of entire interest	Grace
Montana	10	Any rate.	None	No grace.
Nebraska	7	10	Forfeiture of interest and cost	Grace
Nevada	10	Any rate.	None	Grace.
New Hampshire	6	6	Forfeiture of thrice the excess	Grace.
New Jersey	6	6	Forfeiture of entire interest	Grace.
New Mexico	6	12	None	No grace.
New York*	6	6	Forfeiture of principal and interest	Grace.
North Carolina	6	8	Forfeiture of entire interest	Grace
Ohio	6	8	Forfeiture of excess above 6%	Grace.
Oregon	8	10	Forfeiture of principal and interest	Grace.
Pennsylvania	6	6	Forfeiture of excess of interest	Grace.
Rhode Island	6	Any rate.	None	Grace.
South Carolina	7	Any rate.	None	Grace.
Tennessee	6	10	Forfeiture of exc. int. and $100 fine	Grace.
Texas	8	12	Forfeiture of entire interest.	Grace
Utah	10	Any rate.	None.	Grace
Vermont	6	6	Forfeiture of excess of interest.	Grace.
Virginia	6	8	Forfeiture of excess over 6%.	Grace.
Washington Ter'y	10	Any rate.	None	-
West Virginia	6	6	Forfeiture of excess of interest	Grace.
Wisconsin	7	10	Forfeiture of entire interest	Grace.
Wyoming	12	Any rate.	None.	Grace.

* By an amendment of the law New York has legalized any rate of interest upon call loans of $5,000 or upward, on collateral security.

For convenience in computing interest, 30 days are generally considered a month, and 360 days a year.

ART. 318.—To find the interest or amount when the principal, time and rate are given.

ORAL EXERCISES.

1. What is the interest on \$50 for 1 year 4 months 10 days at 6%?

Solution.—The interest for 1 year at 6% is .06 of the principal, or \$3, and for 1 month it is $\frac{1}{12}$ of that sum, or 25 cents, and for $16\frac{1}{3}$ months it is $16\frac{1}{3}$ times 25 cents, or \$4.083.

What is the interest on:

2. \$80 for 2 years at 6%?

3. \$40 for 3 years at 5%?

4. \$100 for 1 year 6 months at 7%?

5. \$200 for 2 years 8 months at 8%?

6. \$300 for 3 years 6 months 24 days at 6%?

7. \$250 for 4 years 4 months 18 days at 6%?

8. \$200 for 3 years 7 months at 6%?

9. \$400 for 2 years 5 months at 6%?

10. \$200 for 1 year 9 months 18 days at 6%?

WRITTEN EXERCISES.

1. What is the interest on \$360 for 3 yr. 8 mo. and 15 days at 8%?

Process.

```
     $360
      .08
 12)28.80 Int. for 1 yr.
    $2.40 Int. for 1 mo.
     44.5
     1200
     960
    960
 $106.800
```

Analysis.—The interest for 1 yr. at 8% is .08 of the principal, and the interest for 1 mo. is $\frac{1}{12}$ of that sum; for 3 yr. 8 mo. (44 mo.) and 15 da. (.5 mo.), it must be 44.5 times \$2.40, which is \$106.80. Therefore, the interest on \$360 for 3 yr. 8 mo. 15 da. at 8% is \$106.80.

ART. 319.—Rule for computing interest. *Multiply the principal by the rate per cent., divide the product by 12, and multiply the quotient by the total number of months and tenths of a month for which interest is required.*

NOTE.—Since 30 days make an interest month, the given number of days divided by 3 gives tenths of a month. Thus, 9 days = .3, 12 days = .4, 13 days = .4⅓, 26 days = .8⅔, and so on.

What is the interest on:

2. $75.50 for 1 yr. 4 mo. at 6%?

3. $84.25 for 2 yr. 6 mo. at 5%?

4. $67.60 for 3 yr. 2 mo. at 7%?

5. $49.54 for 2 yr. 8 mo. at 8%?

6. $125.73 for 4 yr. 3 mo. at 9%?

7. $248.50 for 3 yr. 6 mo. 24 da. at 5%?

8. $310.14 for 2 yr. 5 mo. 18 da. at 6%?

9. $510.00 for 3 yr. 7 mo. 15 da. at 6½%?

10. $628.45 for 2 yr. 9 mo. 10 da. at 8¼%?

11. What is the amount of $317.15 for 3 yr. 1 mo. 9 da. at 4%?

12. What is the amount of $384.23 for 2 yr. 24 da. at 5%?

13. What is the amount of $392.38 for 3 mo. 10 da. at 6%?

14. What is the amount of $421.31 for 1 mo. 20 da. at 7%?

15. What is the amount of $521.37 for 16 yr. 8 mo. at 6%?

16. What is the amount of $492.33 for 24 da. at 8%?

17. A loaned $475, June 30, 1885; how much was due him March 17, 1886, interest at 5%?

18. If you loan $324.75, January 4, 1885, how much will be due you October 1, 1886, interest at 7%?

19. On the 23d of February, 1884, Mr. Brown borrowed $758.84: how much did he owe April 1, 1886, interest at 6%?

ART. 320.—Six Per Cent. Method. 1. What is the interest on $360 for 3 yr. 8 mo. 15 da. at 8%?

Process.

$.06 × 3 =	$.18
$.005 × 8 =	$.04
$.000$\frac{1}{6}$ × 15 =	$.0025
	$.2225
	8
	6)$1.7800
	$.29$\frac{2}{3}$
	360.
	240
	174
	87
Ans.	$106.80

Analysis.—The interest on $1 for 1 yr. at 6% is $.06; the interest for 1 mo. is $\frac{1}{12}$ of $.06 = $.005, and the interest for 1 da. is $\frac{1}{30}$ of $.005 = $.000$\frac{1}{6}$. Hence, the interest for 3 yr. is $.18; for 8 mo., $.04, and for 15 da., $.0025, the sum of which is $.2225. But 8% is $\frac{8}{6}$ of 6%, and $\frac{8}{6}$ of $.2225 is $.29$\frac{2}{3}$; and since the interest on $1 for the given time at 8% is $.29$\frac{2}{3}$, the interest on $360 is 360 times $.29$\frac{2}{3}$ = $106.80.

Rule for finding interest on any principal for any time at any rate per cent. *Multiply $.06 by the number of years; $.005, by the number of months, and $.000$\frac{1}{6}$ by the number of days; add the partial products, take the fractional part of it that the given rate is of 6%, and multiply this by the principal expressed decimally.*

What is the interest on—

2. $75.50 for 1 year 6 months 18 days at 6%?

3. $245.75 for 1 year 5 months 15 days at 5%?

4. $645.25 for 2 years 3 months 20 days at 7%?

5. $749.48 for 6 years 5 months 5 days at 6% ?

6. $1,800.18 for 1 year 7 months 7 days at 5% ?

7. What is the amount of $500 for 2 years 3 months 2 days at 7% ?

8. What is the amount of $480.99 for 11 years 11 months 11 days at 8% ?

9. What is the amount of $75 for 16 years 8 months at 6% ?

10. Illustrate by an original problem the method of calculating interest by the six per cent. method.

ART. 321. Exact Interest. The usual way of computing interest is based on 360 days to the year. By the exact method the actual number of days is found and is regarded as so many 365ths of a year. This rule is the one adopted by banks and the United States Government, and it is growing in favor among business men. When the time in days is less than 1 year, the exact interest is found by first calculating the interest according to the methods already given, and deducting $\frac{1}{73}$ from the result for the common years and $\frac{1}{61}$ for the leap years.

11. What is the exact interest on $234 from July 1, to October 31, at 6% ?

12. What is the exact interest on $563.54 from January 16, 1884, to March 10, of the same year, the rate being 7% ?

13. What is the exact interest on $907.95 from January 1, 1886, to December 1, 1886, the rate being 10% ?

14. What is the difference between common and exact interest on $784.96 at 6% from Christmas, 1886, to July 4, 1887 ?

15. Illustrate by an original problem the method of computing exact interest.

PROBLEMS IN INTEREST.

ART. 322.—To find the principal, when the interest or amount and the rate and time are given.

WRITTEN EXERCISES.

1. What principal will gain \$90 in 2 years 6 months at 6%?

Process.—Interest on \$1 for 2 years and 6 months at 6% equals \$.15; \$90 ÷ .15 = \$600.

Analysis.—The interest of \$1 for 2 years and 6 months at 6% being \$.15, to give \$90 interest, in the same time and at the same rate, it will take as many times \$1 as \$.15 is contained times in \$90, or 600 times \$1, equal to \$600. Therefore, \$600 will gain \$90 in 2 years 6 months at 6%.

2. What sum will amount to \$690 in 2 years 6 months at 6%?

Process.—Amount of \$1 for 2 years 6 months at 6% equals \$1.15; \$690 ÷ 1.15 = \$600.

Analysis.—Since the amount of \$1 for 2 years 6 months at 6% is \$1.15, it will require a principal equal to as many times \$1 as \$1.15 is contained times in \$690. \$1.15 is contained 600 times in \$690. Therefore, \$600 in 2 years 6 months at 6%, will amount to \$690.

ART. 323.—Rule for finding the principal, when the interest or amount and the rate and time are given. *Divide the given interest by the interest of one dollar for the time at the given rate*; or,

Divide the given amount by the amount of one dollar for the given time and at the given rate.

Formulas.—*Principal = interest ÷ (rate % × time).*
Principal = amount ÷ [1 + (rate % × time)].

What principal will gain—

3. \$44.80 in 2 years 4 months, at 6%?

4. \$55.80 in 1 year 6 months 18 days, at 5%?

5. \$47.424 in 1 year 8 months 24 days, at 4%?

6. What principal will amount to \$271.05 in 2 years 7 months 12 days, at 6%?

7. What principal will amount to \$624.546 in 3 years 6 months 21 days, at 7%?

8. What principal will amount to \$946$\frac{2}{3}$ in 2 years 3 months 15 days, at 8%?

9. What principal will amount to \$163.359 in 3 years 3 months 16 days, at 9%?

10. Illustrate by an original problem the method of finding the principal, when the interest or the amount and the rate and time are given.

ART. 324.—To find the time, when the principal, interest and rate are given.

1. In what time will \$500 gain \$30 interest at 4%?

Process.

$\$500 \times .04 = \$20$, the interest for 1 year.
$\$30 \div \$20 = 1.5 = 1$ yr. 6 mo.

Analysis.—The interest of \$500 for 1 year at 4% is \$20, and, to produce \$30 in the same time and at the same rate, it will require as many years as \$20 is contained times in \$30, which is $1\frac{1}{2}$ times.

Therefore, \$500 will gain \$30 interest at 4% in 1 year 6 months.

ART. 325.—Rule for finding the time, when the principal, interest and rate per cent. are given *Divide the given interest by the interest of the principal for* 1 *year at the given rate, and reduce the decimal, if any, to months and days.*

Formula.—*Time = interest ÷ (principal × rate %).*

WRITTEN EXERCISES.

In what time will—

2. \$160 gain \$13.28 at 6% ?
3. \$3,600 gain \$126 at $3\frac{1}{2}$% ?
4. \$340.20 gain \$54.25 at 6% ?
5. \$360 gain \$40.20 at 5% ?
6. \$270 gain \$33.60 at 4% ?
7. \$306 gain \$94.962 at 7% ?
8. \$273 gain \$71.162 at 8% ?

9. In what time will a sum of money double itself at 4% ? at 5% ? at 6% ? at 7% ? at 8% ? at 10% ? at 12% ?

10. Illustrate by an original problem the method of finding the time, when the principal, interest and rate are given.

ART. 326.—To find the rate, when the principal, interest and time are given.

1. At what rate will \$450 gain \$30 in 1 year 8 months ?

Process.

$\$450 \times 1\frac{2}{3} \times .01 = \$7.50$

$\$30 \div \$7.50 = 4$

Analysis.—The interest on \$450 for $1\frac{2}{3}$ years, at 1% is \$7.50 ; to produce \$30 in the same time, it will require a rate equal to as many times 1% as \$7.50 is contained times in \$30, equal to 4 times 1%, or 4%.

Therefore, \$450 will gain \$30 in 1 year 8 months at 4%.

ART. 327.—Rule for finding the rate, when the principal, interest and time are given.—*Divide the given interest by the interest of the principal for the given time at one per cent.*

Formula.—*Rate* = *interest* ÷ (*principal* × 1% × *time*).

WRITTEN EXERCISES.

At what rate will—

2. $250 gain $52.50 in 3 years 6 months?

3. $3,400 gain $391 in 2 years 3 months 18 days?

4. $1,500 gain $151.083⅓ in 1 year 5 months 8 days?

5. $185.80 amount to $313.03 in 7 years 7 months 9 days?

6. $2,500 amount to $3,873⅓ in 6 years 10 months 12 days?

7. $1,350.50 gain $1,350.50 in 12 years 6 months?

8. A lady's investment of $1,600 brings her in a semi-annual dividend of $36: what rate of interest does she receive on her investment?

9. A man built a hotel costing $120,000, and rented it for $6,600 per year: what per cent. interest did his money yield him?

10. A man who has $1,650 in a savings bank receives $41.25 every 6 months: what per cent. interest does he receive on his money?

11. A man who gave his note for $1,250, payable in 8 months, paid at its maturity $1,291.66⅔: what was the rate of interest?

12. A note for $320.25, dated February 20, 1886, was paid September 20, 1886, when the amount due was $333.33: what was the rate?

13. A capitalist who invested $28,000 in mining stock, drew a quarterly dividend of $700: what per cent. interest did his investment yield him?

14. At what rate per annum will any sum double itself in 16 years 8 months? In 10 years? In 8⅓ years? In 6 years?

15. Illustrate by an original problem the method of finding the rate per cent., when the principal, interest and time are given.

Compound Interest.

ART. 328. –Compound Interest is interest on the principal and interest. When the interest is unpaid, the amount forms a new principal on which interest is computed for the next period. Compound interest on debts is illegal and its payment cannot be enforced.

The interest on investments, however, may be compounded annually, semi-annually, quarterly, etc.

WRITTEN EXERCISES.

1. What are the amount and compound interest on $480 for 3 years, 8 months at 6%.

Process.

$480	= 1st principal.
.06	
$28.80	= Int. for 1st year.
$480	
$508.80	= 2d principal.
.06	
$30.53	= Int. for 2d year.
$508.80	
$539.33	= 3d principal.
.06	
$32.36	= Int. for 3d year.
$539.33	
$571.69	= 4th principal.
.04	
$22.87	= Int. for 8 mo.
$571.69	
$594.56	= Amt. for 3 yr. 8 mo.
$480.00	= 1st principal.
$114.56	= Comp. Int.

Analysis.—The amount of $480 for 1 year at 6% is $508.80; taking this as a new principal, it amounts in 1 year to $539.33; in the same manner, $539.33 in one year amounts to $571.69, which in the 8 months remaining, amounts to $594.56. Since $480 is the original principal, the compound interest is the difference between it and $594.56, which is $114.56.

ART. 329.—Rule for finding Compound Interest.—*Find the amount of the principal for the first period on which interest is due, and make it the principal for the second period.*

Find the amount of the new principal for the second period, and make it the principal for the third period Continue in this manner through each period and fraction of a period to the end of the given time.

The given principal subtracted from the last amount, is the compound interest required.

Find the compound interest on—

2. $340 for 3 years 6 months at 6%.

3. $500 for 5 years 9 months 18 days at 5%.

4. $125 for 2 years 9 months at 4% interest, compounded semi-annually.

5. Find the amount of $1,000 for 4 years at 4% compound interest.

6. Find the amount of $75.75 for 3 years 3 months 24 days at 5% compound interest.

7. Find the amount of $1,848 for 5 years 4 months 15 days at 7% compound interest.

8. What is the amount of $1,200 for 2 years at 8%, interest compounded semi-annually?

9. What is the difference between the simple and the compound interest on $180 for 2 years at 8%, if the interest is compounded quarterly?

10. Illustrate compound interest by an original problem.

NOTE—Elaborate tables are published from which the interest and amount of any sum of money, for any given time at any rate, are easily ascertainable Having learned the principles and methods governing these computations the pupils can have recourse to such aid should the necessity arise.

Partial Payments.

ART. 330. A **Promissory Note** is a written promise to pay on demand or at a specified date a certain sum of money.

ART. 331.—The **Face** of the note is the sum named in it. When a note bears interest, its amount is the **Face.**

ART. 332. The **Maker** or **Drawer** of a note is the party that signs it, and thereby promises to pay it.

ART. 333.—The **Payee** of a note is the party to whom, or to whose order, it is payable.

ART. 334.—The **Holder** of a note is its legal owner.

ART. 335.—The **Indorser** of a note is the party who writes his name on its back, and thereby becomes surety for its payment. Should the drawer of a note fail to pay it when due, the indorser is liable.

ART. 336. A **Joint Note** is one signed by two or more persons, who are jointly liable for its payment.

ART. 337. A **Joint and Several Note** is a note signed by two or more parties, who are jointly and singly liable for its payment.

ART. 338. A **Demand Note** is one payable on demand.

ART. 339.—A **Time Note** is a note payable at a future specified time.

ART. 340. A **Negotiable Note** is a note made payable to bearer, or to the order of the payee.

ART. 341.—In some of the States **Three Days of Grace** are allowed on all time notes after the time for payment expires.

A **joint note** reads: "We jointly promise to pay," etc., while a **joint and several note** reads: "We jointly and severally promise to pay," etc. **Demand notes** are payable on presentation without grace, and bear legal interest after a demand has been made. An indorser on a demand note is responsible only for a limited time, variable in different States. Notes falling due on Sunday or on a holiday must be paid the day previous. Notes made on Sunday are void. All notes should contain the words "value received."

ART. 342. Partial Payments are payments in part of notes or written obligations bearing interest.

ART. 343.—An **Indorsement** is a statement of the part payment made and the date of making it. It is written across the back of the note.

The rule prescribed by the Supreme Court of the United States for partial payments, and adopted by many of the States, is based on the following principles:

ART. 344.—*Payments must first be applied to the discharge of interest then due, and the remainder, if any, to the payment of the principal.*

Unpaid interest must not be added to the principal: only unpaid principal can draw interest.

ART. 345. United States Rule. *Find the amount of the principal to the time of the first payment; if the payment equals or exceeds the interest then due, subtract the payment from the amount, and the remainder forms a new principal.*

If the payment is less than the interest, find the amount of the first principal to the time when the sum of the payments equals or exceeds the interest due, and subtract

the sum of the payments from the amount of the principal, using the remainder for a new principal.

Proceed in this manner with the remaining payments until the date of settlement.

WRITTEN EXERCISES.

1.

$800. Austin, Texas, October 1, 1884.

On demand I promise to pay A. S. Manson, or order, Eight Hundred Dollars, with interest at 8 per cent., for value received.

Oscar E. Hazen.

The following payments were indorsed on this note:

Jan. 1, 1885, $80; April 19, 1885, $10; September 1, 1885, $240: what was due Jan. 1, 1886?

Process.

$800 = 1st principal.
16 = Int. from Oct. 1, '84, to Jan. 1, '85 (3 mo.).

$816 = 1st amount.
80 = 1st payment.

$736 = 2d principal.
39.25 = Int. from Jan. 1, '85, to Sept. 1, '85 (8 mo.).

$775.25 = 2d amount.
250. = sum of 2d and 3d payments.

$525.25 = 3d principal.
14.01 = Int. from Sept. 1, '85, to Jan. 1, '86 (4 mo.).

$539.26 = Amount due at settlement.

Analysis.—We first compute the interest on the principal to the date of the first payment. This is $16. Hence, on that day, Oscar E. Hazen owes $816. Instead of paying the note with interest, he paid $80 on it. He then owed $816 − $80 = $736.

The interest on this new principal to the next payment is $17.66, while the payment is only $10. As directed by the Supreme Court of the United States, we merely record the payment of the $10, and use the principal unchanged until the next payment is made.

The interest on $736 to the next payment is $21.59, while the payment is $240, which is in excess of the accrued interest. We therefore add the two interests together, and find they amount to $39.25, which we add to the principal, $736, and find the total indebtedness to be $775.25. From this sum we subtract the sum of the two payments ($10 + $240), obtaining a remainder of $525.25, which is the total indebtedness of Mr. Hazen, September 1, 1885. Instead of settling his indebtedness at that time he waited until January 1, 1886. Hence, he must pay the principal, $525.25, together with the interest to January 1, 1886. This interest is $14.01, which, added to the principal, gives $539.26, the total indebtedness of Mr. Hazen, January 1, 1886.

2. A mortgage of $1,275.50, dated Cincinnati, July 1, 1883, had the following indorsements:

Received, Jan. 1, 1884, $600; Oct. 19, 1884, $250; Jan. 18, 1885, $300; September 18, 1885, $175. It was paid off March 1, 1886: what sum was then due, interest being computed at 6%?

3. A note for $675, dated Milwaukee, March 4, 1884, had indorsements as follows:

Aug. 16, $180; Nov. 30, $15; Feb. 27, 1885, $10; May 11, $240; Dec. 31, $150: what was due April 1, 1886, at 7% interest?

4. A note for $1,450, dated New Orleans, February 3, 1884, was indorsed as follows:

April 3, 1884, $64.10; July 9, 1884, $168.67; Nov. 18, 1885, $20; March 18, 1886, $21: what was due June 14, 1886, interest at 5%?

5. $600. Kansas City, April 1, 1884.

On demand, I promise to pay Warren Grafton, or order, six hundred dollars, with interest at 6%, value received. Hiram H. Watkins.

Indorsements: July 16, 1884, $63.20; Oct. 16, 1884, $58.05; Dec. 30, 1884, $154.90; May 17, 1885, $9; September 29, 1885, $9: what was due January 1, 1886?

6. Illustrate Partial Payments by an original problem.

Notes and interest accounts payable within a year, are generally settled by the

MERCHANTS' RULE.

ART. 346.—*Find the amount of the principal and of each payment to the time of settlement.*

From the amount of the principal, subtract the amount of the payments, and the remainder will be the amount due.

NOTE Interest under the Merchants' Rule is based on 365 days to the year.

7. A note of $500, dated June 1, 1885, had three indorsements: Aug. 1, $120; Oct. 1, $100; Nov. 16, $25: what was due Dec. 28, 1885, interest at 6%?

Process.

Amount of $500, from June 1 to Dec. 28, 210 days....		$517.26
Amount of $120, Aug. 1 to Dec. 28, 149 days	$122.94	
Amount $100, from Oct. 1 to Dec. 28, 88 days	101.45	
Amount $25, from Nov. 16 to Dec. 28, 42 days. ..	25.17	
		249.56
Balance due..............................		$267.70

8. A note of $600, dated May 1, 1885, and drawing interest at 7%, was indorsed as follows: June 20, $75; Aug. 1, $100; Oct. 5, $110: what was due Feb. 1, 1886?

9. A note given for $1,000, dated February 1, 1885, bore the following indorsements: March 1, $200; May 10,

\$100; July 1, \$150; Oct. 15, \$40: what was due Jan. 1, 1886, interest at 6%?

10. Illustrate the Merchants' Rule for Partial Payments by an original problem.

Discount and Present Worth.

ART. 347. **Discount** is a deduction made for the payment of a debt before it is due.

ART. 348. The **Present Worth** of a debt is the debt less the discount.

ART. 349. The **True Present Worth** of a debt, due at a future time without interest, is a sum which, if placed at legal interest, will amount to the debt at the time it becomes due.

ART. 350. The **True Discount** is the debt less the true present worth.

WRITTEN EXERCISES.

1. Find the true present worth and discount of \$330, payable in 1 year and 8 months, at 6%.

Process.

$\$330 \div \$1.10 = \$300$, the present worth.
$\$330 - \$300 = \$30$, the true discount.

Proof. $\$300 \times .10 = \$30$, the interest of \$300, for 1 year and 8 months, at 6%.

Analysis.—Since \$1.10 is the amount of \$1, at interest for 1 year and 8 months at 6%, it will require as many times \$1 as \$1.10 is contained times in \$330, to amount to \$330 in the same time and at the same rate.

ART. 351.—Rule for finding the present worth and the true discount of a debt, payable at a future time.—*Divide the debt by the amount of one dollar for the given time at the given rate, and the quotient is the present worth.*

The difference between the present worth and the debt is the true discount.

Find the present worth and true discount of

2. \$394.40 at 5%, due in 3 years 2 months 12 days.

3. \$428.75 at 7%, due in 1 year 6 months.

4. \$1,100 at 8%, due in 2 years 9 months.

5. \$2,000, due in 93 days, at 6%.

6. Which is the more advantageous for me to buy - molasses at \$24 a hogshead cash, or for \$25 on 6 months' credit, money being worth 6%?

7. Which is worth more, and how much—\$500 due in 12 months, or \$485 cash, money being worth 5%?

8. Mr. Burton bought a bill of goods amounting to \$850 on 8 months' credit, but accepted the merchant's offer of 5% discount for cash. If money is worth 6%, how much would Mr. Burton gain or lose by accepting the proposal?

9. I paid \$1,200 cash for a horse, and sold it to Mr. Work for \$1,300, payable in 1 year. If Mr. Work discounted his own note for that sum at 8%, what was my profit?

10. Mr. Barron bought a bill of goods amounting to \$1,000, of which \$600 was payable in 3 months and \$400 in 6 months. If he decided to pay at once, how much was the bill, discount for cash being at the rate of 6% per annum?

11. I held two notes against Mr. Wilbur; one for \$450, dated January 1, 1886, due in 7 months, and the other for \$680, dated March 1, 1886, due in 1 year. Mr. Wilbur called on me April 1, 1886, and offered to purchase both notes. How much did I receive for them, if they were discounted at $\frac{3}{4}$% a month?

12. Illustrate True Discount by an original problem.

NOTE.—It is usual in business transactions for the seller to deduct a certain per cent. for cash. A payment made within 30 days after purchase is considered cash; "spot cash" is payment when the goods are received and the invoice is found correct.

Bank Discount.

When a person wishes to borrow money from a bank, he submits his note, indorsed by some responsible party. The note is payable at a certain time after date. When discounted at a bank, three days of grace in most States are added to the time it has to run, and the interest on the face of the note for that period is deducted by the bank. The remainder is paid, or is placed to the credit of the one in whose favor the note is drawn.

ART. 352.—**Bank Discount** is interest on the face of a note, deducted in advance, and reckoned from the date of discount to the date of payment including both days.

ART. 353.—**Days of Grace** are the three days added to the time named in the note.

ART. 354. The **Proceeds of a Note** is the face less the discount.

ART. 355. The **Term of Discount** is the time which the note has to run after being discounted.

ART. 356.—A **Protest** is a legal notice of the non-payment of a note when due, and is made by a notary-public to the indorser or indorsers of a note.

When a note bearing interest is discounted by a bank, the sum discounted is the amount of the note at maturity.

ART. 357.—To find the Bank Discount and the Proceeds of a Note.

WRITTEN EXERCISES.

1. What are the proceeds and bank discount on a note of $1,250, payable in 91 days, 3 days' grace being allowed, and the discount being at 5%?

Process.—The interest on $1,250, for 94 days, at 5%, is $16.31, which is the bank discount, the proceeds being $1,250 − $16.31 = $1,233.69.

ART. 358.—Rule for finding the Bank Discount and the Proceeds of a Note. *Compute the interest on the face of the note for the time it has to run. This will give the discount, which subtracted from the face of the note leaves the proceeds.*

2. Find the proceeds of a note of $840, discounted for 85 days at 5%.

3. What is the discount on a note of $150, discounted 93 days before due, at 7%?

4. Find the discount and the proceeds of a note of $280, discounted at 6%, 75 days before due.

5.

$550. St. Louis, Jan. 10, 1886.

Ninety days after date, I promise to pay to the order of W. S. Yard, five hundred fifty dollars, at the First National Bank, value received. George W. Ellis.

Discounted at 6%, January 10: find the proceeds.

6.

$450. Detroit, October 13, 1885.

Three months after date, I promise to pay A. R. Bowdoin & Co., or order, four hundred fifty dollars, value received. Edwin O. Chapman.

Discounted at 6%, Oct. 13: find the proceeds.

7.

$390. New Orleans, December 20, 1885.

Sixty days after date I promise to pay G. C. Meigs & Brother, three hundred ninety dollars, at the First National Bank, value received. Wilmot E. Ellis.

Discounted Dec. 20, at 5%: what is the discount?

8.

$870. New York, January 15, 1886.

Thirty days after date I promise to pay Frederic B. Schell, or order, at the Park Bank, eight hundred seventy dollars, value received. Edwd. B. Bensell.

Discounted at 6%, Jan. 15: what is the discount?

9.

$1,000. Little Rock, Feb. 1, 1886.

Six months after date, for value received, I promise to pay Horatio Alger, Jr., or order, one thousand dollars, with interest at 6%.

Charles A. Fosdick.

Discounted Feb. 1, 1886, at 6%: find the proceeds.

10. What is the difference between the true discount and the bank discount on $500, due in 1 year 8 months, money worth 6%, and not reckoning days of grace?

11. Illustrate Art. 357 by an original problem.

ART. 359.—To find the face of a note, the proceeds, time and rate being given.

12. Mr. Worthington wishes to borrow $800 from a bank for 90 days: for what sum must he give his note, the rate of discount being 6%?

Process.—The bank discount of $1 for 93 days at 6% is $.0155 and the proceeds $.9845. To give $800 proceeds for the same time and at the same rate, will require as many times $1 as $.9845 is contained times in $800, or $812.60.

ART. 360. Rule for finding the Face of a Note when the proceeds, time and rate are given.—*Divide the proceeds by the proceeds of one dollar for the given time and at the given rate: the quotient is the face of the note.*

13. For what sum must a note be drawn at 60 days to net $1,500 when discounted at 5%?

14. If money is worth 5%, for what sum must a note be drawn for 90 days to yield $1,776.75?

15. What must be the face of a note due in 4 months, discounted at 6%, that the proceeds may be $600?

16. What must be the face of a note due in 3 months, discounted at 5%, that the proceeds may be $1,200?

17. Illustrate Art. 359 by an original problem.

ART. 361.—To find the time, the face, rate and proceeds being given.

18. Mr. Dobbins's note of $1,000 discounted at 6% yields $989.64: what is the time?

Process. If the face is $1,000 and the proceeds $989.64, the discount is $1,000 − $989.64 = $10.36. The discount on $1,000 at 6% for 1 day (reckoning 365 days to the year), is $$\frac{60}{365}$ or $$\frac{12}{73}$, and to produce $10.36 it will require as many days as $$\frac{12}{73}$ is contained times in $10.36, or 63 days. 63 days − 3 days grace = 60 days.

NOTE.—The pupil should frame the rule.

19. The proceeds of a note for $500, discounted at 6%, being $494.75, what is the time?

20. The proceeds of a note for $3,000, discounted at a bank at 6%, being $2,953.50, find the time.

21. A note dated January 1, 1886, was discounted April 1, at 7%; the face was $3,500 and the proceeds $3,437.57: what was the time?

22. A note bearing interest at 6%, dated October 1, 1885, at 60 days, was discounted at 6%; the face was $950 and the proceeds $942.97: on what day was it discounted?

23. Illustrate Art. 361 by an original problem.

ART. 362.—To find the rate, when the face, time and proceeds are given.

24. Mr. Waters gave his note for $350 at 60 days; the proceeds at a bank being $347.55, what was the rate?

Process.—The discount of $350 at 1 per cent. for the given time is $.6125, and to produce $2.45 will require a rate equal to the number of times $.6125 is contained in $2.45, or 4. Therefore, the rate is 4%.

NOTE.—The pupil should frame a rule.

25. The proceeds of a note for $400, at 30 days, being $397.80, what is the rate?

26. A merchant bought goods to the amount of $3,413.77, and gave in payment his note for 60 days, discounted at a bank: if the face of the note was $3,450, what was the rate?

27. A note dated July 1, 1886, at three months, was discounted at a bank on July 31, 1886; the face was $2,400 and the proceeds $2,369.66: what was the rate?

28. A note of $1,800, dated November 19, 1885, at 4 months, was discounted December 19, the proceeds being $1,767.45: what was the rate?

29. A note of $7,700 discounted at 50 days gave $7,631.98 proceeds: what was the rate?

30. $800 discounted for 63 days, including three days of grace, yielded $793; what was the rate?

REVIEW QUESTIONS.

What is interest? What is the principal? The rate of interest? The time? The amount?

In computing interest, how many days are generally considered a month? Give the rule for computing interest.

How do you find the principal when the interest or amount, and the rate and time are given? How do you find the time when the principal, interest and rate per cent. are given? How do you find the rate when the principal, interest and time are given?

What is compound interest? Give the rule for computing compound interest.

What is a promissory note? What are days of grace?

What are partial payments? Give the United States Rule for partial payments. Give the Merchants' Rule.

What is discount? Give the rule for finding the present worth and the true discount.

Explain the method of borrowing money from a bank. Give the rule for finding the bank discount and the proceeds of a note.

Exchange.

ART. 363.—Exchange is a method of paying debts to parties at a distance by means of *Drafts* or *Bills of Exchange.*

ART. 364.—Exchange is of two kinds, *Domestic* and *Foreign.*

ART. 365.—Domestic or **Inland Exchange** is that between places in the same country.

ART. 366.—Foreign Exchange is that between different countries.

ART. 367. A **Draft** or **Bill of Exchange** is a written order for the payment of money

ART. 368.—The **Drawer** is the person that makes the order.

ART. 369. The **Drawee** is the one to whom it is addressed.

ART. 370.—The **Payee** is the one to whom the money is payable.

ART. 371.—A Sight Draft is payable when presented. In some States, however, three days of grace are allowed.

ART. 372. A **Time Draft** is payable a certain time after date, or after sight, including 3 days of grace.

ART. 373. The **Indorsement** of a draft is a writing on the back, by which the payee orders the payment of the draft to be made to another party.

ART. 374. A **Letter of Credit** is a letter from one banking house to one or more others in foreign countries, directing them to pay the person in whose favor the letter is drawn, any amount of money not exceeding the sum named in the letter.

ART. 375.—A **Special Indorsement** is an order to pay a draft to a specified party who is called the *Indorsee.*

ART. 376.—The **Acceptance** of a draft is a written agreement of the drawee, made when the draft is presented, to pay it at maturity. This is done by writing across the face "Accepted," with the date and the drawee's signature. The draft then becomes an *Acceptance*, and is of the same nature as a promissory note.

Days of grace are usually allowed on bills of exchange unless a special date is named on which they mature. In New York and several other states no grace is allowed on sight bills.

ART. 377.—The **Rate of Exchange** is the rate above or below par.

ART. 378. The rate of exchange between two places depends chiefly on their relative trade. If the trade between St. Louis and San Francisco is equal, exchange is at par. If St. Louis owes San Francisco, the call in St. Louis for drafts on San Francisco exceeds the demand in San Francisco for drafts on St. Louis; and, as a consequence, the drafts in St. Louis on San Francisco are at a premium, while in the latter city, St. Louis drafts are at a discount.

The cause for this premium and discount is that in St. Louis the banks are forced to the expense of sending the money itself to San Francisco, or be charged with interest on the balance due the San Francisco banks; the latter sell the St. Louis drafts at a discount because they are willing to bear such discount in order to receive the money without delay.

If the drawer finds the course of exchange against him, he sometimes equalizes it by the roundabout method of drawing through several intermediate places, between which the rates are favorable.

ART. 379.—Domestic Exchange.

BANK DRAFT (SIGHT).

First National Bank, St. Louis,
St. Louis, Mo., January 12, 1886.

$5,000.

At sight pay to the order of George W. Smith five thousand dollars.

William M. Cooper, *Cashier.*

To Taylor Bros.,
Detroit, Mich.

Explanation.—George W. Smith, of St. Louis, owed Thorndike Preston, of Detroit, $5,000, and procured the above draft from a bank in St. Louis, by depositing the amount (with premium added or discount deducted, as the case may be), if it did not already stand to his credit. He wrote on the back of the draft, "Pay to the order of Thorndike Preston," and signed his name. He then forwarded it to Thorndike Preston, Detroit, who indorsed it with his own name, and presented it to Taylor Bros., bankers in Detroit, where it was immediately paid, or if he preferred, placed to his credit.

MERCHANT'S DRAFT (TIME).

St. Louis, Mo., May 21, 1886.

$10,000.

Sixty days after sight, pay to the order of W. D. Holt, ten thousand dollars, and charge to account of

Becktold & Co.

To William T. West,
New York City.

Explanation.—Becktold & Co., of St. Louis, are indebted to William D. Holt, of New York, and William T. West, of New York, owes Becktold & Co., of St. Louis. In order to save the expense of shipping money to and from New York, Becktold & Co. draw upon William T. West, and remit the draft to W. D. Holt, of New York. Mr. Holt presents this draft to William T. West,

who accepts it by writing across its face *in red ink*, the words: "Accepted, May 24, 1886, William T. West." The draft now becomes an acceptance, and William T. West is responsible for its payment. Mr. Holt may now present this acceptance at bank for discount, in the same manner as negotiable notes are presented. In order to hold the drawer of a draft responsible for its payment in case the drawee does not accept it, the payee must have it protested for non-acceptance.

ART. 380.—To find the cost of a sight or a time draft.

WRITTEN EXERCISES.

1. What is the cost of a sight draft on Cincinnati for $1,250, at 1¼% premium?

Process.—Since the rate is 1¼%, the course of exchange is $1.01¼ on every $1. The cost of $1,250, therefore, is $1.01¼ × 1,250 = $1,265.63.

NOTE.—The pupil should frame a rule.

2. What is the cost of a draft for $1,000, payable in 60 days after sight, at 5% interest, exchange at ½% premium?

Process.—Since the premium is ½%, the rate of exchange is 1.00½. The bank discount at 5% for 63 days is $.00875; the cost of exchange for $1, therefore, is $1.005 − $.00875 = $.99625, and the cost of $1,000 is $.99625 × 1,000 = $996.25.

NOTE.—The pupil should frame a rule.

Find the cost of—

3. A sight draft for $480, premium 2¼%.

4. A sight draft for $900, discount 1¾%.

5. A sight draft for $800, premium ¾%.

6. A sight draft for $1,885, premium 1⅝%.

7. A draft for $840, premium 1¼%, time 80 days, interest 7%.

8. A draft for $2,500, discount 1¼%, time 90 days, interest 6%.

9. A draft for \$4,700, discount $\frac{3}{8}$%, time 60 days, interest 5%.

10. A draft for \$500, discount $\frac{1}{8}$%, time 30 days, interest 5%.

11. A draft for \$10,000, premium $1\frac{1}{2}$%, time 70 days, interest 7%.

12. Illustrate Art. 379, by an original problem.

ART. 381.—To find the face of a sight or a time draft.

WRITTEN EXERCISES.

1. How large a draft will \$1,989 buy, exchange being 2% premium?

Process.—Since the rate of premium is 2%, the cost of exchange for \$1 is \$1.02. \$1,989, therefore, will buy as many times \$1 as \$1.02 is contained times in \$1,989, equal to \$1,950.

NOTE.—The pupil should frame a rule.

2. What is the face of a draft on New Orleans, at 90 days, bought for \$1,989, exchange being 101, interest 6%?

Process.—Since the course of exchange is 101, the cost of \$1 is \$1.01, payable at sight; but the bank retains the money 93 days, and therefore must allow bank discount for that time. Hence, the cost of exchange is \$1.01 − \$.0155 = \$.9945, and \$1,989 will buy as many dollars as \$.9945 is contained times in \$1,989, equal to \$2,000.

NOTE.—The pupil should frame a rule.

What is the face of a draft that costs—

3. \$1,200, premium $1\frac{1}{4}$%? 6. \$600, premium $\frac{1}{4}$%?

4. \$1,700, premium $1\frac{3}{4}$%? 7. \$900, discount $\frac{1}{5}$%?

5. \$1,300, discount, $\frac{3}{4}$%? 8. \$1,100, discount $\frac{4}{5}$%?

9. What is the face of a 90-day draft costing \$800, premium $1\frac{1}{4}$, interest 6%?

ART. 382.—Foreign Exchange.

£500. New York, March 2, 1886.

Thirty days after sight of this First of Exchange (Second and Third unpaid), pay to the order of J. J. Little, five hundred pounds sterling, for value received, and charge the same to account of

To Harvey & Son, Johnson Brothers.
London, Eng.

The foregoing is the form of the first bill of exchange. Two other bills are forwarded (changed only in the substitution of the words "Second" and "Third" respectively for "First") and, with a view of guarding against loss, each is sent in a different manner. The three form a *Set of Exchange,* and when one is paid the others become valueless.

Most of the foreign exchange is effected through the great commercial centers of London, Paris, Berlin, Antwerp, Frankfort, Hamburg, Bremen, and Amsterdam.

WRITTEN EXERCISES.

1. What is the cost of a bill of exchange on London for £350 5s. 6d., exchange being at $4.87 ?

Process.—£350 5s. 6d. = £350.275; $4.87 × 350.275 = $1,705.84

2. What is the cost of a bill of exchange on Paris for 4,500 francs at $.193 ?

Process.—$.193 × 4,500 = $868.50.

ART. 383. Rule for finding the cost of Exchange.—*Multiply the cost of a unit of the currency in which the bill is given by the face of the bill.*

3. What is the cost of a bill on Berlin for 2,700 marks at $.238 ?

4. What is the cost of a bill on Amsterdam for 3,500 guilders at \$.40½, brokerage ¼%?

5. What is the cost of a bill on Madrid for 4,000 pesetas at \$.193, brokerage ¾%?

6. What is the cost of a bill on Edinburgh for £1,250 at \$4.87, brokerage ⅛%?

7. Find the cost of a bill on Bremen for 1,780 reichsmarks at \$.238, brokerage ⅛%.

8. Messrs Ribsam & Co., of Hamburg, remitted 5,000 reichsmarks to their correspondents in New York. What was the face of the draft, exchange being at \$.238?

9. A gentleman in St. Louis obtained letters of credit from Seligman & Company. In Paris he drew 1,500 francs, and in London £100: what did it cost him, if exchange in Paris was ½% premium, in London 1%, and gold was worth \$1.10?

10. Illustrate by an original problem the method of determining the cost of a bill of foreign exchange.

Equation of Payments.

ART. 384.—Equation of Payments is the method of finding the medium or average time for the payment of several debts due at different times.

ART. 385.—The **Equated Time** is the date on which the several sums due at different times should be paid.

ART. 386.—The **Time of Credit** is the time the debt has to run before it becomes due.

ART. 387.—The **Average Term of Credit** is the time at the end of which the several debts, due at different times, are equitably payable in one sum.

ART. 388.—The **Focal Date** is the date from which the computation is begun in finding the equated time of the several debts and credits.

ART. 389.—To find the equated time where the terms of credit begin at the same date.

WRITTEN EXERCISES.

A owes B \$300 payable in 4 months; \$500 payable in 6 months and \$200 payable in 8 months: when should the whole be paid so that neither may lose?

Process.

$$300 \times 4 = 1,200$$
$$500 \times 6 = 3,000$$
$$200 \times 8 = 1,600$$
$$1,000 \qquad 5,800$$
$$5,800 \div 1,000 = 5\tfrac{4}{5}.$$

Analysis.—The interest on \$300 for 4 months equals the interest on \$1 for 1,200 months; the interest on \$500 for 6 months equals that on \$1 for 3,000 months, and the interest on \$200 for 8 months equals that on \$1 for 1,600 months. Hence, the interest of the several amounts for the several periods equals the interest on \$1 for 5,800 months, and on \$1 000 it will be $\frac{1}{1000}$ of 5,800 months $= 5\frac{4}{5}$ months, which is the average term of credit from the date of the first indebtedness.

ART. 390.—Rule for finding the equated time, when the terms of credit begin on the same date. *Multiply each debt by its term of credit and divide the sum of the products by the sum of the payments.*

NOTE. Whenever cents appear in the debts, they should be rejected if less than 50 and counted as \$1 if equal to or more than 50. If the fraction of a day is less than $\frac{1}{2}$, it is rejected, and if equal to or greater than $\frac{1}{2}$, it is counted as 1 day.

2. Alexander Massey owed John Miller \$4,000, $\frac{1}{4}$ of which was due in 3 months, $\frac{1}{6}$ in 5 months, and the remainder in 8 months: what was the average term of credit?

3. James Gardner owed a merchant \$1,200, due in 3 months; \$900 due in 4 months; \$800 due in $4\frac{1}{2}$ months: what was the average term of credit?

4. John Spencer purchased a bill of goods, January 1, for \$1,600. Of this amount, \$400 was due in 1 month, \$800 due in 3 months, and \$400 due in 6 months: what was the average term of credit?

5. A merchant owed \$3,000, of which $\frac{1}{3}$ was due in $3\frac{1}{2}$ months, $\frac{1}{3}$ in 4 months, and $\frac{1}{3}$ in 6 months: find the equated time of payment.

6. Mr. Graham's debt of \$10,000 was due as follows: $\frac{1}{5}$ in $4\frac{1}{2}$ months, $\frac{2}{5}$ in $5\frac{1}{2}$ months, and $\frac{2}{5}$ in 7 months: what was the equated time?

7. Mr. Braham owed \$1,500.75 due in 10 days; \$300.50 due in 15 days, \$800.25 due in 20 days: what was the average term of credit?

8. Illustrate Art. 389 by an original problem.

ART. 391.—To find the equated time, when the terms of credit begin on different dates.

1. A merchant laid in a supply of goods as follows:

Jan. 1, a bill of \$800, on 3 months' credit;

Jan. 25, a bill of \$600, on 4 months' credit;

March 15, a bill of \$500 on 4 months' credit.

If these bills were all paid at once, on what day should such payment have been made?

Process.

April 1,	800 ×	00 =	00,000
May 25,	600 ×	54 =	32,400
July 15,	500 ×	105 =	52,500
	1,900		84,900

Analysis.—84,900 ÷ 1,900 = $44\frac{13}{19}$, or 44 days, from April 1 (when the first payment was due), which makes the equated time May 15.

ART. 392.—Rule for finding the equated time when the terms of credit begin on different dates. *Take as the focal date the date on which the first debt becomes due, and multiply each debt by the term of credit, reckoned from that date. The sum of these products, divided by the sum of the debts, is the average term of credit, reckoned from the focal date.*

2. Mr. Smith bought, Feb. 10, $500 worth of goods on 3 months' credit; March 20, $450 on 3 months' credit; April 10, $600 on 4 months' credit; May 15, $800 on 4 months' credit: what was the equated time of payment?

3. Mr. Brown bought a bill of goods, on March 20 on 6 months' credit, amounting to $40; March 25 a bill on 4 months' credit, amounting to $30; April 24, on 6 months' credit, a bill of $30; June 17, on 3 months' credit, a bill of $50: what was the equated time of payment?

4. Illustrate Art. 391 by an original problem.

Averaging Accounts.

ART. 393.—Averaging Accounts is the process of finding the average time for the payment of the balance of an account.

1. My ledger contains the following:

Dr. W. R. Glen. Cr.

1885.			1885.		
March 1.	To Mdse. @ 3 mo.	$300	June 10	By Cash	$200
" 15.	" " @ 4 " ..	400	" 20.	"	300
" 30.	" " @ 4 " ..	600	July 1	" ..	500

What is the equated time for the payment of the balance?

Process.—We take as the focal date the earliest date on which any item on either side matures. The latest date, or a date intermediate between the earliest and the latest, may be taken as the focal date, but the earliest is most convenient.

June 1,	300	× 00 =	00	June 10,	200	× 9 =	1800
July 15,	400	× 44 =	17600	June 20,	300	× 19 =	5700
July 30,	600	× 59 =	35400	July 1,	500	× 30 =	15000
	1300		53000		1000		22500
	1000		22500				
	300		30500				

$30500 \div 300 = 101\frac{2}{3}$, or 102 days after June 1, or Sept. 11.

Analysis.—By the process already explained we find that the items on the Dr. side are entitled to the discount of $1 for 53,000 days, while those on the Cr. side are entitled to a discount of $1 for 22,500 days. The difference shows the Dr. side entitled to the discount of $1 for 30,500 days, and $300 therefore is entitled to $\frac{1}{300}$ of 30,500 days, or 102 days. Hence, the balance is due in 102 days after June 1, which is September 11.

ART. 394.—Rule for averaging accounts.—*Take as the focal date the date when the first payment is due, and multiply each item by the number of days it falls due after the focal date. Multiply also the amount of each payment by the number of days it is made after the focal date.*

The difference between these respective products, divided by the difference between the columns of items gives a quotient, which, added to the focal date, is the equated time

If the balances are on opposite sides of the account, the quotient must be subtracted from the focal date.

2. Find the equated time of the following:

Dr. Frederic R. Linden. Cr.

1885.			1885.		
July 1.	To Mdse. @ 2 mo. ..	$600	Sept. 21.	By Cash .	$500
" 21.	" " @ 2 " ..	500	" 25.	"	600
Aug. 20.	" " @ 3 " ..	600	Oct. 1	.	[illegible]
Sept. 10.	" " @ 2 " .	3000			

3. Find the equated time of the following:

Dr. Richard Linn. Cr.

1885.			1885.		
April 10.	To Mdse. on 2 mo. .	$700	July 15.	By Cash .	$450
May 20.	" " " 3 " ..	750	Sept. 10.	"	550
July 30.	" " " 3 " ..	840	Oct. 20.	"	600

4. Find the equated time of the following:

Dr. George H. Streeter. Cr.

1885.			1885.		
July 1	To Mdse. @ 2 mo	$600	Sept. 21..	By Cash	$500
July 21	" @ 2 mo	500	Sept. 25..	" " .	600
Aug 20	" " @ 3 mo...	600	Oct. 1.	"	400
Sept. 10	" @ 2 mo	300			

5. G. D. W. Vroom gave his note drawing interest for the balance of the following account: allowing 3 days grace on each item, what should be the date of the note?

Dr. Cr.

1885.			1885.		
May 1....	To Mdse. on 30 days	$120	May 21	By Mdse. on 30 days.	$180
May 25	" " "	430	May 20	" " "	220
May 30	" " "	100	May 27	" " "	50

6. Edward Perkins gave his note for the balance of the following account, what should the date of the note be?

Dr.				Cr.		
1885.				188 .		
Feb. 6. .	To Mdse. @ 3 mo....	$850		April .	By Cash.........	$50
March 16.	" " @ 30 days.	470		May 12	"	600
April 10..	" " @ 4 mo. .	640		May 28	Note @ 60 days .	350
May 15	" " @ 90 days	730		June 23	By Cash... .	540
June 20..	" " @ 60 days	950		Aug 5	"	600

7. Illustrate by an original problem the method of finding the equated time.

REVIEW QUESTIONS.

What is exchange? What is domestic or inland exchange? Foreign exchange? A draft or bill of exchange? Who is the drawer? The drawee? The payee? How are bills of exchange generally drawn? Why? What do these bills together make? What of days of grace?

What is a sight bill? A time bill? What is an indorsement? The acceptance? How is a bill accepted? What then does the bill become? What of days of grace?

What is said of the date of a bill made payable at some future time? When is a draft at par? Above par? At a discount?

Explain how the rate of exchange between two places is governed. How is the cost of a sight or a time draft found? The face of a sight or a time draft?

Explain the method of foreign exchange. What forms a set of exchange?

How is most of the foreign exchange effected? What is a letter of credit? Give the rule for finding the cost of exchange.

What is equation of payments? What is the equated time? The time of credit? The average term of credit? The focal date? How is the equated time found when the terms of credit begin at the same date? When they begin on different dates? What is averaging accounts? Give the rule.

Review Problems in Percentage.

ORAL EXERCISES

ART. 395.—1. A insured his store for $8,000 at $1\frac{1}{2}\%$: what was the premium?

2. What is the premium for an insurance of $6,400 on a house and furniture at $2\frac{1}{2}\%$?

3. I insured my furniture for $1,600, and my library for $1,400, at $1\frac{1}{2}\%$: what was the premium?

4. If I pay $12 for an insurance of $1,000, what is the rate?

5. What amount of insurance at 3% can be obtained for $60?

6. At $2\frac{1}{2}\%$ what amount of insurance can be obtained for $25?

7. What is the specific duty on 10 cases of shawls containing 50 each, at $1.25 per shawl?

8. What is the duty at 5 cents a pound on 10 sacks of coffee, each containing 50 pounds, tare being 2%?

9. What is the ad valorem duty at 25% on 40 watches, invoiced at $30?

10. What is the ad valorem duty at $33\frac{1}{3}\%$ on 12 chests of tea, each weighing 55 pounds, and invoiced at 40 cents a pound, tare being 5 pounds a chest?

11. What tax must be paid on property worth $8,400 at $1\frac{1}{2}\%$?

12. What tax at 2% must be paid on property assessed at $6,200?

13. What is the amount of A's tax, who pays at the rate of 2% on $10,000 worth of property, and who pays for 3 polls at $1.50 each?

14. The real and personal property of a town is assessed at $850,000: what is the total amount of tax at 2%?

15. If the rate is 5 mills on a dollar, and I pay $50 tax, what is the assessment on my property?

16. B's entire tax was $63. He paid for 2 polls at $1.50 each, and the rate was 6 mills on a dollar: what was the amount of his property?

17. What is the premium at 8% on 50 shares of bank stock?

18. What is the discount at 12% on 100 shares of railroad stock?

19. What is the value of 18 shares of turnpike stock at 5% premium?

20. What must I pay for 120 shares of insurance stock at 5% discount?

21. The premium on 20 shares of mining stock was $100: what was the rate of premium?

22. The discount on 40 shares of rubber company's stock was $120: what was the market value of the stock?

23. I sold 20 shares of stock at 90½, and paid ½% brokerage: how much did I receive?

24. Paid 5½% premium and ½% brokerage for 30 shares of zinc-company stock: what was the total cost?

25. A lady gave $20 for a dress and sold it at an advance of 10%: how much did she gain?

26. Paid $160 for a horse and sold it at a loss of 12½%: how much was received for it?

27. A farmer gave $50 for a cow and sold it at a gain of 7%: what did he receive?

28. A man sold a watch for $72 and thereby lost 28%: what was the cost of the watch?

29. A pony bought for 10% less than its value was sold for 10% more than its value: what was the gain %?

30. If wood is bought for $8 per cord and sold for $9, what is the gain %?

31. If goods are bought for 10% below the market price, and sold at 5% above, what is the gain %?

32. If cloth is bought at 20% below the current price, and sold at 20% above, what is the gain %?

33. What % is gained by selling goods at $\frac{1}{2}$ advance on their cost? At $\frac{1}{3}$ advance? At $\frac{1}{4}$ advance?

34. A man asked $50 for a suit of clothes, which was 20% more than their cost: if he sold the suit for $45, what % did he gain?

35. A merchant bought goods for 20% less than the market rates, and sold them at 10% above: what % did he gain?

36. A man bought a coat for $44, the dealer thereby gaining 10%; after wearing it a few days, the purchaser returned the coat and was allowed 80% of what he gave: what % did the dealer make after selling the coat for 10% less than the original cost?

37. C sold a cow for $48 and thereby cleared $\frac{1}{4}$ of that amount: what % would he have lost by selling it for $30?

38. What was the commission of an agent who charged $2\frac{1}{2}$% for selling a house for $8,000?

39. An agent's commission for buying goods at $1\frac{1}{2}$% was $30: what did he pay for the goods?

40. A collector's commission at 2% was $120: what was the amount of his collections?

41. A factor's commission at $2\frac{1}{4}$% was $27: what was the amount of his sales?

WRITTEN EXERCISES.

1. What is the annual premium for insuring at $1\frac{3}{4}$% a store for $6,800?

2. What is the premium for insuring at $2\frac{1}{4}$%, a house and furniture for $12,400?

3. What will be the cost of insuring my house for \$8,400 at $1\frac{1}{2}\%$, my furniture for \$1,500 at $\frac{3}{8}\%$, and my library for \$1,200 at $\frac{3}{8}\%$?

4. A ship worth \$16,000 was insured for $\frac{3}{4}$ of its value; the premium was \$311.25: what was the rate?

5. The cost of insuring \$10,500 on my factory was \$142.75, including \$2.75 for the policy: what was the rate?

6. An insurance of \$5,600 on a house, \$1,000 on the furniture, and \$1,200 on a barn, for 5 years, amounted to \$341.25: what was the annual rate?

7. If the annual premium for insuring a house at $1\frac{1}{2}\%$ is \$114, what is the amount of insurance?

8. If I pay \$270 for insuring goods at $2\frac{1}{2}\%$ to cover $\frac{3}{4}$ of their value, what is their total value?

9. If a store and goods are worth \$6,860, for what sum must they be insured at 2% to cover the property and premium?

10. What sum must be insured, at $1\frac{3}{4}\%$, on a consignment of coffee that cost \$412.65, to cover property and premium in case of loss?

11. What is the specific duty on 80 drums of figs, each weighing 48 pounds, tare 18 pounds to the hundred weight, at \$12 a hundred weight?

12. What is the duty on 32 hogsheads of sugar, each weighing 5 cwt. 3 qr. 5 lb., tare 18 pounds per hogshead, duty $\$1\frac{1}{2}$ per hundred weight?

13. What is the ad valorem duty, at $2\frac{1}{4}\%$, on 72 boxes of dates, 30 pounds to a box, invoiced at \$.16 a pound, the tare being 4 pounds to a box?

14. The property of a village, assessed at \$650,000, was taxed at 6 mills on the dollar, for public improvements: what amount of tax was raised?

15. What amount of tax must a man pay who is assessed \$12,500 for real estate, and \$4,800 for personal property, if the rate is 2¼%, and he pays for 3 polls at \$1.25 each?

16. A tax of \$6,200 was levied upon the taxable property of a county assessed at \$1,240,000: what was the rate, and how much will B pay on his farm which is assessed at \$7,200?

17. If a tax of \$79 is assessed upon a factory worth \$19,750, what is the assessment on a mill that pays \$44.40 at the same rate?

18. What is the cost of 109 shares of railroad stock at 3% discount?

19. A owns 48 shares of mining stock, at \$50, upon which an assessment of 3¼% was made: how much did he pay?

20. An insurance company declared a dividend of 8% payable in stock: if B owned 25 shares, how many shares did he own after receiving his dividend?

21. I received \$607.50 from a 4½% dividend: how much stock did I own?

22. Mr. C. found after receiving a stock dividend of 20% in a mining company that he owned 36 shares: how many shares did he own at first?

23. A company whose stock was \$830,000, levied an assessment of \$26560: what was the rate?

24. A lady owned 60 shares in a railroad company, whose stock was \$3,500,000, and her dividend was \$390: what was the rate of dividend, and the whole dividend?

25. If I pay \$24 to an agent for selling property for \$6,000, what % commission does he receive?

26. An agent charged \$21 for selling \$840 worth of goods: what was his rate of commission?

27. A broker received $139.49 for buying 145 shares of bank stock at $96\frac{1}{5}$: what was his rate of brokerage?

28. How many barrels of apples at $3 a barrel can be bought with a remittance of $492, after deducting $2\frac{1}{2}\%$ commission?

29. An agent received $8,990 to invest in wood at $5 a cord, after deducting his commission of $3\frac{1}{3}\%$: how many cords did he buy?

30. Bought 40 barrels of flour at $8.50 a barrel and sold it at a gain of 10%: what was the gain and the selling price?

31. What was the loss on 1,200 bushels of wheat, bought at $1.44, and sold at $12\frac{1}{2}\%$ loss?

32. By selling iron at $28 per ton a profit of $16\frac{2}{3}\%$ was realized: what was the profit on 190 tons?

33. A merchant by selling goods at a profit of 15%, cleared $630: what was the cost of the goods?

34. By selling sugar at $5\frac{1}{2}$ cents a pound, a grocer lost $8\frac{1}{3}\%$: what was the loss on 96 pounds?

35. A drover bought some cattle for $3,250, and sold them at an advance of 15%: for how much did he sell them?

36. A man gave $1,800 for a farm. He offered it for sale at an advance of 20%, but accepted 95% of the asking price: how much did he receive?

37. A dwelling house that cost $7,400 was sold for $8,140: what % was gained?

38. A dealer gained $121 on some coal, which was 11% of the cost: what was the cost?

39. A grocer paid $935 for 110 barrels of flour: for what must he sell it per barrel to gain 20%?

40. An agent bought a piano marked $440 for $33\frac{1}{3}\%$ off, and sold it at a gain of 30%: how much did he receive for it?

41. A merchant bought 48 yards of cloth at \$1.20 per yard, and sold it at a gain of $12\frac{1}{2}$%: how much did he gain?

42. A dealer bought 40 bushels of potatoes at \$.60 a bushel, and sold them so as to gain 25%: how much did he receive?

43. How must I mark cloth which cost \$2 a yard, so that I can fall $16\frac{2}{3}$%, and still gain 25%?

44. A merchant's income is \$6,000, which is $12\frac{1}{2}$% of his invested capital: what would be his income if it was 20% of his capital?

45. An owner gained 9% by selling a lot for \$3,270: how much would he have received if he had lost 9%?

46. Two farms were sold for \$4,600 each. On one the loss was 15%, and on the other the gain was 15%: what was the total gain or loss?

47. Goods bought at 20% below the market price were sold at 20% above the market price: what was the gain %?

48. A leak in a vessel ruined $12\frac{1}{2}$% of a cargo of 800 bushels of wheat; 20% of the remainder was sold: what % of the original cargo remained?

49. By selling some stocks for \$720, the owner cleared $\frac{1}{6}$ of that amount: what % would have been gained by selling them for \$700?

50. I asked \$240 for my horse, but fell \$20 and still made 10%: what did I pay for the horse?

51. A farmer asked 28% advance on the cost of his farm, but was obliged to sell at 25% less than was asked: if he received \$9,600, how much was his loss?

52. If 5 men can do a piece of work in 12 days, what % of the work can be done by 4 men in one day?

53. A boy having $\frac{4}{5}$ of a dollar, gave $\frac{1}{6}$ of it to his playmate, and spent $\frac{1}{2}$ of what remained: what % of the first sum did he then have?

54. A man sold a horse for \$240, which was $\frac{3}{4}$ of twice the sum he paid for it: what % did he gain?

55. A son's age is $\frac{2}{5}$ of his father's age: what % of the son's age is the father's age?

56. A man bought a watch and chain for \$120, and $\frac{1}{2}$ the cost of the chain was $\frac{1}{6}$ the cost of the watch: if the cost of the chain had been \$10 more than it was, what % would it have been of the cost of the watch?

57. A man and his two sons earned \$60 a week; the man earned twice as much as the elder son, and three times as much as the younger son: what % of the father's wages was earned by his two sons?

58. A pipe will fill a cistern in 4 hours, and another will empty it in 5 hours: if both pipes are left running, what % of the cistern will be filled in one hour?

59. Find the interest on \$960 for 5 years 6 months 20 days at 5%.

60. What is the amount of \$99 for 10 years 8 months at 6%?

61. How much money does a man borrow at 7%, if he pays an annual interest of \$161?

62. The interest on A's money for 3 years at 5% is \$270: how much money has he?

63. The amount of a certain principal for 1 year at 5% is \$3,675, and the amount at 7% is \$3,745: what is the principal?

64. At what rate will \$250 in 3 years 3 months gain \$40.625 interest?

65. A ninety-day note for \$450, was discounted 11 days after date at 7%: find the proceeds.

66. What is the difference between the true discount and the bank discount of \$130 for 90 days at 6%, no allowance being made for days of grace?

REVIEW QUESTIONS.

What does per cent. mean? What is percentage? The base? The rate per cent.? The amount? The difference?

How do you find the percentage when the base and rate per cent. are given? Give the formula. The rate, when the base and percentage are given? Give the formula. The base, when the rate and percentage are given? Give the formula.

What is meant by profit and loss? When is an article at a premium? When at a discount? What does the gain or loss equal? The rate? The cost?

What is an agent? By what other names is he known? What is a consignor? The consignment? The consignee or correspondent? What is commission? Brokerage? The net proceeds?

What does the commission or brokerage equal? Give the formula. The rate? Give the formula. The base? Give the formula.

What is insurance? What is life insurance? Health or accident insurance?

Who is the underwriter? What is the policy? The premium? How is the premium computed in property insurance? In life insurance?

What does the premium equal? Give the formula. The rate? Give the formula. The amount insured? Give the formula. The property and premium? Give the formula.

What are taxes? Why are taxes necessary? What is a poll or capitation tax? A property tax? Real estate? Personal property? An assessor? What does the tax equal? Give the formula. The rate? Give the formula. The amount of taxes collected? Give the formula.

What are duties and customs? Ad valorem duty? Specific duty? Tare? Leakage? Breakage? An invoice or manifest?

What is capital? A company? What is its charter? What is its stock? How is it divided? When is stock above par or at a premium? What is a dividend? An assessment? A stock broker? What is a bond?

What does the assessment or dividend equal? The rate? The discount or premium? The market value? The par value?

Aliquot Parts.

ART. 396.—An **Aliquot Part** of a number is any number, either integral or mixed, which will exactly divide it.

The aliquot parts of a dollar are:

$\$.50 = \$\frac{1}{2}$.	$\$.12\frac{1}{2} = \$\frac{1}{8}$.
$\$.33\frac{1}{3} = \$\frac{1}{3}$.	$\$.10 = \$\frac{1}{10}$.
$\$.25 = \$\frac{1}{4}$.	$\$.08\frac{1}{3} = \$\frac{1}{12}$.
$\$.20 = \$\frac{1}{5}$.	$\$.06\frac{1}{4} = \$\frac{1}{16}$.
$\$.16\frac{2}{3} = \$\frac{1}{6}$.	$\$.05 = \$\frac{1}{20}$.

ORAL EXERCISES.

1. What is the cost of 28 grammars at 50 cents apiece?

Solution.—Since one grammar costs \$.50, equal to $\$\frac{1}{2}$, the cost of 28 grammars is 28 times $\$\frac{1}{2}$, or $\$\frac{28}{2}$, equal to \$14.

2. If you pay 20 cents apiece for 35 chickens, how much do they all cost?

3. At $16\frac{2}{3}$ cents a yard, what cost 48 yards of gingham?

4. At $6\frac{1}{4}$ cents a quart, what cost 84 quarts of milk?

5. How much must I pay for 24 yards of muslin at $8\frac{1}{3}$ cents a yard?

6. A farmer sold 64 pumpkins at $12\frac{1}{2}$ cents apiece: how much did he receive?

7. A stationer bought 120 bottles of ink at 25 cents a bottle: how much did he pay?

8. A grocer sold 16 quarts of sirup at $33\frac{1}{3}$ cents a quart: how much did he receive?

9. A farmer's son sold 38 quarts of chestnuts at $12\frac{1}{2}$ cents a quart: how much did he receive?

10. What is the cost of 124 fowls at 50 ¢ each?

11. What are 36 quarts of nuts worth at $12\frac{1}{2}$ cents a quart?

12. How much must I pay for 81 yards of sheeting at $33\frac{1}{3}$ cents a yard?

13. How many bottles of ink at $6\frac{1}{4}$ cents each can be bought for $1?

WRITTEN EXERCISES.

1. A dealer sold 273 pairs of rubber shoes at $33\frac{1}{3}$ cents a pair: how much did he receive?

2. If a man saves $16\frac{2}{3}$ cents a day, how much will he save in 18 days?

3. What cost 840 histories at 25 cents each?

4. What must be paid for 624 bushels of potatoes at 50 cents a bushel?

5. What is the cost of 768 copy books at $6\frac{1}{4}$ cents each?

6. What will 340 posts cost at $33\frac{1}{3}$ cents each?

7. What is the cost of 18 pieces of calico, each containing 24 yards at $6\frac{1}{4}$ cents a yard?

8. Find the cost of 48 yds. of carpet at $1.25 per yard.

9. What is the cost of 120 hats at $1.50 each?

10. What cost 45 shovels at 75 cents each?

11. What cost 8 bales of Sea Island cotton, each containing 300 pounds, at $16\frac{2}{3}$ cents a pound?

12. A farmer purchased 12 yards of calico at $8\frac{1}{3}$ cents a yard, 24 pounds of sugar at $6\frac{1}{4}$ cents a pound, and 6 yards of cloth at $\$1.16\frac{2}{3}$ a yard: what was his bill?

13. A dealer sold 15 tons of coal at $5.20 a ton: how much did he receive?

14. A grocer bought 144 gallons of kerosene at $\$.12\frac{1}{2}$ a gallon: how much did he pay?

15. I sold 64 cords of wood at $\$4.06\frac{1}{4}$ a cord: how much did I receive?

16. What must be paid for 40 barrels of flour at $8.05 per barrel?

17. What is the cost of 448 books at $\$1.12\frac{1}{2}$ each?

Ratio and Proportion.

ART. 399.—**Ratio** is the relation that two similar numbers bear to each other: thus, the ratio of 6 to 3 is 2; of 9 to 3 is 3; of 12 to 2 is 6.

ART. 400.—Ratio can exist between like things only.

ART. 401.—The **Sign** of ratio is the colon (:); thus, 18 : 6 means the ratio of 18 to 6. Ratio is sometimes expressed in the form of a fraction. Thus, $\frac{6}{2}$ expresses the ratio of 6 to 2.

ART. 402.—The **Terms** of a ratio are the two numbers compared; thus, in the ratio 24 : 8, the terms are 24 and 8.

ART. 403.—The **Antecedent** is the first term.

ART. 404.—The **Consequent** is the second term.

ART. 405. The antecedent and the consequent together form a *Couplet.*

ART. 406. A **Ratio** is found by dividing the antecedent by the consequent.

ART. 407.—A **Simple Ratio** is the ratio of two numbers; as 4 : 2, and 10 quarts : 4 quarts.

ART. 408.—A **Compound Ratio** is a ratio whose terms are the products of the corresponding terms of two or more simple ratios. It is generally expressed thus $\left\{\begin{matrix} 6 : 18 \\ 5 : 20 \end{matrix}\right\}$. This is 30 : 360, or reduced to its simplest terms, 1 : 12.

ORAL EXERCISES.

Find the ratio of—

1. 16 to 4; 14 to 7.

2. 18 to 6; 6 to 18.

3. 16 to 8; 8 to 16.

4. 24 to 6; 6 to 24.

5. 7 to 21; 21 to 7.

6. 27 to 9; 9 to 27.

7. 2 to 20; 10 to 2; 12 to 4; 3 to 21.

8. If the antecedent is 9, the consequent 36, what is the ratio?

9. If the consequent is 7, the antecedent 28, what is the ratio?

10. The consequent is 16, the antecedent 32: what is the ratio?

11. The consequent is 10, the ratio 5: what is the antecedent?

12. The antecedent is 9, the ratio 7: what is the consequent?

13. The consequent is 33, the antecedent 11: what is the ratio?

14. The antecedent is 15, the consequent 75: what is the ratio?

15. The antecedent is 13, the ratio 5: what is the consequent?

16. The antecedent is $\frac{1}{2}$, the ratio 3: what is the consequent?

17. The consequent is $\frac{4}{5}$, the antecedent [illegible]: what is the ratio?

18. The consequent is $\frac{2}{3}$, the ratio $\frac{1}{3}$: what is the antecedent?

19. What is the value of the ratio 14.3 : 42.9? 16.25 : 3.25?

NOTE. Since the antecedent represents the dividend and the consequent the divisor, factors common to both terms may be cancelled.

Simple Proportion.

ART. 409.—A **Proportion** is an equality of ratios.

The equality of two ratios may be expressed in the usual way, by the sign of equality, but the use of the double colon (: :) for this purpose is more common. Thus, 6 : 30 : : 8 : 40 means that 6 : 30 = 8 : 40, that is, the ratio of 6 to 30 equals the ratio of 8 to 40, each being equal to $\frac{1}{5}$.

In reading such a proportion we say, "The ratio of 6 to 30 equals the ratio of 8 to 40," or, "6 is to 30 as 8 is to 40."

ART. 410. The **Terms** of a proportion are the numbers compared.

ART. 411. The first and fourth terms are called the *Extremes;* the second and third terms are called the *Means.*

ART. 412. The **Couplets** are the equal ratios, hence, the first couplet must be the first and second terms, and the second couplet the third and fourth terms.

ART. 413.—**Simple Proportion** is an equality of simple ratios.

ART. 414. **Compound Proportion** is an equality of ratios of which one or both are compound.

When the first three terms of a proportion are given, the fourth can readily be found.

To show this, let us take the simple proportion 3 : 9 : : 4 : 12. We see that the first term, or antecedent, is multiplied by 3 to make the second term. It follows, therefore, that since 4 bears the same ratio to the last term that 3 does to the second term, we have only to multiply the third term by 3 to obtain the last term.

By similar reasoning, the first term can be ascertained when the others are given. In general, when any three terms are known, the remaining one may be found.

Find the missing term in the following:

1. 7 : 21 : : 6 : (?)

2. 4 : 20 : : 11 : (?)

3. (?) : 30 : : 12 : 72.

4. (?) : 6 : : 5 : 15.

5. 8 : 24 : : (?) : 45.

6. 12 : (?) : : 6 : 36.

7. $\frac{3}{4} : 2\frac{1}{4} : : \frac{3}{5} : (?)$

8. $\left\{\begin{matrix} 3 : 6 \\ 5 : 10 \end{matrix}\right\} : : 24 : (?)$

9. $\left\{\begin{matrix} 4 : 8 \\ 3 : 18 \end{matrix}\right\} : : (?) : 1728.$

10. $\left\{\begin{matrix} 4 : (?) \\ 7 : 35 \end{matrix}\right\} : : 40 : 200.$

11. $\left\{\begin{matrix} 5 : 25 \\ 6 : (?) \end{matrix}\right\} : : 64 : 576.$

It follows from what has been stated that:

ART. 415.—1. *The product of the means equals the product of the extremes.*

2. *Either extreme equals the product of the means divided by the other extreme.*

3. *Either mean equals the product of the extremes divided by the other mean.*

WRITTEN EXERCISES.

1. What is the cost of 5 yards of broadcloth, if 16 yards cost $64?

Process.

$$\begin{matrix} \text{yd.} & \text{yd.} & & \$ & \$ \\ 16 & : 5 & : : & 64 & : x \end{matrix}$$

$$\frac{5 \times \overset{4}{\cancel{64}}}{\cancel{16}} = \$20$$

Analysis.—The fourth term, which is the one we are seeking must be a *certain number of dollars*, for the problem demands the *number of dollars* that 5 yards of cloth will cost.

For convenience, we represent the unknown term by x. Then, as ratio *can exist only between like things* the other term of the latter couplet must be *dollars*; and since $64 is the only number of dollars named in the problem, we place that for the third term.

If 16 yards cost $64, 5 yards must cost *a less sum*. Consequently x, the fourth term which we are seeking will be *less* than the third term and to preserve the equality of ratios the first consequent must be *less* than the first antecedent Furthermore, 16 yards, the number

bought with \$64, must bear the same ratio to 5 yards that the *cost of 16 yards* (\$64) bears *to the cost of* 5 *yards*. Hence, we make the first consequent *less* than the first antecedent. Arranged thus we have:

yds.		yds.			\$		\$
16	:	5	:	:	64	:	x

Multiplying the means together and dividing by the first extreme, we obtain \$20 for the other extreme, which is the required cost of 5 yards of broadcloth.

ART. 416. Rule for Simple Proportion.—*Write for the third term the number which is of the same kind as the number sought.*

Then from the nature of the problem, consider whether the fourth term will be greater or less than the third term. If it will be greater, arrange the remaining two terms so that the second will be greater than the first. If the fourth term will be less than the third, make the second less than the first.

Multiply the second and third terms together and divide the product by the first: the quotient is the fourth term, or term required.

TO TEACHER.—Proportion, formerly known as the Rule of Three, is a form of mathematical logic so admirable that it has been called the Golden Rule. The reason for every step taken is so plain that a young child can readily understand it, and, once fully explained to him, it will never be forgotten. The teacher, therefore, should make certain that by continued drill and exercises the pupils comprehend the fundamental truths of Proportion.

2. What is the cost of 40 bushels of potatoes, if 30 bushels cost \$45?

3. What is the cost of 60 hhd. of molasses, if 45 hhd. cost \$2,500?

4. What are 16 horses worth, if 8 are worth \$1,800?

5. What are 320 acres of land worth, if 60 acres cost $5,000?

6. If $200 gains $14 in 1 year, how much will $500 gain in the same time?

7. If 30 men eat 48 loaves of bread in a certain time, how many loaves will 70 men eat in the same time?

8. If in a certain time 30 men eat 48 loaves, how many men will be required to eat 80 loaves in the same time?

9. If 25 cows cost $1,250, how many cows can be bought for $3,000.

10. If a clerk earns $75 in 5 weeks, how much can he earn in 32 weeks?

11. If a staff 8 feet high casts a shadow 11 feet in length, how many feet of shadow will a pole 23 feet high cast at the same time of day?

12. If 5 lb. 8 oz. of butter cost $1.50, how much will 12 lb. 4 oz. cost?

13. If 5 acres will pasture 10 cows a certain time, how many cows can be pastured on 14 acres for the same time?

14. If $\frac{3}{4}$ of a yard of satin cost $1.75, what will $2\frac{1}{4}$ yards cost?

15. If 12 men can build a wall in 7 days, how long would it take 15 men to build the wall?

16. If $\frac{5}{8}$ of the value of a ship is $11,000, what is $\frac{2}{3}$ of its value?

17. If $7\frac{1}{2}$ tons of coal cost $37.50, how many tons can be bought for $75?

18. If an express averages 8 miles in 9 minutes, how long will it take to go 48 miles?

19. If there are 96 men in a single company, how many men compose a regiment composed of 10 companies?

20. A bankrupt merchant pays 80 cents on a dollar: if he owes A $1,500, how much should A receive?

21. If $\frac{5}{8}$ of a bushel of potatoes cost $\$\frac{4}{5}$, what will $\frac{1}{6}$ of a bushel cost?

22. If 14 ounces of silver make 5 tea-spoons, how many will 7 pounds of silver make?

23. If 48 men build a house in $104\frac{1}{2}$ days, how long will it take 12 men to build a similar house?

24. If A can saw a cord of wood in 5 hours, and B in 8 hours, how long will it take both together to saw a cord?

25. A man can dig a ditch in 6 hours, and with the help of a boy he can dig it in 4 hours: how long will it take the boy to dig it?

26. If the interest of $550.75 is $33.045 for 1 year, how much will the interest of $110.15 be for the same time?

27. If 36 yards of carpet $\frac{3}{4}$ of a yard wide will cover my office, how many yards $\frac{1}{2}$ yard wide will be required to cover it?

28. If 10 masons build a wall 40 ft. long, 2 ft. wide and 12 ft. high, in 8 days, how long will it take them to build a wall 18 ft. long, 1 foot wide and 10 ft. high?

29. If a block of marble 8 ft. long, 4 ft. 6 in. broad and 3 feet deep, weighs 1,500 lb., what is the weight in tons of a block 9 ft. long, 5 ft. 4 in. broad and 4 ft. 8 in. deep?

30. Illustrate by an original problem, the method of solving examples in Simple Proportion.

Compound Proportion.

WRITTEN EXERCISES.

ART. 417. 1. If 8 men dig a trench 40 rods long in 12 days, how many men can dig a trench 100 rods long in 20 days?

Process.

$$\left.\begin{array}{ccc} \text{rds.} & & \text{rds.} \\ 40 & : & 100 \\ \text{days} & & \text{days} \\ 20 & : & 12 \end{array}\right\} :: \begin{array}{c}\text{men} \\ 8 : x\end{array}$$

Analysis.—In this problem, the answer must express a number of men: hence the third term is 8 men.

Now, if 8 men can dig 40 rods in *a certain time*, it will take the same number of men a *longer* time to dig 100 rods. Hence, basing the problem on the *question of rods*, the fourth term will be *greater* than the third, and the couplet will be 40 rods : 100 rods.

If 8 men can do the work in 12 days, it will take a *less number* of men to do it in 20 days. Hence, the last term will be *less* than the third term, and the couplet will be 20 days : 12 days.

Multiplying the means together and dividing the product by the product of the first terms, canceling when possible, we obtain 12, as the fourth term, which represents the number of men required.

Therefore, etc.

ART. 418. Rule for Compound Proportion. *Write for the third term, the number that is of the same kind as the answer required.*

Then arrange in turn the remaining ratios as though the answer depended upon each of them and the third term alone.

Multiply the second and third terms together, and divide their product by the product of the first terms. The quotient is the fourth term, or answer required.

NOTE.—Compound Proportion offers abundant opportunities for cancellation.

2. If \$400 gain \$10 in 6 months, how long will it take \$300 to gain \$5?

3. If \$500 in 3 yr. 6 mo., at 6%, gain \$105 interest, how much will \$600 gain in 4 yr. 9 mo., at 8%?

4. If \$250 gain \$17.50 in 12 months, what will \$450 gain in 8 months?

5. If a man travels 145 miles in 5 days, when he travels 12 hours a day, how many days will it take him to travel 435 miles, if he travels only 6 hours a day?

6. If a bin that is 9 feet long, 6 feet wide, and 9 feet deep, holds 320 bushels of wheat, how many bushels will a bin hold that is 15 feet long, 8 feet wide, and 12 feet deep?

7. If 18 men, working 10 hours, finish a task in one day, how many days will it require for 10 men to do the same work, by working 9 hours a day?

8. If 32 men have enough provisions to last them 40 days, how long will twice the quantity last 75 men?

9. If a regiment of 1,000 soldiers consume 10,400 pounds of bread in 16 days, how many pounds will 5 regiments of the same size consume in 10 days?

10. If a five-cent loaf of bread weigh 12 ounces, when flour is $8 a barrel, how much should a three-cent loaf weigh when flour is $9 a barrel?

11. If 10 yards of muslin, $1\frac{1}{4}$ yards wide, cost $1.30, what is the cost of 12 yards of the same quality $1\frac{1}{8}$ yards wide?

12. If 24 men can build 40 rods of wall in 16 days, in what time can they build 64 rods, if 10 men quit work when 20 rods are finished?

13. If 8 horses eat 42 bundles of hay in 30 days, how many bundles will 16 horses eat in 28 days?

14. If it takes 3 men 24 days to lay a pavement 40 feet long and 3 yards wide, how long will it take 5 men to lay a pavement 50 feet long and $2\frac{1}{2}$ feet wide?

15. If 24 men, working 10 hours a day, build a wall 180 feet long, 2 feet wide and 8 feet high, how many men, working 12 hours a day, will it take to build a wall 144 feet long, 1 foot wide and 8 feet high in the same time?

16. If 10 men excavate a cellar 12 feet long, 10 feet

wide and 5 feet deep, in 3 days, how many men will be required to excavate a cellar 18 feet long, 12 feet wide and 5 feet deep in 2 days?

17. If a bicyclist rides 360 miles in 4 days of 9 hours each, how many miles will he ride in 6 days of 8 hours each?

18. If a man walk 84 miles in 4 days of 12 hours each, how many miles will he walk in 18 days of 10 hours each?

19. If 6 men dig a ditch in 12 days of 10 hours each, in how many days of 9 hours each will 8 men perform the same work?

20. If 6 acres will pasture 8 cows for 21 days, how long will 9 acres pasture 12 cows?

21. If 2,000 bricks, each 8 inches long and 4 inches wide, will make a pavement, how many flags of stone, each 3 feet long and 1 foot 6 inches wide, will be required to make a pavement of the same dimensions?

22. If 3 men build a fence 40 rods long and 4 rails high in 2 days, working 8 hours a day, how long will it take 4 men to build a fence 50 rods long, 5 rails high, working 10 hours a day.

23. A contracted to build a wall 500 feet in length in 20 days; 20 men built 300 feet in 15 days: how many additional men must be employed to complete the wall in the required time?

24. If 22 men, working 9 hours a day, build a stone wall 36 yards long, 2 feet wide and 6 feet high, in 8 days, how many men, working 12 hours a day, can build a wall 2 rods long, $1\frac{1}{3}$ feet wide and 8 feet high, in 6 days?

25. Illustrate by an original problem the method of solving questions in Compound Proportion.

Partnership.

ART. 419.—**Partnership** is the association of two or more persons with a joint capital, for the transaction of business.

ART. 420.—The **Capital** consists of the money or property invested by the partners, and constitutes the *Interest* or *Joint Stock*.

ART. 421.—The **Resources** or **Assets** of a firm are the property which it owns, and the debts due it.

ART. 422.—The **Liabilities** of a firm are its debts.

ART. 423.—The **Net Capital** is the excess of the resources over the liabilities.

WRITTEN EXERCISES.

1. A, B and C engaged in business with a capital of \$12,000, of which A invested \$3,000, B \$5,000, and C \$4,000. At the end of a year they had gained \$3,000: how should it be divided among them?

Process.

A invested $\frac{3000}{12000}$, or $\frac{1}{4}$ of the capital, and was therefore entitled to $\frac{1}{4}$ of \$3,000 = \$750.

B invested $\frac{5000}{12000}$, or $\frac{5}{12}$ of the capital, and was therefore entitled to $\frac{5}{12}$ of \$3,000 = \$1,250.

C invested $\frac{4000}{12000}$, or $\frac{1}{3}$ of the capital, and was therefore entitled to $\frac{1}{3}$ of \$3,000 = \$1,000.

2. Suppose A put in \$3,000, and at the end of 6 months added \$1,000 to his investment; B put in \$5,000, and at the end of a year withdrew \$1,000; C put in \$4,000, and at the end of a year and 4 months withdrew \$500. At the close of two years they had gained \$5,800: how should the gain be apportioned among the partners?

$$\text{A's}\left\{\begin{array}{l}\$3,000 \times 6 = \$18,000 \\ 4,000 \times 18 = 72,000\end{array}\right\} = \$90,000 \text{ for 1 mo.}$$

$$\text{B's}\left\{\begin{array}{l}5,000 \times 12 = 60,000 \\ 4,000 \times 12 = 48,000\end{array}\right\} = 108,000 \text{ for 1 mo.}$$

$$\text{C's}\left\{\begin{array}{l}4,000 \times 16 = 64,000 \\ 3,500 \times 8 = 28,000\end{array}\right\} = 92,000 \text{ for 1 mo.}$$

$$\$290,000 \text{ for 1 mo.}$$

gained \$5,800; or \$1 for 1 mo. gained $\frac{5800}{290000}$ = \$.02.

$$\begin{array}{l}\$.02 \times 90,000 = \$1,800, \text{ A's share of the gain.} \\ \$.02 \times 108,000 = 2,160, \text{ B's ” ” ”} \\ \$.02 \times 92,000 = 1,840, \text{ C's ” ” ”} \\ \qquad\qquad\qquad \overline{\$5,800}\end{array}$$

Analysis.—In problem 1, the capital of each partner was invested for the same time. The division of profits or losses, therefore, is based simply on the *amount* of each one's capital.

In problem 2 the investments are for different times. A's \$3,000 for 6 months should earn the same as \$18,000 for 1 month, and his \$4,000 for 18 months is the same as \$72,000 for 1 month. Hence, A's investment is equivalent to \$90,000 for 1 month.

In like manner, B's capital is equivalent to an investment of \$108,000 for one month, and C's to an investment of \$92,000 for 1 month.

The investments being now reduced to investments for the same time, that is, for 1 month, the gain is divided as in problem 1.

ART. 424. Rule for Partnership. *Distribute the gains or losses in proportion to the amount of each one's capital, multiplied by the time it was invested.*

3. A, B, C and D form a partnership, A contributing \$3,000; B \$2,500; C \$3,500; D \$1,000. At the end of a year they had lost \$3,000: how should it be apportioned?

4. A, B and C associated themselves in business. A invested \$4,000; B, \$6,000 and C was allowed a fifth interest for managing the business. The year's balance sheet showed they had gained \$2,500: what was each one's share?

5. Messrs. Brown, Jones and Robinson formed a partnership in the wholesale grocery business. Brown put in \$3,000 and added \$1,500 more at the end of the first year; Jones put in \$5,000 and at the end of 8 months withdrew \$1,000; Robinson invested \$4,000 and at the end of a-year-and-a-half withdrew \$800. At the close of two years they found they had gained \$8,556: how should it be apportioned?

6. A man failing in business owed C \$1,500, D \$1,800, and E \$2,000; his assets were \$2,650: how much did each receive?

7. A and B hired a pasture together, for \$40; A put in 20 cattle for 3 months, and B 30 cattle for 4 months: what should each pay?

8. Button Brothers find themselves obliged to suspend with liabilities amounting to \$25,000, while their resources foot up only \$15,000. They owe A \$4,000, B \$3,000, C \$6,400, D \$6,500: what did each creditor receive?

9. A, B and C went into the coal business, A investing \$6,000, B \$5,000 and C \$4,000. At the close of the year they sold out for \$13,365 cash: what was each one's share?

10. Mr. Smith left \$6,000 to his wife, \$4,600 to his son and \$4,000 to each of his two daughters, but upon settling his estate it amounted to only \$12,400: how much did each receive?

11. A and B engaged in a cotton speculation; A invested \$4,800, and B \$5,600; they lost $\frac{1}{8}$ of their stock by fire, and gained on the remainder $\frac{1}{4}$ of its cost: what was the gain of each?

12. A, B and C engaged in business; A furnished \$4,000, B \$6,000, and C managed the business, they gained \$1,800: what was the share of each, if C received as much as A and B together?

13. A merchant fails in business with debts amounting to \$35,000, and resources to \$16,500. He owes A \$9,000, B \$7,500, and C \$11,000: what should each receive?

14. Illustrate by an original problem the method of finding the gain or loss when the shares of the several partners are invested for unequal periods of time.

15. Illustrate by an original problem the method of finding the shares in the gain or loss of three partners in business, when their shares of capital are unequal and a full account of resources and liabilities is taken.

REVIEW QUESTIONS.

What is ratio? Can it exist between unlike things? What is the sign of ratio? What are the terms of a ratio? The antecedent? The consequent? What form a couplet? How is a ratio found? What is a simple ratio? A compound ratio?

What is a proportion? What are the terms? The extremes? The means? The couplets? What is simple proportion? Compound proportion? Give the rule for simple proportion. For compound proportion.

What is partnership? Of what does capital consist?

What are the resources of a firm? The liabilities? The net capital? Give the rule for Partnership.

Arithmetical Analysis.

ART. 425.—An understanding of the conditions of a problem will make clear the method of solution.

In the analysis of problems all solutions are referred from one to many parts, and from many to one part. One whole or one part is the key to all solutions.

ORAL EXERCISES.

1. A can do a piece of work in 6 days and B in 9 days: in what time can both do the work together?

Analysis.—Since A can do the work in 6 days, he can do $\frac{1}{6}$ of it in one day; and since B can do it in 9 days, he can do $\frac{1}{9}$ in one day, and, in one day, both can do $\frac{1}{6}+\frac{1}{9}=\frac{5}{18}$ of the work. To do all the work will take them as many days as $\frac{5}{18}$ is contained times in $\frac{18}{18} = 3\frac{3}{5}$ days.

2. A can do a piece of work in 10 days and B in 20 days: in what time can both do the work together?

3. A, B and C can do a piece of work in 6 days: A can do it alone in 18 days, and B can do it alone in 24 days: in what time can C do it alone?

Analysis.—Since A, B and C can do $\frac{1}{6}$ of the work in 1 day, and B and C can do the sum of $\frac{1}{18}$ and $\frac{1}{24}$ in 1 day, C can do as much in 1 day as the difference between $\frac{1}{6}$ and $(\frac{1}{18}+\frac{1}{24}) = \frac{5}{72}$. Hence C would require as many days to do the work as $\frac{5}{72}$ is contained times in $\frac{72}{72}$, which is $14\frac{2}{5}$ times, or $14\frac{2}{5}$ days.

4. A, B, and C can do a piece of work in 12 days, and A and B can do it in 20 days: how long will it take C to do it?

5. The head of a fish is 10 inches long. Its tail is as long as the head and half the body, and its body is as

long as the head and tail together: what is the length of the fish?

Analysis.—The length of the head is 10 inches, the length of the tail is 10 inches + ½ the body and the body is 10 inches + 10 inches + ½ of itself. Then 20 inches must be the other ½ of the body and $\frac{2}{2}$ of the body is 40 inches. The tail is 20 inches + 10 inches = 30 inches, and the length of the fish is 10 + 30 + 40 inches = 80 inches.

6. The head of a fish is 8 inches long; the tail is as long as the head plus ½ the body, and the body is as long as both the head and tail: what is the length of the body?

7. What is the time of day, if the time from now till noon is one-half of the time from noon to midnight?

Analysis.—The time from noon to midnight is 12 hours, and ½ of that is 6 hours, which is the time from now till noon, or 6 A.M.

8. What is the time, if one-third of the time past noon is one-sixth of the time to midnight?

Analysis.—The time between noon and midnight is 12 hours, which, if divided into two parts so that they are to each other as ⅓ is to ⅙, will give the time it lacks of being midnight, = 4 P.M.

9. Ten years ago I was 3 times as old as my son, and now I am 2 times as old: what are our respective ages?

Analysis.—Ten years ago my son was one times his age, and now he is 1 times his age at that time + 10 years, and I am 3 times his age at that time + 10 years. Twice his age + 10 years or 2 times + 20 years = 3 times his age + 10 years. Then 1 times his age 10 years ago = 10 years.

10. A man dying left an estate worth $42,000. He provided that if his son returned from a foreign land he should receive twice as much as his wife, but if the daughter returned she should receive one half as much

as his wife: if both son and daughter returned, how should the estate be divided?

Analysis.—By the conditions of the problem, the daughter should receive one half as much as the wife, and the wife one half as much as the son. Then the daughter should receive 1 part, the wife 2 parts, and the son 4 parts, or $\frac{1}{7}$, $\frac{2}{7}$, and $\frac{4}{7}$, or $6,000, $12,000, and $24,000 respectively.

11. A hare is 100 leaps ahead of a hound and takes 4 leaps to 3 of the hound; but 2 of the hound's leaps equal 3 of the hare's leaps: how many leaps must the hound take to catch the hare?

Analysis.—Since 2 of the hound's leaps equal 3 leaps of the hare, 3 leaps of the hound equal $4\frac{1}{2}$ leaps of the hare; but these 3 leaps of the hound are taken in the same time that the hare takes 4 leaps; hence the hound gains $\frac{1}{2}$ a leap in every 3 leaps, and he must take 6 leaps to gain 1 whole leap of the hare; to gain 100 leaps, he will have to take 600 leaps.

12. A, B and C have $7,800 invested in business together, their interest being in the ratio of $\frac{1}{3}$, $\frac{1}{2}$ and $\frac{3}{4}$. B dies, and the remaining partners pay his widow $3,900 cash for his interest in the business: what is each one's share of the remaining capital?

Analysis.—After paying for B's interest, the remaining capital is $7,800 – $3,900 = $3,900, which divided in the ratio of $\frac{1}{3}$ to $\frac{3}{4}$ gives $1,200 for A, and $2,700 for C.

13. A merchant bought goods at 36 cents a yard, which he sold at a net profit of $33\frac{1}{3}$% after allowing his customers a deduction of 20%: what were the goods marked?

Analysis.—Since he gained $33\frac{1}{3}$% or $\frac{1}{3}$, he received 48 cents per yard, and since this is a discount of 20% or $\frac{1}{5}$ of his asking price, it is $\frac{4}{5}$ of his asking price, = 60 cents.

14. When gold is worth 50% premium, how much gold is a dollar bill worth?

Analysis.—Since a gold dollar is worth 150 cents in paper money, a dollar in paper money is worth $\frac{100}{150}$, or $\frac{2}{3}$, = $66\frac{2}{3}$ cents in gold.

WRITTEN EXERCISES.

1. I bought $12\frac{1}{2}$ cords of wood at $\$4\frac{1}{2}$ per cord, and paid for it with flour at $\$5\frac{5}{8}$ per barrel: how many barrels were required?

2. A man owning $\frac{3}{4}$ of a factory, sold $\frac{2}{3}$ of his share for \$4,000: at that rate what was the value of the factory?

3. What must I ask per acre for land which cost \$90, so as to sell it at 25% less than the asking price and still make 10%?

4. I sold goods at \$2.40 per yard and lost 20%: for what price should I have sold it to gain 20%?

5. A, B, C and D invested \$25,000 in a mining speculation; A's gain was \$1,600, B's \$2,400, C's \$2,800, and D's \$3,200: what was the capital of each?

6. E invested 50% of his money in a house and 25% of the remainder in furniture; the difference between the sums was \$600: what was his total investment?

7. If 8 men plough 12 acres in 9 days, how many men can plough 8 acres in 12 days?

8. If 20 horses eat 186 bushels of oats worth \$92 in 13 days, how long will it take 3 horses to eat one-half as much?

9. If 18 men can do a piece of work in 44 days, in how many days can they do twice as much work with the assistance of 8 more men?

10. If 11 men can cut 147 cords of wood in 7 days by working 14 hours a day, in how many days can 5 men cut 150 cords of the same kind of wood, by working 10 hours each day?

11. A stock company has 360 shares equally divided among 9 stockholders. If A sells 10 shares to B and B sells $\frac{1}{5}$ of his stock to C, what % of A's share is B's?

12. A merchant bought corn at 30 cents a bushel. It

having lost $16\frac{2}{3}\%$ in weight, he sold $\frac{1}{2}$ of it at 36 cents a bushel: at what rate must he sell the remainder so as to realize a net profit of 20%?

13. B owes C \$1,324; C offers to allow 4% for ready money. If B pays \$900 immediately, how much does he still owe?

14. A man bought 10 books at the rate of \$9 a dozen, and sold them for 85 cents each: what % was gained?

15. A man bought 60 tons of coal at $\$5\frac{1}{4}$ a ton, $16\frac{2}{3}\%$ of which was lost by the sinking of the boat: for how much per ton must he sell the remainder so that he may neither lose nor gain?

16. The premium on an insurance of \$1,250 was \$22.50: what was the rate?

17. A and B rented a pasture for \$18.66. A put in 20 cows for 11 days, and B 23 cows for 31 days: how much rent should each pay?

18. A, B and C enter into partnership; A puts in \$500 for 8 months, B \$600 for a year, and C \$800 for a year-and-a-half. They lose \$512: how should the loss be apportioned?

19. C and D form a partnership; C put in \$4,000, and at the end of 6 months put in \$2,000 more; D put in \$8,000, and at the end of 8 months withdrew \$3,000. At the close of the year they had gained \$1,320: what was each one's share?

20. A certain piece of work was to be done by 240 men in 64 days, but a number having been sent away, it was performed in 120 days: what was the number of men sent away?

21. How much more is the compound interest than the simple interest of \$1,500 for 5 years at 6%?

22. A cistern containing 480 gallons can be emptied by two pipes in 4 and 5 minutes respectively: if both pipes are left open, in what time will they empty the cistern?

23. When 9 barrels of onions are worth \$33.75, what are 28.50 barrels worth?

24. A man bought 358 bushels of corn at 78 cents a bushel; he distributed 38 bushels among his poor neighbors, and sold the rest at 93 cents a bushel: how much did he make?

25. Bought a house and a farm of 48 acres for \$9,450. The house cost \$4,320: what was the value of the land per acre?

26. A farmer sold 143 acres of land at \$45.75, and 125 acres at \$61: what was the average price to the nearest cent of each acre?

27. I bought land for \$2,450.75, and sold 48.5 acres at \$35.50 an acre. The value of the remaining land was \$36.45 an acre: how many acres did I buy?

28. If 24,500 bricks are sold for \$134.75, what is the price per M.?

29. What is the cost of 4,250 feet of planking at \$35 per M.?

30. What are 1,480 melons worth at \$8.75 per hundred?

31. A man bought 4,000 bushels of wheat at \$1.10 per bushel; he sold 10% at 5% loss, 40% at 10% gain, and the remainder at 8% gain: what did he gain by the transaction?

32. If you sell a house for \$5,000 and lose $12\frac{1}{2}$%, for what price should you sell another at an advance of $12\frac{1}{2}$%, so as to cover the loss on the former house?

33. Sold property for \$2,400, 25% of which was gain. I was able to collect only 80% of the proceeds of the sale: what was the actual gain?

34. A grocer after losing 10% of his apples had 153.9 barrels left: if they cost \$3.50 a barrel, for what must they be sold so that he may neither gain nor lose?

35. What is the difference in net cash value between a bill of \$1,200 less a discount of 25% and 10% off the remainder, and the bill less a discount of 35%?

36. I wish to purchase a 5% bond, so as to make the investment yield me 7%: how much can I pay for the bond, including brokerage at $\frac{1}{8}$%?

37. A piece of property was worth \$5,000 in gold, when the premium on gold was 12%: what was the value of the same in currency?

38. A man borrowed \$5,000, which he invested in real estate. Six months later he sold the property on a credit of 12 months, with interest, for \$7,500: money being worth 6%, what was his profit?

39. A man sold a quantity of wheat at \$1.08 per bushel, at a profit of 20%; he afterward sold a lot of the same for \$31.05, at a profit of 50%: how many bushels were there in the last lot?

40. A man sold at a gain of 30% and took a note for the amount of the sale. He sold the note at 15% less than its face value: what was his gain per cent.?

41. A farmer allows one acre of pasture for every 5 sheep, and one acre of plowed land for every 8 sheep: how many sheep can be kept on 325 acres?

42. If a gallon measure is $\frac{1}{24}$ too small, how much will a dealer filch from his customers in a year, if he sells 150 gallons of milk daily at 13 cents a gallon?

43. A certain man owns $\frac{3}{5}$ of a tract of land, and sells $14\frac{2}{7}$% of it to one man, 25% of the remainder to another, and $28\frac{4}{7}$% of what he then had to a third person, when he has 15 acres remaining: how many acres had he at first?

44. A merchant bought on 12 mo. time, 5 casks of wine, each containing 125 gallons, at 90 cents a gallon, and sold it at 95 cents a gallon on 6 mo. time. If money is worth 10%, how much does he gain?

45. A farmer sold a number of cows and had 21 left, which was $\frac{1}{3}$ of the number sold; if the number sold be divided by $\frac{4}{5}$ of $8\frac{3}{4}$, the quotient will be $\frac{2}{9}$ of the price of each cow: how much did he receive for his cows?

46. A, B and C invested jointly \$10,252. A's capital was in the business 10 mo., B's, 14 mo., and C's, 15 mo. They gained \$6,300, which was divided so that A received \$4 as often as B received \$5 and C, \$3. B drew out \$4,329 and absconded. What was each man's share of the stock? How much did A and C gain or lose by B's withdrawal?

47. A boy bought an equal number of apples at 2 for 1 cent, and 3 for 1 cent, and, selling them at 5 for 2 cents, thereby lost 1 cent. How many apples did he buy?

48. A, B, C and D agree to do a piece of work for \$152. A, B and C can do it in 5 days; B, C and D, in $3\frac{1}{4}$ days; C, D and A, in 4 days, and D, A and B, in $4\frac{2}{7}$ days. How long will it take them all to do it? How long will it take each of them to do it? If they work together, how much should each receive?

49. At what time after 4 o'clock are the hour and minute hands of a clock together?

SUGGESTION.—The minute-hand gains $\frac{11}{12}$ of the distance it passes over.

50. A, B and C were employed to do a piece of work for \$29.50. A can do it in 4 days working 9 hr. a day; B in 3 days working 7 hr. a day, and C in 4 days working 6 hr. a day. If they all work together, what part of the money should each man receive?

51. A merchant sold 20 stoves for \$180. He received \$19 for the largest size; \$7 for the middle size, and \$6 for the smallest size. How many stoves of each size did he sell?

Involution.

ART. 425. Every number is the first power of itself; when multiplied once by itself, the product is the *square* or *second* power of the number; when the number is used three times as a factor, the product is the *cube* or third power; when the number is used four times as a factor, the product is the fourth power, and so on.

ART. 426. An **Exponent** is a small figure placed to the right and slightly above a number, and shows the power to which the number is to be raised. Thus, 7^2 means that 7 is to be raised to the second power; 9^3 that 9 is to be raised to the third power; 13^4 that 13 is to be raised to the fourth power.

ART. 427. The **Root** or **Base** of a number is one of the *equal factors* of the number.

ART. 428. **Involution** is the process of finding the powers of numbers.

Find the value of:

1. 67^2.	**11.** 32^4.
2. 49^3.	**12.** 25^3.
3. 26^2.	**13.** 121^2.
4. 23^4.	**14.** 101^3.
5. 42^3.	**15.** $(\frac{1}{2})^2$.
6. 120^2.	**16.** $(\frac{3}{4})^3$.
7. 1.5^3.	**17.** $(\frac{2}{3})^2 \times (\frac{1}{2})^3$.
8. $(3.2)^2$.	**18.** $(.25)^3 \times 2^4$.
9. $(.44)^3$.	**19.** $(.02)^2 \times 5^2$.
10. $(.01)^4$.	**20.** $(\frac{4}{5})^2 \times 3^2$.

Evolution.

ART. 429.—Evolution is the process of finding any required root of a number.

ART. 430. A **Root** of a number is one of its equal factors.

ART. 431. The **Square Root** of a number is one of its two equal factors. Thus, 6 is the square root of 36, because 6 is one of its two equal factors. 4 is the cube root of 64, because 4 is one of its three equal factors.

ART. 432. The **Sign** generally used to denote that some root of a number is to be extracted is $\sqrt{}$, it is called the *Radical Sign*, and means that the square root is to be extracted.

ART. 433. The **Index** of the root is the small figure placed in front and above the radical sign. It indicates what root is to be extracted. Thus, $\sqrt[3]{64}$ means that the third or cube root of 64 is to be extracted.

ART. 434. Evolution is the reverse of involution. Let us now learn the relation of a number to its square root.

$$25^2 = (20 + 5)^2$$

$$\begin{array}{l} 20 + 5 \\ 20 + 5 \\ \hline 20^2 + 20 \times 5 \\ \qquad\quad 20 \times 5 + 5^2 \\ \hline 20^2 + 2(20 \times 5) + 5^2 = 625 \end{array}$$

The square of 25 is 625. In squaring 25, we first square the tens, which gives 400. We next multiply the units (5) by the tens (2), which gives 100. We next multiply the tens (2) by the units (5), which gives 100. We next square the units, which gives 25. The sum of these several products is 625, which is the square of 25.

By analyzing the foregoing we find that 625, when compared with its square root, *contains the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

$9^2 = 81$
$99^2 = 9801$
$125^2 = 15625$

From the squares in the margin we may infer that *the square of any number contains twice as many figures as the number itself, or twice as many less one.* Hence,

If a number be separated into periods of two figures each, beginning at the right, there will be as many periods as there are figures in its square root.

The left hand period may contain only one figure.

WRITTEN EXERCISES.

1. Required the square root of 1,024.

Process.

10'24(32
900
62)124
124

Analysis.—The square root of 1,024 consists of two figures, since the number contains two periods. The greatest square contained in 1,000 is 3 tens, which being squared and subtracted, leaves 124.

This remainder must be equal to twice the tens by the units, plus the square of the units, since we have subtracted the square of the tens.

Twice the (3) tens is 6 tens, which we place on the left as the first figure of the trial divisor. 6 tens is contained in 12 tens, 2 units times which we write for the second figure of the root. For convenience we write it also on the left as the second figure of the trial divisor.

Now when 62 is multiplied by 2, we have multiplied twice the tens in the root by the units, and we have also squared the units, and no remainder is left.

For convenience, we omit, in extracting the root, the ciphers in the several partial products, and treat each figure of the root as a simple digit.

2. Extract the square root of 55,225 : also of 31,449,664.

Process.

```
  5'52'25(235
  4
 43)152
    129
465)2325
    2325
```

Process.

```
    31'44'96'64(5608
    25
  106)644
      636
11208)89664
      89664
```

ART. 435. -Rule for the extraction of the square root. *Beginning at the right hand, separate the number into periods of two figures each.*

Find the greatest square contained in the left hand period, and place its root at the right for the first figure of the root.

Subtract the square of this quotient from the left hand period, and to the remainder annex the second period for a dividend.

Double the root already found, and place it on the left of the dividend as a trial divisor. Find how many times it is contained in the dividend (excluding the right hand figure of the dividend). The quotient will be the second figure of the root.

Annex the second figure of the root to the trial divisor for the true divisor and multiply by the last quotient figure; subtract the product from the dividend, and annex the third period to the remainder for the next dividend.

Double the root already found for a second trial divisor, find the third figure of the root as before, and so continue until all the periods have been used.

NOTES (*a*). If the product of a divisor by the last quotient is greater than the dividend, the figure in the root must be diminished by a unit.

(*b*). If a cipher appears in the root, a cipher must be annexed to the trial divisor, the next period brought down, and the process continued as before.

(*c*). If the number is not a perfect square, the exact root cannot be found. It may be approximated, however, by annexing periods of ciphers.

(*d*). In separating into periods a decimal or a mixed decimal number, we must begin with the units. If the decimal places are uneven, a cipher should be annexed.

(*e*). To find the square root of a common fraction, find the square root of each term. When these are not perfect squares, the exact square root cannot be found. In such cases the fraction may be reduced to a decimal, and its square root approximated.

Extract the square root of:

3. 576081.	**13.** 7569687.	**23.** 10.5625.
4. 784684.	**14.** 8464746.	**24.** 9.1809.
5. 964965.	**15.** 9801897.	**25.** 2043.04.
6. 1296892.	**16.** 15129683.	**26.** 40.4496.
7. 1936467.	**17.** 59049987.	**27.** 2550.25.
8. 2304965.	**18.** 12321465.	**28.** $\frac{169}{576}$.
9. 2809004.	**19.** 166464890.	**29.** $\frac{324}{2025}$.
10. 3136864.	**20.** 250000764.	**30.** 15.625.
11. 4096968.	**21.** 366025892.	**31.** 1.0201.
12. 6889763.	**22.** 102019654.	**32.** .000004.

APPLICATIONS OF SQUARE ROOT.

ART. 436.—To find the side of a square whose area is given.

Since the area of a square is the product of two equal factors expressing the length of its sides, it follows that the square root of the number expressing its area represents each of the sides.

1. What is the side of a square field which contains 9,859,600 square feet?

2. A blackboard contains 10,800 square inches, and its length is 3 times its breadth: what are its dimensions?

3. A general seeking to mass his army in a square, found that by placing 236 men on a side, he lacked 196 men to form the square: how many men were in his army?

4. How much will it cost at \$.75 a rod to inclose 10 acres of land in the form of a square?

5. A certain square room contains 1,296 square feet, and another 729 square feet: how much longer is the side of the first than the side of the second?

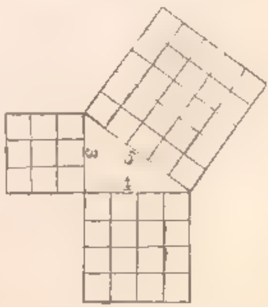
6. A man having a garden 420 yards square, extended it so that it was four times as large: how many yards square was it then?

7. The owner of a tract of land containing 1,200 acres divided it into four equal square farms: what was the length of one of their sides?

8. Illustrate the application of square root by an original problem.

ART. 437. To find any side of a right-angled triangle when the other two sides are given.

Principles.—1. *The square described upon the hypotenuse of a right-angled triangle equals the sum of the squares on the other two sides.*



2. *The square on a side adjacent to the right angle equals the square on the hypotenuse less the square on the other side.*

1. The base and perpendicular of a right-angled triangle are 15 inches and 20 inches respectively: what is the hypotenuse?

2. The hypotenuse of a right-angled triangle is 45 inches, and the base 27 inches: what is the perpendicular?

3. The perpendicular of a right-angled triangle is 57 feet, and the hypotenuse 95 feet: what is the base?

4. Two vessels sail from the same port, one south 4 miles an hour, and the other east 5 miles an hour: how far apart are they at the end of 12 hours?

5. If a ladder 48 feet long touches the side of a building at a point 36 feet above the ground, how far is the bottom of the ladder from the base of the building?

6. The height of a tree on the bank of a river is 96 feet, and a line reaching from its top to the opposite bank is 120 feet: what is the width of the river?

7. A flagstaff 80 feet high casts a shadow 60 feet in length: what is the distance from the top of the staff to the extremity of the shadow?

8. A tree was broken 36 feet from the bottom, and fell so that the end struck 42 feet from the foot: what was the height of the tree?

9. What is the distance from a lower corner to the upper opposite corner of a room 40 ft. long, 30 ft. wide and 20 ft. high?

10. Illustrate Art. 437 by an original problem.

ART. 438.—Comparison of Similar Figures.

Principles.—1. *The areas of similar figures are to each other as the squares of their like dimensions.*

2. *The dimensions of similar figures are to each other as the square roots of their areas.*

1. One side of a square is 15 feet, and one side of another square is 20 feet: how many times the area of the first is the area of the second?

2. A certain square field contains 144 rods, and another similar field contains 1,296 rods: how many times the length of the side of the first is the side of the second?

3. If a pipe $2\frac{1}{2}$ inches in diameter can empty a tank in 1 hour, how long will it take a pipe 2 inches in diameter to empty it?

4. The area of a circle, whose diameter is 20 feet, is 314.16 square feet: what is the diameter of a circle whose area is 157.08 square feet?

5. If the area of a circle, whose diameter is 20 feet, is 314.16 square feet, what is the diameter of a circle whose area is 2,827.44 square feet?

6. What is the length of the edge of a cubical block whose entire surface is 4,704 square feet?

7. Find the edge of a cube whose entire surface is 33,750 square feet.

8. Illustrate Art. 438 by an original problem.

Cube Root.

WRITTEN EXERCISES.

Multiplying the square of any number, as 35, by the number itself gives the cube of the number. Doing this in the manner already explained,

$$\begin{array}{lll} 35^2 & = & 30^2 + 2\,(30 \times 5) + 5^2 \\ \underline{35} & = & \underline{30 \;\; + 5} \\ & & 30^3 + 2\,(30^2 \times 5) + \;\;\;(30 \times 5^2) \\ & & \qquad\;\;\, (30^2 \times 5) + 2\,(30 \times 5^2) + 5^3 \\ \end{array}$$

$$35^3 = 30^3 + 3\,(30^2 \times 5) + 3\,(30 \times 5^2) + 5^3 = 42,875.$$

Hence, the cube of any number consisting of tens and units equals the cube of the tens, plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.

$2^3 = 8.$
$5^3 = 125.$
$12^3 = 1728.$
$99^3 = 970299.$

From the cubes in the margin we may infer that *the cube of any number contains three times as many figures as the number itself, three times as many less one, or three times as many less two.* Hence,

If a number be separated into periods of three figures each, beginning at the right, there will be as many periods as there are figures in its cube root.

The left hand period may contain one, two, or three figures.

1. Extract the cube root of 42,875.

Process.

		42'875(35
		27
Tens² × 3	2700	15875
Tens × units × 3	450	
Units ² =	25	
	3175	15875

Analysis.—Separating the number into periods in accordance with the principle developed above, we learn that there will be two figures in the cube root. The first figure of the root is found by taking the cube root of the greatest cube in the left hand period. This root is 3, and we write it as the tens' figure of the root. Placing its cube under the left hand period and subtracting, we have a remainder of 15, to which the next period is annexed to form a new dividend. From the composition of the cube of 35, as explained above, it is clear that this dividend, 15,875, is what is left after subtracting the cube of the tens in 35 from the cube of 35. But *tens² × 3 × units* or 2,700 times the units, forms almost all this dividend; hence, if we divide by 3 times the square of the tens, or 2,700, and remember that the quotient is likely to be too large, since the divisor is too small, we shall obtain the units' figure. The trial divisor 2,700 is now increased by 3 times the tens × units, and by the square of the units, and the sum is multiplied by the units. The product 15,875 being equal to the dividend, the operation is complete.

ART. 439.—Rule for extracting the cube root of a number.—*Beginning at the right, separate the number into periods of three figures each.*

Find the greatest cube contained in the left hand period, and place its cube root at the right hand for the first term of the root. Subtract its cube from the left hand period, and annex the next period to the remainder for a dividend.

Take three times the square of the first term of the root regarded as tens, for a trial divisor, divide the dividend by it, and the quotient will be the second term of the root. Add to the trial divisor three times the product of the first part of the root, considered as tens, by the last term of the root; also the square of the last term. The sum is the complete divisor.

Multiply the complete divisor by the second term of the root; subtract this sum from the dividend, annex the next period to the remainder for the second dividend, and proceed as before.

NOTE.—Should the trial divisor at any time not be contained in the dividend, write a cipher in the root, annex two ciphers to the trial divisor, bring down the next period, and proceed as before.

Extract the cube root of—

3. 12167.

4. 15625.

5. 35937.

6. 970299.

7. 2571353.

8. 34328125.

9. 131096512.

10. 633839.779.

11. .000512.

12. $49\frac{8}{27}$.

13. $\frac{343}{12167}$.

14. 2.

15. 3.

16. 6.

ART. 440.—Applications of Cube Root

1. A cubical box contains 474,552 cubic inches: what is the length of one of its sides?

2. How many square feet in the surface of a cube whose volume is 389,017 cubic inches?

3. What is the area of one of the sides of a cube which contains 185,193 cubic inches?

4. What is the depth of a cubical cistern whose capacity is 6,591 cubic feet?

5. Find the side of a cubical vat whose capacity is 5,545,233 cubic inches.

6. A certain apartment of cubical form contains 1,953,125 cubic feet of air: what is the height of the room?

7. To excavate a cubical cellar, 450 cubic yards and 17 cubic feet were taken out: what was the length of one side of the cellar?

8. What is the side of a cube of stone equal in volume to a monument 8 feet square and 81 feet high?

9. What is the length of a cubical pile of stone equal in volume to a rectangular pile whose length is 64 feet, breadth 24 feet, height 9 feet?

10. What is the side of a cube equal to a pile of wood 1,024 feet long, 4 feet wide, 28 feet high?

11. A rank of wood 120 feet long, 8 feet wide, and 15 feet high, is placed in a cubical pile: how high is it?

12. Two blocks of granite are together equal to another of cubical form; the first is 18 feet long, 9 feet wide and 6 feet deep; the second is 33 feet long, 18 feet wide, and 9 feet deep: what is the side of the cube?

13. A certain room is twice as long as high, and the width is equal to the height: what is the height, if the capacity of the room is 3,456 cubic feet?

14. Illustrate Art. 440 by an original problem.

ART. 441.—Comparison of Similar Solids

Principles. 1. *The contents of similar solids are to each other as the cubes of their corresponding dimensions.*

2. *The corresponding dimensions of similar solids are to each other as the cube roots of their contents.*

1. The side of a cubical block is 4 inches and the side of another is 6 inches: how many times is the bulk of the second greater than that of the first?

2. If a globe of gold 2 inches in diameter is worth $900, what is the value of one 5 inches in diameter?

3. The diameters of two spheres are respectively 3 and 7 inches: how many times the volume of the smaller sphere is the larger?

4. If a man 6 feet high weighs 144 pounds, what is the weight of a boy of similar build whose height is 4 feet?

5. A stack of hay 14 feet high contains 9 tons: how high must a similar stack be to contain 36 tons?

6. What is the diameter of a ball whose contents equal that of two balls that are respectively 4 and 5 inches in diameter?

7. If a cylindrical cistern 5 feet in diameter contains 24 hogsheads of water, how much will a similar cistern contain whose diameter is 18 feet?

8. A cubical box contains 343 cubic inches: what are the dimensions of the box inside?

9. The side of a cubical vessel is 2 feet: find the side of another cubical vessel that shall contain 5 times as much.

10. How many balls 5 inches in diameter equal in volume a ball whose diameter is 20 inches?

11. Illustrate Art. 441 by an original problem.

Arithmetical Progression.

ART. 442.—An **Arithmetical Progression** is a series of numbers that increase or decrease by a constant difference; as 2, 4, 6, 8, and 12, 10, 8, 6, 4, 2.

ART. 443.—The **Terms** of a series are the numbers that compose it.

ART. 444.—An **Ascending Series** is one in which each term is greater than the preceding term; as 3, 5, 7, 9, 11, etc.

ART. 445.—A **Descending Series** is one in which each term is less than the preceding term; as 15, 12, 9, 6, 3.

ART. 446.—The **Extremes** are the first term and the last term.

ART. 447.—The **Means** are the terms between the extremes.

ART. 448.—The **Common Difference** is the difference between two consecutive terms.

ART. 449.—The **Sum of the Terms** is the number obtained by adding together all the terms.

ART. 450.—In an arithmetical series five elements are concerned: the first term, the last term, the common difference, the number of terms, and the sum of the series. When any three of these are given, the others may be found.

1. If the first term of an ascending arithmetical progression is 2, the common difference 2, and the number of terms 12, what is the last term?

Solution.—The statement of this question shows that 2, the common difference, is to be added 11 times to the first term in order to make the 12th or last term. Hence, the last term is 2 + (2 × 11), which is 24.

2. If the first term of a descending series is 30, the common difference 3, and the number of terms 10, what is the last term?

Solution.—In this question it is clear that the common difference, 3, is subtracted 9 times from the first term in order to make the last term. The last term then is $30 - (9 \times 3)$, that is $30 - 27 = 3$.

3. If the last term is 60, the common difference 3, and the number of terms 20, what is the first term?

Solution.—In order to make 60 the last term, the common difference, 3, must have been added to the first term 19 times. 3, taken 19 times is equal to 57, which, in order to make 60, must be added to the difference between 60 and 57, or 3. Hence, 3 is the first term.

4. If the first term is 5, the last term 80, and the number of terms 26, what is the common difference?

Solution.—The statement of this question shows that the first term has been increased to the extent of 75, by 25 equal additions; consequently, each addition was $75 \div 25 = 3$, which must be the common difference.

5. If the first term is 4, the last 94, and the common difference 5, what is the number of terms?

Solution.—The first term, 4, has been increased to the extent of 90 by adding the common difference, 5, a certain number of times. How often must 5 be added in order that it may amount to 90? Evidently $90 \div 5$, or 18, is the number of terms exclusive of the first; the number of terms, therefore, is 19.

6. If the first term is 4, the last term 25, and the number of terms 8, what is their sum?

Solution.—The common difference found as in example 4, is 3.

4	7	10	13	16	19	22	25
25	22	19	16	13	10	7	4
29	29	29	29	29	29	29	29

Writing out the series in direct and in reverse order and adding the two, we obtain 29 as many times as there are terms. That is, twice the sum of the series is $29 \times 8 = 232$. Hence, $232 \div 2$, or 116, is the sum of the series.

It follows, therefore, that *the sum of a series equals the sum of the extremes multiplied by half the number of terms.*

7. If the first term of a decreasing series is 100 and the common difference 3, what is the 32d term?

8. If John sold 25 apples at the rate of half a cent for the first, 1 cent for the second, $1\frac{1}{2}$ cents for the third and so on, what price was received for the last apple?

9. Caspar walked 10 days, traveling 3 miles farther each day than on the preceding day. On the last day he walked 33 miles: how far did he walk the first day?

Geometrical Progression.

ART. 451. A **Geometrical Progression** is a series of numbers that increase or decrease by a common multiplier; as, 3, 9, 27, 81 or 36, 12, 4.

ART. 452. The **Ratio** is the common multiplier; thus, in the foregoing series the rate is 3 and $\frac{1}{3}$ respectively. In a descending series the ratio is a fraction.

WRITTEN EXERCISES.

1. If the first term is 3, the rate 4 and the number of terms 6, what is the last term?

Solution.—The second term = 3×4; the third = 3×4^2; the fourth = 3×4^3, etc. That is, the last term equals the first term multiplied by the ratio raised to a power whose exponent is one less than the number of terms. Hence, the 6th term of the series equals $3 \times 4^5 = 3 \times 1024 = 3072$.

2. If 3 is the first term, 4 the rate, and the number of terms 5, what is the sum of the terms?

Solution.—The series written out is 3, 12, 48, 192, 768.

Multiplying by 4, the ratio, and reversing terms, 3072, 768, 192, 48, 12.
Subtracting original series reversed, 768, 192, 48, 12, 3.

Gives (series × 4) − series = series × 3 = 3072 − 3 = 3069.

Since 3 times the series = 3069, 3069 ÷ 3, or 1023, = sum of series.

Hence, *the sum of a series is found by multiplying the last term by the ratio, and dividing the difference between the product and the first term by the ratio less* 1.

3. The first term is 4, the rate 3: what is the sixth term?

4. The first term of a descending series is 48, and the second 24: what is the 8th term?

5. The first term is 5, the rate 5 and the last term 15,625; what is the sum of the terms?

6. A man accepted an offer to buy 15 dictionaries, on the condition that he should pay 2 cents for the first volume, 4 cents for the second, 8 cents for the third, and so on; what did he owe for the 15 books?

7. A father agreed to deposit 1 cent in a savings bank for his son when he was a year old, and to double the deposit on each succeeding birthday until the son was 21 years of age. Find the sum of the deposits.

NOTE.—Few realize the enormous increase of a number when expanded by geometrical ratio. If a man should agree to buy a horse on the condition that he paid 1 cent for the first nail in its shoes, 2 cents for the second, 4 cents for the third, and at that rate until the whole 32 nails had been paid for, the sum required (as the pupil can learn) would be over $40,000,000.

REVIEW QUESTIONS.

What is meant by the square or second power of a number? The cube? What is an exponent? The root or base of a number?

What is involution? Evolution? The root of a number? The square root? What is the index of a root?

What is evolution? Give the rule for the extraction of the square root of a number. How do the areas of similar figures compare with each other? How do the corresponding dimensions of similar figures compare?

Give the rule for the extraction of the cube root of a number. How do similar volumes compare with each other? How do the corresponding dimensions of similar volumes compare?

What is an arithmetical progression? An ascending series? A descending series? What are the terms of a series? The extremes? The means? What is the common difference? The sum of the terms? What is a geometrical progression? The ratio?

Mensuration.

ART. 453.—Mensuration treats of the measurements of angles, lines, surfaces and volumes.

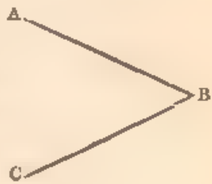
ART. 454.—A **Line** has length only.

ART. 455.—A **Straight Line** is one that does not change its direction: it is the shortest distance between two points.

ART. 456.—A **Curved Line** is one that changes its direction at every point.

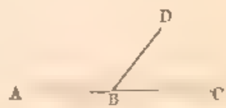
ART. 457.—**Parallel Lines** are those that have the same direction.

ART. 458.—An **Angle** is the opening between two straight lines that meet. Thus, the opening between the lines A B and C B is an angle.



ART. 459.—A **Right Angle** is an angle formed by one line perpendicular to another. Thus, the angles A B C and C B D are right angles.

ART. 460.—An **Acute Angle** is an angle less than a right angle; as D B C.



ART. 461.—An **Obtuse Angle** is greater than a right angle; as A B D.

ART. 462. The **Vertex** of an angle is the point where the sides meet.

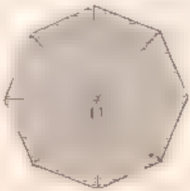
ART. 463.—A **Surface** is that which has length and

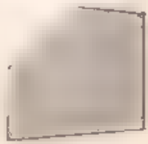
breadth but no thickness; it may be either plane or curved.

ART. 464. A **Plane Surface** is a surface such that if any two of its points be joined by a straight line, every part of the line will touch the surface.

Measurement of Lines and Surfaces.

ART. 465.—A **Plane Figure** is a figure all of whose parts are in the same plane.





ART. 466.—A **Polygon** is a portion of a plane bounded by straight lines.

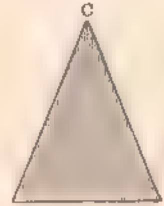
ART. 467. The **Perimeter** of a polygon is the sum of its sides, or the distance around it.

ART. 468. The **Diagonal** of a polygon is a line connecting the vertices of two angles not adjoining each other.

ART. 469. The **Area** of a plane figure is the number of square units in its surface.

ART. 470. A **Triangle** is a polygon with three sides; a **Quadrilateral** is a polygon with four sides; a **Pentagon** is a polygon with five sides; a **Hexagon** is a polygon with six sides; etc.

THE TRIANGLE.



A B C represents a triangle.

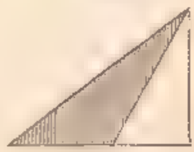
ART. 471. The **Base** is the side on which it is supposed to stand.

ART. 472. The **Altitude** is a perpendicular line drawn from the vertical

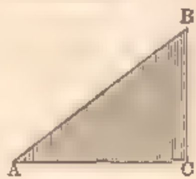
angle to the base: thus, C D is the altitude of the triangle A B C.

ART. 473.—An **Equilateral Triangle** is a triangle that has all its sides equal.

ART. 474.—An **Isosceles Triangle** is a triangle that has two of its sides equal.



ART. 475. A **Scalene Triangle** is a triangle that has no two sides equal.



ART. 476. A **Right-Angled Triangle** is a triangle that has one right angle. Thus, A C B is a right-angled triangle, because the angle A C B is a right angle. The side A B, opposite the right angle is the hypotenuse, the side A C the base, and the side B C the altitude.

A right-angled triangle is equal to one half a rectangle having the same base and altitude.

ART. 477. Rule for finding the area of any triangle.—*Multiply the base by one half the altitude.*

WRITTEN EXERCISES.

1. What is the area of a right-angled triangle whose base is 8 feet and altitude 6 feet?

2. What is the area of a right-angled triangle whose base is 30 rods and altitude 20 rods?

3. How many square feet in a triangle whose base is 40.5 feet and its altitude 30.25 feet?

NOTE.—When the three sides are given, and not the altitude, from one-half the sum of the sides subtract each side separately, multiply the half the sum and the remainders together, and extract the square root of the product.

4. What is the area of a triangle whose sides respectively are 30, 40 and 50 chains?

5. The base of a right-angled triangle is 6 feet and the height 5 feet: what is the hypotenuse?

NOTE.—It has been shown that the square of the hypotenuse equals the sum of the squares of the other two sides; the difference between the square of the hypotenuse and the square of one of the sides is the square of the remaining side. Hence, when any two sides of a right-angled triangle are given, the third can be found.

6. If the hypotenuse is 10 feet and the altitude 8 feet, what is the base?

7. If the hypotenuse is 9 feet and the base 8 feet, what is the altitude?

8. A certain room is 60 feet long and 40 feet wide: what is the distance between the opposite corners?

9. A ladder 32 feet long reached a window 24 feet above ground: how far was the base of the ladder from the side of the house?

10. A flagstaff was broken 40 feet from the ground by the wind, and fell so that the top rested 30 feet from the foot of the staff: how long was the flagstaff before it was broken?

11. A surveying expedition separated into two parties, one traveling north at the rate of 3 miles an hour and the other west at the rate of $3\frac{1}{4}$ miles an hour: how far were they from each other at the end of 12 hours?

12. Illustrate by an original problem the measurement of triangles.

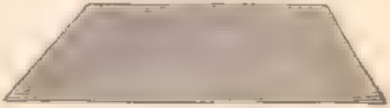
THE QUADRILATERAL.

ART. 478.—A **Quadrilateral** is a polygon having four sides.

ART. 479.—A **Parallelogram** is a quadrilateral whose opposite sides are equal and parallel.

ART. 480.—When the parallelogram is right-angled, it is called a *rectangle*; when the four sides are equal, it is a *square*, when not right-angled, it is a *rhomboid*; when the sides of the rhomboid are equal, it is a *rhombus*.

ART. 481.—A **Trapezoid** is a quadrilateral having two of its sides parallel.



ART. 482.—A **Trapezium** is a quadrilateral having no two of its sides parallel.

ART. 483.—The **Altitude** of a parallelogram or of a trapezoid is the perpendicular distance between its parallel sides.

ART. 484.—**To find the area of a parallelogram.** *Multiply the base by the altitude.*

WRITTEN EXERCISES.

1. What is the area of a parallelogram 30 feet long and 16 feet wide?

2. What is the difference between the areas of two lots, one of which is 250 rods long, 60 rods wide, and the other 140 rods long and 80 rods wide?

3. Illustrate by an original problem the method of measuring parallelograms.

ART. 485. **To find the area of a trapezoid.** *Multiply one half the sum of the parallel sides by the altitude.*

20

WRITTEN EXERCISES.

1. What is the area of a trapezoid whose parallel sides are 160 feet and 100 feet respectively, and whose altitude is 80 feet?

2. What is the area of a trapezoid whose parallel sides are 140 and 80 feet, and whose height is 65 feet?

3 A field in the form of a trapezoid has two parallel sides, 75 and 100 rods respectively, and the distance between them is 60 rods: what is the area of the field?

4. Illustrate by an original problem the method of measuring a trapezoid.

ART. 486. To find the area of a trapezium *Divide the trapezium into two triangles by a diagonal, find the area of the triangles separately, and add the two together.*

WRITTEN EXERCISES.

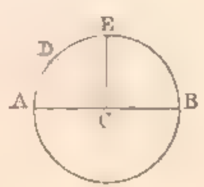
1. What is the area of a trapezium whose diagonal is 80 inches, and the altitudes of whose 2 triangles, measuring from the diagonal, are 40 and 50 inches respectively?

2. In a trapezium the sides in order are 30, 32, 34 and 36 feet, and the length of the shorter diagonal is 49 feet: what is the area? (Let the pupil draw the figure.)

3 Illustrate by an original problem the method of finding the area of a trapezium.

THE CIRCLE.

ART. 487.—A Circle is a plane figure bounded by a curved line called the *circumference*, every point of which is equally distant from a point within called the *center*.



ART. 488.—The **Diameter** of a circle is a straight line beginning at the circumference,

passing through the center and ending in the circumference on the other side. Thus, AB is the diameter of the circle in the margin.

ART. 489.—The **Radius** of a circle is the distance from the center to the circumference. Thus, AC is the radius of the circle in the margin.

Illustration.—Take as large a cylinder as can be obtained, measure its diameter and circumference accurately, and divide the circumference by the diameter. The quotient, if the measurements are accurate, is 3.1416, which is the ratio of the diameter to the circumference of any and every circle. Hence—

ART. 490.—To find the circumference of a circle. *Multiply the diameter by* 3.1416. **To find the diameter of a circle.** *Divide the circumference by* 3.1416.

WRITTEN EXERCISES.

1. What is the circumference of a circle whose diameter is 30 inches?

2. If the circumference of a circle is 36 feet, what is the radius?

3. Find the circumference of a circle whose diameter is 45 feet.

SUGGESTION.—Since the surface of a circle consists of a triangle whose base is the circumference and whose altitude is the radius, its area equals $3.1416 \times \text{Diameter} \times \frac{1}{4}$ of Diameter $= D^2 \times .7854$. (The convex surface of a cone is also a triangle. See **Art. 505.**) Hence—

ART. 491. To find the area of a circle.—*Multiply the square of the diameter by* .7854.

WRITTEN EXERCISES.

1. What is the area of a circle whose diameter is 20 feet?

2. What is the area of a circle whose diameter is 30 inches?

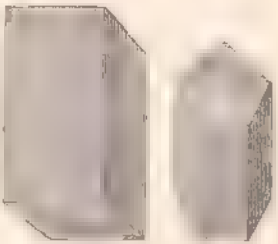
3. What is the area of a circle whose radius is 10 feet?

4. What is the area of a circle whose circumference is 120 feet?

Measurement of Volumes.

ART. 492. A **Volume** has length, breadth and thickness.

ART. 493.—A **Prism** is a volume whose ends are equal and parallel polygons, and whose sides are parallelograms.

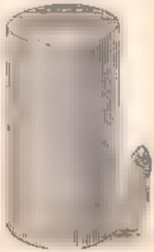


Quadrangular Prism. Pentagonal Prism.

ART. 494. The **Bases** of a prism are its ends.

ART. 495. A **Cylinder** is a round volume of equal diameter throughout, whose ends are circles.

ART. 496. The **Convex Surface** of a prism or of a cylinder is its entire surface less the surface of the two bases.



ART. 497.—The **Altitude** of a prism or of a cylinder is the perpendicular distance between the bases.

ART. 498. **To find the convex or the entire surface of a prism or of a cylinder.**—*Multiply the perimeter of the base by the altitude:* to find the entire surface, *add the area of the bases.*

WRITTEN EXERCISES.

1. What is the convex surface of a prism whose base is square, each of whose sides is 4 feet, and whose height is 15 feet?

2. What is the entire surface of the prism described in the foregoing question?

3. What is the convex surface of a cylinder 30 feet long and 10 feet in diameter?

4. What is the entire surface of the cylinder described in the foregoing question?

5. Illustrate by an original problem the method of finding the surface of a prism.

6. Illustrate by an original problem the method of finding the surface of a cylinder.

ART. 499. To find the contents of a prism or of a cylinder.—*Multiply the area of the base by the altitude.*

WRITTEN EXERCISES.

1. Find the contents of a square prism, each of whose sides is 5 feet and whose height is 7 feet.

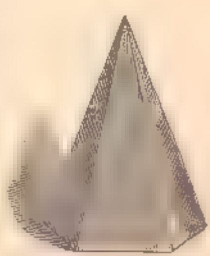
2. Find the contents of a triangular prism, each of whose sides is 8 inches and whose height is 1 foot.

3. Find the contents of a cylinder 50 feet long and 6 feet in diameter.

4. Illustrate by an original problem the method of finding the contents of a prism or cylinder.

THE PYRAMID.

ART. 500.—A **Pyramid** is a volume bounded by a polygon and several triangles that meet at a common vertex.



ART. 501. — The polygon is the **Base**, and the triangles form the convex surface.

ART. 502. To find the convex surface of a pyramid.—*Multiply the perimeter of the base by one half the slant height.*

WRITTEN EXERCISES.

1. What is the convex surface of a square pyramid whose base is 10 feet 6 inches, and whose slant height is 20 feet?

2. What is the convex surface of a triangular pyramid whose sides are each 5 feet, and whose slant height is 24 feet?

3. Illustrate by an original problem the method of finding the convex surface of a pyramid.

ART. 503. To find the contents of a pyramid. *Multiply the area of the base by one third the altitude.*

WRITTEN EXERCISES.

1. Find the contents of a pyramid whose base is 6 feet square, and whose altitude is 51 feet.

2. Find the contents of a pyramid whose base is a triangle, each side of which is 8 feet, and whose altitude is 40 feet.

3. Illustrate by an original problem the method of finding the contents of a pyramid.

THE CONE.

ART. 504.—A **Cone** is a volume whose base is a circle, and whose convex surface tapers uniformly to a point called the vertex.

ART. 505. To find the convex surface of a cone. *Multiply the circumference of the base by one half the slant height.*

WRITTEN EXERCISES.

1. Find the convex surface of a cone, the diameter of

whose base is 4 feet and whose slant height is 20 feet 8 inches.

2. Find the convex surface of a cone, the circumference of whose base is $10\frac{1}{2}$ feet and whose slant height is 40 feet.

3. Illustrate by an original problem the method of finding the convex surface of a cone.

ART. 506. To find the contents of a cone.—*Multiply the area of the base by one third the altitude.*

WRITTEN EXERCISES.

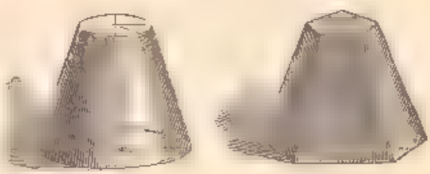
1. Find the contents of a cone 2 feet in diameter and 6 feet high.

2. Find the contents of a cone 9 feet high and 12 feet in circumference.

3. Illustrate by an original problem the method of finding the contents of a cone.

THE FRUSTUM OF A PYRAMID OR CONE.

ART. 507. — The **Frustum** of a pyramid or of a cone is that portion which remains after cutting off the upper part by a plane parallel to the base.



ART. 508. — To find the convex surface of a frustum.— *Multiply the sum of the perimeters of the two bases by one half the slant height.*

WRITTEN EXERCISES.

1. What is the convex surface of the frustum of a square pyramid whose slant height is 18 feet, the side of the upper base 5 feet and of the lower base 12 feet?

2. What is the convex surface of a cone whose slant

height is 14 feet, whose upper base is 3 feet in diameter and whose lower base 7 feet in diameter?

3. Illustrate by an original problem the method of finding the convex surface of **a frustum.**

ART. 509.—To find the contents of a frustum.—*To the area of the two bases, add the square root of their product, and multiply the sum by one third the altitude of the frustum.*

WRITTEN EXERCISES.

1. Find the contents of the frustum of a square pyramid the sides of whose bases are 3 and 4 feet respectively, and whose height is 12 feet.

2. Find the contents of the frustum of a cone 30 feet long, the diameter of one base being 2 feet and of the other 5 feet.

3. A stump 10 feet high was 3 feet in diameter at the base, and 30 inches at the top: find its cubical contents.

4. Illustrate by an original problem the method of finding the contents of a frustum.

THE SPHERE.

ART. 510.—A Sphere is a volume bounded by a curved surface, every point of which is equally distant from a point within called the center.

ART. 511.—The Diameter of a sphere is a straight line beginning at the surface, passing through the center and terminating in the surface on the other side.

ART. 512.—The **Radius** of a sphere is a straight line extending from the center to the circumference.

ART. 513.—To find the surface of a sphere.—*Multiply the square of the diameter by* 3.1416, *or multiply the circumference by the diameter.*

WRITTEN EXERCISES.

1. Required the surface of a sphere whose diameter is 20 inches.

2. Required the surface of a sphere whose circumference is 8 feet.

3. Required the surface of a sphere whose diameter is 1 foot.

Illustration.—Take a cube and a sphere made of the same material, and let the edge of the cube equal the diameter of the sphere. Weigh them carefully: divide the weight of the sphere by the weight of the cube, which gives .5236, the ratio of their volumes. Hence—

ART. 514.—To find the volume of a sphere.—*Multiply the cube of the diameter by* .5236.

WRITTEN EXERCISES.

1. What is the volume of a sphere whose diameter is 30 inches?

2. What is the volume of a globe whose diameter is 18 inches?

3. What is the volume of a sphere whose radius is 10 inches?

4. The diameter of a sphere is 6 feet: how many cubic feet does it contain?

5. The circumference of a sphere is 3.1416 inches: find its cubical contents.

6. Find the cubical contents of a spherical vessel whose diameter is 3 feet.

7. The volume of a sphere is 2 cubic feet: what is its diameter?

8. Illustrate by an original problem the method of finding the volume of a sphere.

GAUGING.

ART. 515.—Gauging is the method of finding the capacity of casks, barrels, and similar vessels.

A barrel differs from a cylinder by the expansion in the middle. One half of the middle, or bung diameter, added to one half the end diameter is the mean diameter of the barrel, or of a cylinder of equal capacity.

The following rule is in general use:

ART. 516.—*Multiply the square of the mean diameter in inches by the length in inches, and the product by .0034. The result will be the capacity in wine gallons.*

WRITTEN EXERCISES.

1. How many gallons in a cask whose head diameter is 26 inches, bung diameter 32 inches, and length 42 inches?

2. What is the capacity of a cask whose bung diameter is 36 inches, head diameter 30 inches, and length 3 feet 8 inches?

3. What is the capacity of a cask whose length is 40 inches, and the diameters 24 and 28 inches respectively?

4. How many gallons will fill a barrel 4 feet long, head diameter 24 inches, bung diameter 28 inches?

5. Illustrate by an original problem the method of gauging.

The Metric System.

ART. 517.—The Metric System was adopted by France in 1795. The commission to whom was entrusted the work of constructing a system of weights and measures, divided the distance from the equator to the poles into 10,000,000 equal parts, and named one of these parts a meter. This they made the unit of length, and from it the entire system is derived.

Multiples and subdivisions are indicated by prefixes to the names of the fundamental units. The prefixes denoting multiples are *deca*, ten; *hecto*, hundred; *kilo*, thousand; *myria*, ten thousand. Those denoting subdivisions are *deci*, tenth; *centi*, hundredth; *milli*, thousandth.

Reduction from one denomination to another is performed by simply moving the decimal point to the right or left as required.

The principal metric units are:—

ART. 518.—Meter for length.

ART. 519.—Square Meter and the **Are** for surface.

ART. 520.—Cubic Meter or **Stere** for large volumes.

ART. 521.—Liter for smaller volumes.

ART. 522.—Gram for weight.

Measures of Length.

ART. 523.—The **Meter** is the unit of length. It equals 39.37 inches or 3.28 feet.

10 millimeters (mm.)	= 1 centimeter, cm.
10 centimeters	= 1 decimeter, dm.
10 decimeters	= 1 meter, M.
10 meters	= 1 decameter, DM.
10 decameters	= 1 hectometer, HM.
10 hectometers	= 1 kilometer, KM.
10 kilometers	= 1 myriameter, MM.

WRITTEN EXERCISES.

1. Write 7 DM, 7 M, 4 cm, 9 mm, as meters.
2. Write 3 MM, 5 HM, 4 M, 8 dm, 3 mm, as meters.
3. Write 6 KM, 4 DM, 3 dm, 7 cm, as meters.
4. Change 7.46 HM to decimeters.
5. Reduce 78.46 KM to meters.
6. Reduce 4.28 MM to centimeters.
7. Reduce 400,326.39 centimeters to decameters.

Measures of Surface.

ART. 524.—The **Are** is the unit of surface. It is 10 meters square, and contains 100 square meters.

10 milliares (ma)	=	1 centiare, ca.
10 centiares	=	1 deciare, da.
10 deciares	=	1 are, A.
10 ares	=	1 decare, DA.
10 decares	=	1 hectare, HA.
10 hectares	=	1 kilare, KA.
10 kilares	=	1 myriare, MA.

WRITTEN EXERCISES.

1. Reduce 1,217 ares to hectares.
2. Reduce 9 kilares to deciares.
3. Reduce 1,848.362 milliares to ares.
4. What is the surface of a ceiling 15.8 M long and 12.7 M wide?
5. At $1.25 a sq. M, how much will it cost to carpet a room 8.25 M long and 6.5 M wide?

Measures of Volume.

ART. 525. The cubic meter, called a *stere*, is the unit of volume. It equals 35.3166 cubic feet, or 1.308 cubic yards.

TABLE.

1,000 cubic millimeters	=	1 cubic centimeter, cu. cm.
1,000 cubic centimeters	=	1 cubic decimeter, cu. dm.
1,000 cubic decimeters	=	1 cubic meter, cu. M.

A cord is equal to 3.6 *steres* nearly.

WRITTEN EXERCISES.

Change:

1. 1840 cubic centimeters to cubic decimeters.

2. 392.7 cubic decimeters to cubic centimeters.

3. 1654 cubic decimeters to steres.

4. What will it cost to excavate a cellar 15.3 M long, 8.4 M wide and 2.1 M deep, at $.75 a stere?

5. How many loads of coal, each 2.5 steres, will fill a vault 8 M long, 5 M deep and 3.25 M wide?

Measures of Capacity.

ART. 526.—The **Liter** is the unit of capacity. It equals a cubic decimeter, or 1.0567 qt. liquid measure, or .908 qt. dry measure.

TABLE.

10 milliliters (ml)	=	1 centiliter, cl.
10 centiliters	=	1 deciliter, dl.
10 deciliters	=	1 liter, L.
10 liters	=	1 decaliter, DL.
10 decaliters	=	1 hectoliter, HL.
10 hectoliters	=	1 kiloliter, KL.
10 kiloliters	=	1 myrialiter, ML.

The *liter* is principally used in measuring liquids in small quantities; the decaliter, in large quantities; the hectoliter in measuring grain. Four liters are slightly more than a gallon; 35 liters are almost a bushel.

WRITTEN EXERCISES.

1. Reduce 17 deciliters to decaliters.

2. Reduce 7.002 liters to centiliters.

3. Reduce 349.2 milliliters to myrialiters.

4. Reduce 16.084 kiloliters to milliliters.

5. A tank 4.6 M long, 3.4 M wide, and 4.6 M deep contains how many liters?

Measures of Weight.

ART. 527. The **Gram** is the unit of weight, and is the weight of 1 cubic centimeter of distilled water at the temperature of melting ice. The *gram* equals 15.432 Troy grains, or .03527 of an ounce avoirdupois.

TABLE.

10 milligrams (mg)	=	1 centigram, cg.
10 centigrams	=	1 decigram, dg.
10 decigrams	=	1 gram, G.
10 grams	=	1 decagram, DG.
10 decagrams	=	1 hectogram, HG.
10 hectograms	=	1 kilogram, KG or K.
10 kilograms	=	1 myrigram, MG.
10 myrigrams } 100 kilos }	=	1 quintal, Q.
1000 kilos	=	1 tonne, T.

The kilogram, or kilo, is the common unit of weight. It is about $2\frac{1}{5}$ pounds avoirdupois.

WRITTEN EXERCISES.

1. Reduce 32.1 grams to decagrams.

2. Reduce 413.7 decigrams to decagrams.

3. Reduce 563.04 hectograms to centigrams.

4. Reduce 7.001 milligrams to grams.

5. Reduce 2005.0904 kilos to decigrams.

6. If a kilogram of butter is worth 45 cents, how much will 5 hectograms cost?

ART. 528.—Table of Equivalents.

MEASURES OF LENGTH.

1 inch	=	2.54 centimeters.
1 foot	=	.3048 meter.
1 yard	=	.9144 meter.
1 rod	=	5.029 meters.
1 mile	=	1.6093 kilometers.

MEASURES OF SURFACE.

1 square inch	=	6.4528 square centimeters.
1 square foot	=	.0929 square meter.
1 square yard	=	.8361 square meter.
1 square rod	=	25.293 centars.
1 acre	=	40.47 ares.
1 square mile	=	259 hektares.

MEASURES OF VOLUME.

1 cubic inch	=	16.387 cubic centimeters.
1 cubic foot	=	28.317 cubic decimeters.
1 cubic yard	=	.7645 cubic meter.
1 cord	=	3.6243 steres.

MEASURES OF CAPACITY.

1 fluid ounce	=	.02958 liter.
1 liquid quart	=	.9465 liter.
1 gallon	=	3.785 liters.
1 dry quart	=	1.101 liters.
1 bushel	=	.35243 hectoliter.

MEASURES OF WEIGHT.

1 grain Troy	=	.0648 gram.
1 ounce Troy	=	31.324 grams.
1 pound Troy	=	.37324 kilo.
1 ounce Avoirdupois	=	28.35 grams.
1 pound Avoirdupois	=	.4536 kilo.
1 ton	=	.907 tonne.

REVIEW QUESTIONS.

Of what does mensuration treat? What is a line? A straight line? A curved line? Parallel lines? An angle? A right angle? An acute angle? An obtuse angle? A surface? A plane surface?

What is a plane figure? A polygon? The perimeter of a polygon? The diagonal of a polygon? The area of a plane figure? A triangle? Draw a triangle and show the base; the altitude.

What is an equilateral triangle? An isosceles triangle? A scalene triangle? A right-angled triangle? Give the rule for finding the area of a right angled triangle. What relation does the hypotenuse of a right-angled triangle bear to the sum of the other sides?

What is a quadrilateral? A parallelogram? A rectangle? A square? A rhomboid? A rhombus? A trapezoid? A trapezium? What is the altitude of a parallelogram or trapezoid?

How do you find the area of a parallelogram? Of a trapezoid? Of a trapezium?

What is a circle? What is the center? The diameter? The circumference? The radius? How do you find the circumference of a circle? The diameter? The area of a circle?

What is a volume? A prism? The bases of a prism? The convex surface? A cylinder? The altitude of a prism or cylinder? How do you find the convex surface of a prism or cylinder? The contents of a prism or cylinder?

What is a pyramid? The base? How do you find the convex surface of a pyramid? The contents of a pyramid?

What is a cone? How do you find its convex surface? Its contents? What is the frustum of a pyramid or cone? How do you find the convex surface of a frustum? The contents of a frustum?

What is a sphere? Its diameter? Its radius? How do you find the surface of a sphere? The contents of a sphere?

What is gauging? What rule is in general use?

What country originated the metric system? What was fixed upon as the basis? What was this called? What is the unit of length? Of surface measure? Of capacity? Of the measure of solidity? The unit of weight?

How are the decimal multiples and subdivisions indicated? What are the prefixes denoting multiples? What is said of those denoting subdivisions?

How do you add, subtract, multiply, and divide by the metric system?

Test Problems.

ORAL EXERCISES.

1. Joseph and William each had the same sum; Joseph spent $\frac{1}{3}$ of his and William spent $\frac{1}{2}$ of his: what part of Joseph's money then equaled William's?

2. If 9 men can do a piece of work in $7\frac{7}{8}$ days, how long will it take 7 men to do it?

3. Mary divided 30 yards of ribbon among her classmates, giving to each $3\frac{3}{4}$ yards: how many classmates had she?

4. Bonsall bought 7 yards of cloth worth $\$3\frac{1}{8}$ a yard and paid for it with apples worth $\frac{5}{8}$ of a dollar a bushel: how many bushels were required?

5. Robert's bicycle cost \$95, and $\frac{5}{19}$ of its cost is $\frac{5}{13}$ of the cost of his sister's sewing machine: what was the cost of the sewing machine?

6. What part of 11 cents is $\frac{4}{5}$ of a dime?

7. How many ounces in $8\frac{2}{3}$ pounds Troy?

8. If 7 grains of medicine cost 35 cents, how much will 2 scruples cost?

9. What is the difference between 7 feet square and 7 square feet?

10. Which months have 30 days each? Which have 31 days?

11. How much will 10 sheets of paper cost at 24 cents a quire?

12. What number is that to which if you add $\frac{2}{3}$ of itself, the sum will be 45?

13. David's age, increased by twice $\frac{3}{4}$ of his age equals 40 years: what is his age?

14. Mr. Walters being asked his age, replied that the difference between $\frac{3}{4}$ and $\frac{1}{3}$ of his age added to his age would give 68 years: what was his age?

15. Beulah skipped the rope 60 times, which was $\frac{3}{4}$ of $\frac{4}{5}$ of twice the number of times Maria skipped the rope: how many times did Maria skip the rope?

16. Susannah expended $\frac{2}{3}$ of her money. She then earned $\frac{4}{5}$ as much as she had at first, and then had 34 cents: how many had she at first?

17. $\frac{4}{9}$ of John's money plus \$8 equals \$36: how much money has he?

18. Timothy's age diminished by $\frac{1}{3}$ of $\frac{2}{3}$ of his age is 14 years: what is the age of Timothy?

19. A's house is 20 rods from B's, and $\frac{4}{5}$ of that distance lacks 6 rods of being $\frac{11}{12}$ the distance from B's house to C's: find the distance from B's house to C's.

20. Sarah spent $\frac{1}{3}$ of her money for nuts, 10 cents for candy, and had 14 cents left: how much had she at first?

21. Richard had a certain number of apples. He gave away $\frac{1}{4}$ of them, bought 10, and then had 34: how many had he at first?

22. A horse and carriage cost \$300, and $\frac{3}{8}$ of the cost of the carriage plus \$9 equaled \$84: what was the cost of each?

23. A certain number increased by 15 is 4 times the same number: what is the number?

24. 5 times a certain number, less 20, is three times the number: what is the number?

25. Samuel's age, less 6 more than one half, equals 4 years: what is his age?

26. Harry spent 9 cents more than $\frac{1}{2}$ his money, and then had $\frac{1}{4}$ as much as at first: how much had he at first?

27. 5 times a number, less 10, equals three times the number, plus 4: what is the number?

28. 4 times a certain number, plus $\frac{2}{3}$ the number, plus 3, equals 5 times the number: what is the number?

29. Spencer is 4 times as old as Allen, and the sum of their ages is 55 years: what is the age of each?

30. A house and barn cost \$7,000. If the barn cost $\frac{2}{5}$ as much as the house, what was the cost of each?

31. John had 30 cents more than Frank: if 3 times Frank's money equaled John's, how much had each?

32. A man willed his wife twice as much as his daughter, and his son twice as much as his wife: how much did each receive, if he left \$14,000?

33. Tracy is $\frac{1}{3}$ as old as Warren, and Warren is $\frac{1}{3}$ as old as Mr. Brown: if the sum of their ages is 104 years, how old is each?

34. The age of Watson is $\frac{3}{4}$ of the age of Warren, and the sum of their ages is 35 years: how old is each?

35. The sum of two numbers is 20, and $\frac{1}{2}$ the smaller equals $\frac{1}{3}$ the greater: what are the numbers?

36. Divide 45 so that $\frac{3}{4}$ of the one part will equal $\frac{6}{7}$ of the other.

37. The difference between two numbers is 11, and $\frac{1}{2}$ of the one equals $\frac{1}{3}$ of the other: what are the numbers?

38. If \$340 be divided into two parts that are to each other as $\frac{2}{3}$ to $\frac{3}{4}$, it will give $\frac{1}{6}$ of A's and $\frac{1}{10}$ of B's money: how much money has each?

39. If 5 boys spend 50 cents in 5 days, how long, at the same rate, will 24 cents last 3 boys?

40. If it takes 4 men 7 days to dig a cellar, how long will it take 14 men to do twice as much?

41. If 3 boys can pick $\frac{3}{4}$ of a bushel of cherries in 1 hour, how long will it take 5 boys to pick $2\frac{1}{2}$ bushels?

42. How many books can be bought for \$20, if 3 books are worth 6 gold pens, and 4 gold pens are worth \$8?

43. If a party of 8 men have enough provisions to last 7 days, how long will they last if 1 man leaves after half the provisions are eaten?

44. How many pounds in $5\frac{3}{4}$ tons?

45. How many grains in $9\frac{7}{8}$ pwt?

46. How many pints in $12\frac{3}{4}$ gal.?

47. How many rods in $\frac{4}{5}$ of a mile?

48. How many square inches in $7\frac{1}{2}$ square feet?

49. How many cubic inches in a block 6 inches long, 8 inches wide, and 7 inches thick?

50. The side of a certain cube is 11 inches square: find the contents of the cube.

51. A square ceiling contains 110.25 square feet: how long is one side of the ceiling?

52. How many pens in a box containing $\frac{1}{3}$ of a gross?

53. How many sheets of paper in $4\frac{1}{2}$ reams?

54. What part of a peck is $\frac{3}{8}$ of a bushel?

55. What part of an hour is $\frac{1}{48}$ of a day?

56. What part of the cost equals the gain at $6\frac{1}{4}$%? $8\frac{1}{3}$%? $16\frac{2}{3}$%? 25%? $33\frac{1}{3}$%?

57. Jane bought 8 yards of cambric for $1.60 and sold it so as to gain 20%: how much did she receive a yard?

58. A owned 40 horses and sold 25% of them; B owned twice as many as A, and sold 50% of them: how many horses had each left?

59. John bought a dog for $5 and sold it for $$4\frac{1}{2}$: what was his loss per cent.?

60. Stewart broke 20% of a score of eggs, and sold 25% of what was left: how many remained?

61. What % of $\frac{2}{3}$ is $\frac{1}{6}$? Of $\frac{5}{7}$ is $\frac{1}{14}$? Of $\frac{5}{6}$ is $\frac{2}{18}$?

62. Orson sold his pony for $60 and thereby lost $\frac{1}{4}$ of what he gave for it: what % would he have gained by selling it for $90?

63. If a grocer sold potatoes at 75 cents a bushel and thereby gained 20%, what per cent. would he have lost by selling them at 50 cents a bushel?

64. Garwood gains 10% by selling apples at 22 cents a peck: what per cent. would he gain by selling them at $1 a bushel?

65. Hammond gains 40% by selling his boat for $20 more than it cost: how much did the boat cost?

66. Lawrence sold his wagon for $24 less than its value, and lost $33\frac{1}{3}$%: how much per cent. would he have gained had he sold it for $96?

67. $24 is $16\frac{2}{3}$% of the sum of A's and B's money, and A's is 3 times B's money: how much has each?

68. I sent my agent $2,100 to buy goods after deducting his commission of 5%: what was his commission?

69. What is the interest on $75 for 4 years at 5%?

70. What is the interest on $200 for 3 years at 4%?

71. What is the interest on $150 for 2 years at 6%?

72. What is the interest on $400 for 5 years at 3%?

73. What is the interest on $300 for 2 years at 4%?

74. What is the interest on $250 for 3 years at 8%?

75. What is the interest on $100 for 1 year 6 months at 6%?

76. What is the interest on $300 for 1 year 4 months at 8%?

77. What is the interest on $200 for 2 years 8 months at 6%?

78. What is the interest on $500 for 3 years 4 months at 3%?

79. What is the interest on $200 for 2 years 6 months at 9%?

80. What is the interest on $10 for 4 years 4 months at 4%?

81. What is the interest on $1,000 for 2 years 2 months 12 days at 5%?

82. What is the interest on $700 for 1 year 10 months 24 days at 6%?

83. What is the interest on $400 for 1 year 10 months 15 days at 7%?

84. What is the interest on $600 for 2 years 9 months 18 days at 4%?

85. What is the interest on $300 for 1 year 10 months 20 days at 8%?

86. If I sell a house to Mr. Jones at a loss of 10%, and he sells it at a gain of 10%, how does his selling price compare with the price I paid for the house?

87. When a gold dollar is worth $1.25 paper money, how much gold is a paper dollar worth?

88. What principal in 1 year 2 months 12 days at 5% will amount to $530?

89. What will it cost at 20 cents a cubic yard, to have a cellar dug 33 feet long, 27 feet wide and 6 feet deep?

90. What will it cost to carpet a room, 24 feet long, 13½ feet wide, at $1.50 a square yard?

91. A and B have together $800, and A has 3 times as much as B: what is the amount of each sum for 4 years at 5%?

92. If the amount of $3,000 for 5 years at 6% be divided into two parts as 6 to 7, it will respectively give 9 times C's and 7 times D's money: how much money has each?

93. The amount of ¾ of A's and ⅘ of B's money for 4 years at 5% is $8,400: how much has each if ½ of A's equals ⅔ of B's?

WRITTEN EXERCISES.

1. Divide 3,380,321 by MDCCXCIX and express the quotient in Roman notation.

2. Change .013 to an equivalent fraction whose denominator is 135.

3. Find the greatest common divisor of 315, 357 and 504.

4. How many men would be required to hoe a field of $2\frac{5}{8}$ acres in $5\frac{1}{2}$ days of 10 hours each, if each man completed 77 square yards in 3 hours?

5. 5 cubic feet of gold weigh 98.2 times as much as a cubic foot of water, and 2 cubic feet of copper weigh 18 times as much as a cubic foot of water: how many cubic inches of copper will weigh as much as $\frac{7}{8}$ of a cubic inch of gold?

6. Find the least common multiple of the numbers 456, 608 and 760.

7. A wins 9 games of chess out of 15 when playing against B, and 16 out of 25 when playing against C: at that rate, how many games out of 118 should C win when playing against B?

8. A and B run a race, their rates of running being as 17 to 18. A runs $2\frac{1}{3}$ miles in 16 minutes 48 seconds, and B runs the entire distance in 34 minutes: what was the entire distance?

9. A and B can do a piece of work in 4 hours, A and and C in $3\frac{3}{5}$ hours, B and C in $5\frac{1}{4}$ hours: in what time can A do it alone?

10. A and B worked together for 17 days and earned $72.25, how much should each receive if $\frac{2}{3}$ of A's daily pay was equal to $\frac{3}{4}$ of B's?

11. John and William start from the same corner of a rectangular field $\frac{1}{2}$ a mile long and 36 rods wide; William walks straight across to the opposite corner at the rate of 3 miles an hour, while John walks around to the same corner at the rate of 4 miles an hour: who reaches the corner first, and how far is the other from it?

12. If ten men earn \$40 in 3 days, working 8 hours a day, how much will three men earn in 2 days, working 10 hours a day?

13. A cistern is filled by 2 pipes. The first fills it in 45 minutes, and the second in one hour: how long would it require them together to fill the cistern?

14. What is the cube root of .0125?

15. How long must \$650 be on interest at 4% to yield \$273?

16. What is .004 of .004?

17. .03 is .006 of what number?

18. A and B each had the same number of sheep. A sold 30% of his, and B sold 50% of his number, after which the sum of their remaining sheep was 288: how many had each at first?

19. Add $3\frac{1}{3}$, $2\frac{5}{8}$, $\frac{7}{10}$, $10\frac{3}{25}$, $\dfrac{9\frac{3}{4}}{2\frac{3}{5}}$.

20. A man has two herds, one of sheep and one of cows. One half of three fifths of the number of sheep is 9, which is one third of three fourths of the number of cows: how many are in each herd?

21. A newsboy earns $\$3\frac{1}{2}$ per week, and saves $\frac{3}{10}$ of it: how long will it take him to save enough to buy a suit of clothes worth \$10.80?

22. Find the value of $\left(3\frac{1}{4} + \dfrac{6}{2\frac{2}{3}} - \dfrac{7}{3\frac{1}{4}}\right) \div \left(\dfrac{2\frac{1}{2}}{3\frac{1}{6}} \text{ of } \dfrac{6\frac{1}{4}}{9\frac{1}{8}}\right)$.

23. A man has a number of blocks each 3 inches in length, breadth and height; how many of them in one cubic foot?

24. If I sell \$15,000 worth of my property, the remainder will be worth 85 per cent. of the value of the whole property: what is the value of the whole?

25. I bought 10 barrels of apples; $\frac{1}{4}$ of them proving worthless, I sold the remainder at \$3.96 per barrel, and thus gained 10 per cent. on the whole cost: what was the cost per barrel?

26. How much does a man gain or lose in a year by borrowing \$20,000 from a bank at 6 per cent. discount and lending the proceeds at $6\frac{1}{2}$ per cent. interest?

27. What is the equated time for paying \$200 due in five months, \$80 due in four months, and \$60 due in two months, the date of each indebtedness being July 4, 1885?

28. What will a 90-day note for \$3,000 yield, if discounted at a bank, the rate of discount being 6%?

29. What is the interest on a note of \$1,000 for one year, compounded monthly at $1\frac{1}{2}$ per cent. a month?

30. If apples are bought 3 for 2 cents, and sold 2 for 3 cents, what per cent. is made?

31. A tower 30 yards high has at its foot a ditch 40 yards wide: what is the distance from the top of the tower to the outside of the ditch?

32. If $3\frac{1}{8}$ yards of cloth $1\frac{1}{3}$ yards wide cost $\$3.62\frac{1}{2}$, what cost $30\frac{1}{4}$ yards $1\frac{1}{2}$ yards wide?

33. Find the value to five decimal places of $1.725 \times .38 \times 1.02 \div 1.821$.

34. The wheel of a carriage is 4 feet 6 inches in diameter: how many revolutions does it make in $1\frac{1}{2}$ hours, the rate of traveling being $7\frac{1}{3}$ miles per hour?

35. What principal at $7\frac{1}{2}$ per cent. per annum, will in 9 months and 20 days amount to \$650?

36. I discounted at a bank a note due in 60 days, and the proceeds was \$1,000: what was the face of the note, if the rate of discount was 6%?

37. Three carpenters, A, B and C, in 15 days, built a shed, for which they received \$75. A's share was \$25; B's share was \$35, and C's the remainder: how many days did each man work, all receiving the same wages?

38. The greatest common divisor of 110, 385 and 770, is contained how many times in their least common multiple?

39. One fourth per cent. of \$28,000 is $16\frac{2}{3}$ per cent. more than what sum?

40. Find the value of a bin of wheat 4 meters long, 3 meters wide, and 3 meters high at \$3.75 a hectoliter.

41. A and B can do a piece of work in $\frac{3}{4}$ of a day, and with C's assistance in $\frac{2}{3}$ of a day: how long will it take C alone to perform the work?

42. Divide $\frac{4}{5} + 3\frac{1}{3}$ by $4\frac{1}{6} - 3\frac{2}{3}$.

43. How many feet of flooring 5 inches wide will be required for a floor 26 feet by 22 feet 6 inches?

44. A and B together can do a piece of work in 20 days: in what time can each do it separately, if A does three times as much as B?

45. What fraction is that to which if $\frac{2}{7}$ of $\frac{5}{9}$ be added, the sum will be 1?

46. In what time, at 12 per cent., will \$1,728 amount to \$3,853.44?

47. If I sell cloth at \$7 per yard, and thereby gain 75 per cent., what per cent. do I gain or lose if I sell the same at \$3 per yard?

48. How many pounds of hay, worth \$18 per ton, can be bought for \$15?

49. If \$1 in gold is worth $\$1.02\frac{1}{2}$ in legal tender, what is \$1 in legal tender worth in gold?

50. A man dug a ditch 57 feet long, 3 feet wide, and 4 feet deep and received in payment 1 peck of wheat per cubic yard: how much did he get for the work, if he sold his wheat for \$1.10 per bushel?

51. Find the sum of $\frac{2\frac{1}{3}}{6}$, $\frac{3\frac{3}{4}}{4\frac{1}{8}}$ and $\frac{5\frac{1}{4}}{\frac{3}{4}}$.

52. A box contains 50 square feet of glass: the panes are 10 inches by 12 inches: how many are there?

53. If I buy cloth at \$1.20 a yard, less 25 per cent., and sell for \$1.35 a yard, what per cent. profit do I make?

54. The sum of two fractions is $\frac{11}{17}$ and their difference is $\frac{14}{35}$: what are the fractions?

55. After cutting 2 yds. 2 qrs. from a piece of cloth, there remained 8 yds. 3 qrs.: what part of the whole piece remained?

56. A man sold two buildings for \$7,200 each; on one he gained $12\frac{1}{2}$ per cent., and on the other he lost $12\frac{1}{2}$ per cent.: did he gain or lose, and how much?

57. What is the cube root of $\frac{59319}{68921}$?

58. Bought a hogshead of molasses containing 126 gallons for \$75.60, but 16 gallons having leaked out, the remainder was sold at \$.80 a gallon: what was the gain per cent.?

59. Reduce $\left(\frac{1\frac{2}{3}}{2\frac{1}{2}} \div \frac{2\frac{1}{5}}{5\frac{1}{2}}\right) \times \frac{1}{2}$ of $\frac{3}{10}$ to a decimal.

60. Borrowed \$500, and at the end of 5 years 6 months and 18 days the debt was \$694.25: what was the rate of interest?

61. An agent receives \$5,740 to expend for flour; after deducting his commission at $2\frac{1}{2}$ per cent., how many barrels of flour at \$9 a barrel will he be enabled to purchase?

62. An agent receives \$64.890 with which to purchase cotton; if he deducts his commission at 3 per cent. on the cost of the cotton that he buys, and with the remainder purchases 700 bales of cotton, how much will be the cost per bale?

63. What is the entire surface of a cube whose volume is 91.125 feet?

64. If Brown sells his horse for \$220, he gains 10 per cent.: what per cent. will he gain or lose if he sells him for 10 per cent. less than \$220?

65. The brokerage for selling or buying is $\frac{1}{4}$ per cent.: what will it cost to sell \$10,000 worth of bonds, and invest the net proceeds in stocks, both at par?

66. General Winfield S. Hancock was born Feb. 14, 1824, and died Feb. 9, 1886: what was his age in days?

67. I bought goods to the amount of \$650: for what sum shall I draw a 60-day note, so that when discounted at 7 per cent. it will yield that amount of cash?

68. Illustrate and explain by an original problem the difference between true and bank discount.

69. If $121\frac{1}{4}$ yards of linoleum, 2 yards wide, cost \$250, what is the cost of $171\frac{1}{2}$ yards, $1\frac{1}{2}$ yards wide, and 20 per cent. cheaper per square yard than the first?

70. If a ball 6 inches in diameter weighs 8 pounds, what will be the weight of a ball 12 inches in diameter?

71. The hypotenuse of a right-angled triangle is 100 feet, the other sides being equal: what is the area?

72. The longitude of Boston being 71° 4', and that of Chicago 87° 36' 42" west: what is the time in Chicago when it is 6 o'clock P.M. in Boston?

73. I have a room to carpet that is 30 feet long and 27 feet wide: which is cheaper, and how much, to buy yard-wide carpet, at \$1.25, or carpet $\frac{3}{4}$ yard wide at \$1?

74. What will it cost to shingle a roof 112 feet long, each side being 25 feet wide, 8 shingles covering a square foot, the shingles costing \$6.50 per thousand?

75. Hood, Bonbright & Co. sold goods for \$150, and lost 10 per cent., whereas they designed to gain 30 per cent.: how much were the goods reduced?

76. The interest on a note for 2 years 6 months at 7 per cent., was \$118.23: what was the face of the note?

77. For what sum must a note be made, payable in 60 days, so that when discounted at 6 per cent., the proceeds may be $593.70 ?

78. If it costs $30 to pave a cellar whose width is 16 feet, what will it cost to pave a similar one whose width is 24 feet ?

79. For a house worth $5,620 I have received in 2 years 3 months 15 days, $1,545.50 rent : what rate of interest have I received ?

80. If a piece of silk cost 72 cents per yard, at what price shall it be marked that the merchant may sell it at 10 per cent. less than the marked price, and still make 25 per cent. ?

81. John Jones divided his estate among 2 sons and 3 daughters, the latter sharing equally with one another. The younger son received $2,200, which was $\frac{5}{12}$ of the share of the elder, who received $\frac{11}{36}$ of the whole estate : what was the share of each daughter ?

82. Multiply $\frac{2}{5}$ by .000015 and divide the product by twelve hundred-millionths.

83. At $2.50 per yard in length, what will be the cost of a carpet $\frac{5}{8}$ of a yard wide, that will cover the floor of a room $40\frac{1}{4}$ ft. long and $20\frac{3}{4}$ ft. wide ?

84. Bought cloth at $5 per yard : what must be the asking price, in order to fall 10 per cent. and still make 10 per cent. on the purchase ?

85. A 9-months note, discounted at 6 per cent., yields $477.25 : required the face of the note.

86. Purchased goods as follows : April 2, a bill of $300 on 2 months' credit ; April 4, a bill of $400 on 3 months' credit ; May 5, a bill of $500 on 4 months' credit : what is the average term of credit, and the equated time of payment ?

87. It is required to lay out 58 A. 175 sq. rods of land in the form of a rectangle whose length shall be 125 rods: what will be its width?

88. How many square feet of surface has a cube whose solidity is 15,625 cubic feet?

89. In how many days can a person travel 690 miles, if he goes the first day 25 miles, and the last 67 miles, each day's journey being greater than that of the preceding day by the same number of miles?

90. What is the area of a circle whose radius is 3 feet?

91. Reduce $\dfrac{2\frac{1}{2} \times \frac{3}{5}}{\frac{5}{8} \text{ of } 3\frac{1}{5}}$ to a decimal.

92. A B and C own a mill; A owns $\frac{53}{154}$ of it; B $\frac{3}{22}$ of it more than C: what part of the mill do B and C respectively own?

93. What is the face of a note at 60 days, the proceeds of which when discounted at bank at 8 per cent. are $961.35?

94. What is the least common multiple of 18, 30, 54 and 70?

95. If A owes $500 due in 6 months, $400 due in 4 months, and $300 due in 7 months, and pays $\frac{2}{3}$ of the whole in 3 months, when ought the remainder to be paid?

96. What must be paid in Boston for a draft on New York for $4,500 at 30 days, discounted at 6 per cent., exchange being $\frac{1}{2}$ per cent. premium?

97. It is 80 rods between the opposite corners of a square field: how many acres in the field?

98. A man wishes to tether his horse to a stake, so that he may graze on one acre: what must be the length of the rope?

99. I sent to Liverpool 5,000 bushels of wheat that cost me $1.25 per bushel; 25 per cent. of the wheat was thrown overboard and the remainder was sold at $2 a bushel: what was gained on the wheat?

100. A merchant sold a quantity of lumber and received a note payable in six months; he had his note discounted at a bank at 6 per cent. and received \$4,572.40: what was the amount of the note?

101. The triangular gable of a certain building has a base of 40 feet and an altitude of 15 feet: how many square feet of boards will cover it?

102. What principal in 9 years 10 months and 12 days, at 5 per cent., will give \$3,875.824 interest?

103. I can sell my horse for \$280 cash, or \$300 on a credit of 1 year 6 months; I choose the latter: how much do I lose, money being worth 6 per cent.?

104. A, B and C can mow a field in 6 days, and A and B in 9 days; after the three had worked 2 days, C left: how long did it require A and B to finish it?

105. A dealer in horses sold 24 for \$150 each. On one half of them he gained 30 per cent., and on the remainder he lost 20 per cent. Did he gain or lose on the whole, and how much?

106. A can do a piece of work in $\frac{2}{3}$ of a day, and B can do the same in $\frac{3}{5}$ of the time that it takes A: in what time can they perform it together?

107. Explain how, in the metric system, the liter is derived from the meter.

108. A merchant losing $\frac{1}{5}$ of his stock by fire, sold the remainder so that $\frac{1}{11}$ of his sales was profit. What % of the cost of an article was its selling price? What % of the selling price was the cost?

109. How much does the circumference of a hind-wheel of a carriage exceed that of a fore-wheel, their diameters being 4 feet 6 inches and 4 feet respectively?

110. Write in correct form a non-negotiable joint note payable to yourself; also a note on which A. P. Flint is security, payable at the National Park Bank.

111. Required the side of a cube equal to a rectangular solid 40 inches long, 25 inches wide and 25 inches high.

112. If 19 men, in $3\frac{3}{4}$ days, working $8\frac{3}{4}$ hours a day, reap $26\frac{1}{4}$ acres of grain, how many acres will 26 men reap in $12\frac{2}{3}$ days, working $10\frac{3}{14}$ hours a day?

113. If it cost $800 to inclose a section of land in the form of a square, what will it cost to fence $\frac{1}{4}$ of a section in the same form?

114. I bought a farm of 185 acres the first payment amounted to $\frac{3}{8}$ of the price, and the second to $\frac{2}{5}$ of the remainder. The first payment exceeded the second by $2,220: what did I pay per acre?

115. A can do a piece of work in 12 days; B can do the same in 16 days; and C in 24 days. They all worked 3 days, when C left; 2 days afterward B left: how long was A in finishing it?

116. A pole stands $\frac{1}{4}$ of its length in mud, and $\frac{1}{3}$ in water: the part out of the water casts a shadow 16 feet long: how long is the pole, if a part 4 feet long casts a shadow 6 feet long?

117. Show why, in division of fractions, we invert the divisor and proceed as in multiplication.

118 $\dfrac{.5 \times .006}{\frac{9}{15} + \frac{1}{5} \times (\frac{1}{4})^2} + \dfrac{\frac{4}{5} - \frac{15}{16} \times (\frac{2}{3})^3}{1.6 \times .625} = ?$

119 Paid $108 for grain, $\frac{3}{10}$ of it being barley at $\$.62\frac{1}{2}$ per bushel, and $\frac{3}{8}$ of it wheat at $\$1.87\frac{1}{2}$ per bushel, and the remainder oats at $\$.37\frac{1}{2}$ per bushel: how many bushels of grain were bought?

120. Three men rented a pasture, agreeing to pay $38.40 for its use. The first put in 18 cows for 3 months, the second 24 cows for 4 months, and the third 12 cows for $3\frac{1}{2}$ months: how much of the rent ought each to pay?

121. If 180 men, in 6 days of 10 hours each, dig a trench 200 yards long, 3 yards wide, 2 yards deep, in how many days can 100 men, working 8 hours a day, dig a trench 180 yards long, 4 yards wide and 3 yards deep?

122. What is the entire surface of a cube whose volume is 91.125 feet?

123. If 284 men in $5\frac{1}{2}$ days, of 11 hours each, dig a trench $232\frac{1}{2}$ feet long, $3\frac{2}{3}$ feet wide and $2\frac{1}{3}$ feet deep, in how many days of 9 hours each, will 24 men dig a trench $337\frac{1}{2}$ feet long, $5\frac{3}{8}$ feet wide and $3\frac{1}{3}$ feet deep?

124. A commission merchant's regular charge for selling goods is $1\frac{1}{2}$%. By selling goods for me for $5,000, I find that my total loss is $125. If the commission merchant shares one half this loss, what is his actual rate of commission, and what did the goods cost me?

125. The cashier of a bank took from the safe, on the opening of business, 10% of the money and put it in the cashier's drawer. During the day he paid out 10% of this money and received twice as much as was paid out. At the close of business, there was $5,500 remaining in the drawer. How much money was in the safe at the beginning of business?

126. A man spends $33\frac{1}{3}$% of his capital for real estate, $50 more than one third of the remainder for merchandise, and then had $5,000 remaining: how much had he at first?

127. I have goods marked 383.33\frac{1}{3}$: what per cent. can I deduct from this price and still make 15%, the goods costing $300?

128. If $100 will produce $15 in 1 year 6 months at a certain rate per cent., what will produce $25 in 2 years at the same rate?

129. A tobacconist bought 1,000 pounds of tobacco at $1.50 per pound. From the tobacco, cigars were made and sold to a retailer at the rate of $2.25 per hundred. The retailer sold them at 6 for 25 cents. If there was no waste, and 6 cigars weighed an ounce, how much did each party gain?

130. A can do a piece of work in 6 days; B and C working together in 4 days, and C in 5 days: how many days will it take B to do it, and how long will it take A, B and C working together.

131. If $\frac{15}{16}$ of a pound of coffee cost 30 cents, what would $126\frac{3}{8}$ pounds sell for at an advance of 4%?

132. I buy goods for $400 and mark them so as to gain 15%: what % can I deduct from this marked price so as to gain $9\frac{1}{4}$%?

133. Divide $121 among 4 boys so that A will have $3 to B's $4; B will have $5 to C's $6 and C will have $7 to D's $8.

134. How many cubic inches of wood are there in the plain frame of a mirror 6 feet by 4 feet, the frame being 4 inches wide and 2 inches thick?

135. If A can cut a cord of wood in $\frac{3}{4}$ of a day and B in $\frac{4}{5}$ of a day, in what time can they together cut a cord?

136. The difference between two numbers is 8, and if 7 be added to the less, the sum will be 90% of the greater: what are the numbers?

ANSWERS TO WRITTEN PROBLEMS.

No answers are given to the oral exercises and the abstract work, and, in some instances, answers to the written problems are omitted in order that the pupil may learn to rely on himself.

Page 25.

14. 20,128,348.
15. 1,019,863,210.
16. $99,651 67.
17. 2,012 yrs.

Page 26.

18. 4,008 cattle.
19. 2,902,437,000 bu.
20. $10 44.
21. $10,537.
22. $37,815.
23. $25,250.
24. $1,759.
25. $3[illegible],250.
26. $75,100.

Page 34.

23. $2,131.
24. $563.15.
25. $66,534.

Page 35.

26. $19,115.
27. $899.
28. 1,379,269.
29. 1,418,813.
30. 23,838,156.
31. $1,510 14.
32. $95,913.13.
33. $41,023.13.
34. 2,258.
35. 5,005.
36. 1,403,996.
37. $6,375.
38. 45 cents.
39. $990.
40. $991.

Page 37.

1. 57 years.
2. Lost $100.
3. $2[illegible].
4. $[illegible]4.
6. 65,075.
7. $1,382.

Page 38.

8. $12,500.
9. $636.
10. $1,500.
11. $1,256.
12. $184.
13. $41,526.
14. 6,850.
15. 1,173,400 sq. mi.
16. $261.
17. $30.75.

Page 50.

46. 35,787 miles.
47. $10,530.
48. 4,284 miles.
49. $1,710.

Page 51.

50. $310.50.
51. 38,112 miles.
52. $255,532.
53. $35,493,304.
54. $1,694,105.
55. $578,125.
56. $208.
57. $236,632.
58. $4,695.
59. 52,585,104 stalks.
60. 1,210,412 bricks.
61. 1,041,600 seconds.
62. 18,432 panes.

Page 66.

50. 2[illegible].
51. [illegible].
52. $92\frac{1}{4}$.
54. 300 acres.
55. 217 [illegible] days.
56. 15[illegible] miles.
57. [illegible] sq. ft.
58. $0.729[illegible].

Page 68.

1. $32
2. [illegible] miles.
3. [illegible]
4. [illegible]
5. [illegible]
6. [illegible]
7. [illegible]
8. [illegible]
9. 1,34[illegible]
10. 31.

Page 69.

11. $15,000.
12. 90 years.
13. 9,844 bushels.
14. 58,441[illegible].
16. 87 bushels.
17. A.D. 1790.
18. 714 years.

Page 70.

19. 420,000,000.

Page 75.

3. 2.
4. 2.
5. 6
6. 12.
7. 25
8. 2
9. 10
10. 25
11. 6
12. 320
13. 4.
14. 8.
15. 512.

Page 76.

16. 3 feet.
17. 81 acres.
18. 12 cords.

Page 77.

2. 3,744
3. 3,024
4. 720
5. 1,3[illegible]0,480.
6. 17,670,120.
7. 7,260,480.

Page 78.

8. 376,216.
9. 1,825,824.
10. 223,344.
11. 1,008.
12. 1,440.
13. 2,016
14. $1 50
15. $360
16. 180 bushels.
17. 300 feet.
18. 720 acres.
19. 900 gallons.
20. 720 oranges.
21. $960
22. 120 hours.

Page 81.

11. 6 dozen.
12. 4 bushels.
13. 40 lbs.
14. 15 cords.
15. 6 chickens.
16. $140.
18. 98 cents.
19. $7\frac{1}{2}$ tons.
20. 25 suits.
21. 64 bushels.
22. 48 cents.

Page 82.

23. 20 boxes.
24. 5 bales.
25. 13 days.
26. 48 cents.
27. 77 bushels.
28. 100 bbl.
29. 120 lbs.
30. 12 bushels.
31. 12 pieces.
32. 6 cents.

Page 89.

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]
6. [illegible]
7. [illegible]
8. [illegible]

Page 90.

9. [illegible]
10. [illegible]
11. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

Page 91.

1. [illegible]
2. 10, 10, [illegible]
3. [illegible] 24.
4. [illegible]
5. [illegible]
6. [illegible]

Page 97.

6. 30[illegible] acres.
7. 344[illegible] lbs.
8. [illegible]
9. 500[illegible] tons.

Page 98.

10. $\$163\frac{3}{4}$.
11. $264\frac{4}{5}$
12. $\$201\frac{3}{25}$.
13. [illegible]
14. [illegible]
15. $161\frac{11}{80}$ miles.
16. $\$85\frac{7}{..}$.
17. $\$8,585\frac{5}{8}$
18. $629\frac{187}{160}$ acres.
19. $161\frac{1}{3}$ yards.
20. $\$110\frac{3}{16}$.

Page 101.

20. $110\frac{3}{4}$ bushels.
21. $\$12\frac{1}{20}$.
22. [illegible]
23. $\frac{1}{3}$ of a yard.
24. $136\frac{1}{3}$ bushels.
25. $108\frac{3}{16}$ tons.

Page 103.

1. $46\frac{1}{8}$
2. $307\frac{3}{8}$ bushels.
3. $\$12\frac{3}{8}$.

Page 104.

4. $43\frac{4}{15}$ acres.
5. $10\frac{4}{15}$ yards.
6. $\$1\frac{1}{7}$.
7. $\$35\frac{1}{20}$.
8. $1\frac{11}{25}$ tons.
9. 17 gallons.
10. $\frac{1}{10}$
11. $\frac{13}{80}$.
12. $44\frac{1}{8}$.
13. $\$773\frac{3}{4}$.
14. $149\frac{3}{16}$ tons.

Page 107.

2. $142\frac{4}{5}$; 151.
3. 178 ; $93\frac{1}{8}$.
4. $359\frac{1}{4}$; $122\frac{1}{3}$.
5. $401\frac{1}{4}$; $255\frac{1}{3}$.

Page 108.

6. $1851\frac{1}{2}$; $7644\frac{3}{8}$.
7. $5920\frac{5}{8}$; $2213\frac{1}{8}$.
8. [illegible]
9. $1033\frac{1}{2}$; $5509\frac{1}{8}$
10. $3071\frac{9}{17}$; 16714.
11. $10\frac{5}{9}$; $7\frac{29}{29}$
12. $14\frac{32}{37}$; $10\frac{88}{119}$.
13. $20\frac{23}{28}$; $4\frac{39}{117}$
14. $15\frac{7}{81}$; $7\frac{13}{17}$.
15. [illegible]
16. [illegible]
17. $169\frac{4}{7}$; $7624\frac{1}{5}$.
18. $379\frac{7}{10}$; $6989\frac{1}{24}$.
19. $6\frac{9}{16}$.
20. $104\frac{1}{3}$.
21. $1,372\frac{8}{9}$ cents.
22. $\$337\frac{7}{8}$.
23. $7\frac{1}{3}$ miles.
24. 72 + cents.

Page 111.

10. $26\frac{2}{11}$.
11. $1\frac{3}{11}$.
12. $1\frac{5}{12}$.
13. $\frac{5}{8}$.
14. $\frac{4}{15}$.
15. $1\frac{3}{16}$.
16. $\frac{5}{123}$.
17. $\frac{5}{8}$
18. [illegible]
19. [illegible]
20. $\frac{27}{850}$
21. 6 lbs.

Page 112.

22. [illegible]
23. [illegible]
24. 6 tons.
25. [illegible] days.
26. $\$3\frac{3}{4}$
27. $\$1\frac{1}{4}$
28. 21 pieces.
29. 20 days.
30. [illegible]
31. [illegible]
32. [illegible]
33. $74\frac{4}{5}$ lbs.
34. 16 tons.
35. $\$15,555\frac{5}{9}$
36. $75\frac{1}{2}$ bushels.

Page 119.

1. [illegible]
2. [illegible]
3. [illegible] miles.
4. [illegible] yards.
5. 27 bushels.
6. $54
7. $3\frac{3}{4}$ yards.
8. $260\frac{4}{21}$.

Page 120.

9. $\frac{1}{4}$ a day.
10. 3 score.
11. 11 tons.
12. 33 miles.
13. 100 lbs.
14. 121 [illegible].
15. 8 oranges.
16. $\$11\frac{1}{4}$
17. 10 acres.
18. [illegible]
19. [illegible]
20. 4 tons.
21. 625 acres.
22. [illegible] days.
23. $5,000
24. [illegible]

Page 121.

25. $16,000
26. $24,000
27. [illegible] cents.
28. [illegible]
29. [illegible] baskets.
30. $2,160.
31. $43\frac{21}{55}$ feet.
32. 21.

Page 128.

2. 1867.5867.
3. 126.84137.

4. 6781.9772.
5. 39931.419.
6. 1,001,042 028851.
7. 304,565.001888.
8. 74,037 041759.
9. 4620.014.

Page 129.

10. $207.79.
11. $10[illegible].95.
12. $373.38.
13. $6[illegible]3.99.
14. $1,068.44.
15. $37.96.

Page 130.

2. 4.447
3. 25.0751.
4. 902.0749.
5. 910.328746.
6. 68786.909891.
7. .6999993.
8. 2.000118.
9. .049995.
10. $12 044.
11. $11 927.
12. $8 019.
13. $485.37.
14. $2.40.

Page 131.

2. 6.2625.
3. 1 44108.
4. .067266.
5. 245 0584.
6. 6199.986.
7. 96 128032.
8. .584.
9. 386 2
10. 38417.37.
11. 1908.303.
12. 10.
13. 4453.818.
14. .1125.
15. 1.0201.
16. 77[illegible]75.
17. 603.606036.

18. 3026.2136.
19. 4142.118024.
20. 961.6201.
21. 1992.30243.
22. .3214.
23. 103.896.

Page 132.

24. .8125.
25. 1.2204.
26. 71.0556.
27. 3.44.
28. 2640.
29. .00024.
30. $13991.25.
31. $1.84.
32. $171.
33. $7.12.

Page 133.

2. 44.38[illegible].
3. 2.
4. 10 193[illegible].
5. [illegible]
6. [illegible]
7. .111[illegible].
8. 1.[illegible]
9. 27392$\frac{1}{2}$.
10. 1000.
11. 54[illegible].
12. .004[illegible].
13. .00026[illegible].
14. .00001.
15. .001$\frac{1}{4}$.
16. .41$\frac{2}{3}$.
17. 10000.
18. 12 tons.

Page 134.

2. $89.13.
3. $36.33.
4. $105 31.
5. $127 80.
6. $28 69.
7. $15 91.
8. $91.31.
9. $32 36.
10. $16.17.

Page 136.

1. $138.18.
2. $270.30.
3. $10.5 09.
4. $2 87$\frac{1}{2}$.
5. $20 62$\frac{1}{2}$.
6. $98 10.
7. $14 50.
8. $17 50.
9. $191.25.

Page 137.

10. 2282.81\frac{1}{4}$.
11. 36.3 bushels.
12. $3823.25.
13. 23+ weeks.
14. $1 30.
15. $136.98 +.
16. $ 63.
17. $111.65.
18. $136 10.
19. 60 tons.
20. $27 36.

Page 138.

21. Gained $27.55.
22. Gained $216.
23. 24 days.
24. 143.8 tons.
25. $18.48.
26. $15 84.
27. $79 76.
28. $535.50.
29. $6.
30. $ 25.

Page 140.

1. $1747.

Page 141.

2. $105.05.
3. $78.55.

Page 142.

4. $54.23.
5. $44.40.

Page 143.

6. $41.91.

Page 144.

7. $23.92.

Page 148.

1. 9,550 lbs.
2. 192,320 oz.
3. 9 T. 4 cwt. 62 lbs.
4. 16 cwt. 78 lbs.
5. $3.36
6. 49 lbs.
7. 14$\frac{1}{2}$ bbls.

Page 149.

1. 204 grs.
2. 238,354 grs.
3. 2 oz. 11 pwt. 16 gr.
4. 4 lbs. 2 oz.
5. 2160 pwt.
6. 1 lb. 4 oz. 4 pwt. 1 gr.

Page 150.

1. 1,848 ℈.
2. 6,000 grs.
3. 1 lb.
4. 29 ℔ 2 ℥ 6 ʒ 2 ℈.
5. 6,321 grs.
6. 1 ℔ 3 ℥ 7 ʒ 1 ℈ 2 gr

Page 151.

1. 6,747 far.
2. 5,612 far.
3. 515 d.
4. £7 2s. 8d. 2 far
5. 48,000 far.
6. 26 sov. 0 cr. 0 fl. 2d.
7. £80.
8. £650
9. 34,080 far.

Page 154.

1. 5,280 feet.
2. 82$\frac{1}{2}$ yards.
3. 35 yds. 1 ft. 8 in.
4. 9 mi. 254 rds. 2 ft. 4 in.
5. 318,166 in.

Page 155.

1. 64,000 links.
2. 4,840 fathoms.
3. 63,360 inches.
4. 152 yards.
5. 508 eighths.
6. 14,740 fathoms.

Page 157.

1. 43,560 sq. ft.
2. 42$\frac{3}{4}$ acres.
3. 77,863$\frac{1}{2}$ sq. ft.

Page 158.

4. 18,818,064 sq. in.
5. 2 acres.
6. 6,311,988 sq. in.

Page 159.

1. 17,280 cu. in.
2. 12$\frac{1}{2}$ cu. yds.
3. $\frac{3}{4}$ of a cord.
4. 5 cords.
5. 132 cords.

Page 160.

1. 26 qts.
2. 160 pts.
3. 1,131 pts.
4. 75$\frac{31}{32}$ bush.

Page 161.

5. 120 qts.
6. 82 pts.
1. 2,039 gills.
2. 6,053 gills.
3. 1 hhd
4. 5$\frac{1}{16}$ bbls
5. $\frac{35}{28}$ hhd.
6. 19$\frac{44}{63}$ hhds.

Page 163.

1. 3$\frac{1}{3}$°.
2. 1,008″.
3. 7 0° +.
4. 587,700″.
5. 148,271″.

Page 165.

2. 1 cwt. 52 lbs. 14.016 oz.
3. 1 pk. 7 qts. .000256 pt.
4. 270 rd. 4 yd. 2 ft. 9.7 + in.
5. 2 qts. 3.07296 gi.
6. 74 sq. rd. 29 sq. yd. 6.894 sq. ft.
7. 7 oz. 3 pwt. 15.1296 gr.

Page 166.

9. 15 cwt. 55 lb. 8$\frac{4}{5}$ oz
10. 38 gal. 3 qts. $\frac{3}{13}$ pt.
11. 10 mos.
12. 8 oz. 6 pwt. 3$\frac{9}{13}$ gr.
13. 10 s. 8 d.
14. 91 sq. rd. 12 sq. yd. 8$\frac{13}{28}$ sq. ft.

Page 167.

16. .704875 T.
17. .984375 bush.
18. .558 [illegible] + A.
19. .390277 + mi.
20. .003769 + hhd.
21. $87.49\frac{7}{9}$ [illegible].
23. $\frac{53}{64}$ bush.
24. [illegible] C.
25. [illegible] A.
26. [illegible] mi.
27. [illegible] l.l. l.
28. $\frac{397}{320}$ £.

Page 168.

2. £507 6s. 7d. 1 far.
3. 112 mi. 115 rds. 5 yds. 3 in.
4. 201 bush. 1 pk. 1 qt. 2 gills.
5. 16 S. 16° 31′ 44″.
6. 8 A., 74 sq. rds. $10\frac{3}{4}$ sq. yds.
8. 3 cwt. 33 lb. $9\frac{1}{3}$ oz.
9. 26 sq. rd. 28 sq. yd. 4 sq. ft. 45 sq. in.

Page 169.

2. 23 lbs. 19 pwt. 1 gr.
3. 58 gal. 1 qt. 1 pt.
4. 21 mi. 317 rds. 4 yds. 1 foot 5 in.
5. 2 lbs. 4 pwt. 23 gr.
6. 3 mo. 6 days.
7. 100 yrs. 9 mo. 4 days.

Page 170.

8. 6 days 7 hours 45 min.
9. 374 yrs. 8 mo. 8 days.
10. 37 yrs. 2 mo.
11. 1 yr. 8 mo. 19 days.
12. 3 yrs. 11 mo. 28 days.
13. 43 yrs. 0 mo. 24 days.
14. 6 yrs. 4 mo. [illegible] days.
16. 88 yrs. 14 days.
17. 33 yrs. 7 mo. 29 days.

Page 171.

2. 4 T. 8 cwt. 78 lb. 7 oz.
3. 110 lb. 8 oz. 3 pwt. 6 gr.
4. 148 hhd. 19 gal.
5. 13 yrs. 6 mo. 27 days, 17 hrs. 25 min.
6. 140 ℔ 10 ℥ 3 ℨ 12 gr.
7. 104 C. 80 cu. ft. 207 cu. in.
8. 2 gal. 1 qt.
9. 22 mi. 30 rd. 3 yds.

Page 172.

2. £11 9s. 1d. 3 far.
3. 10 lb. 11 oz. 18 pwt. $11\frac{3}{5}$ grs.
4. 14 T. 17 cwt. 10 lb. 2 oz.
5. 10 hhd. 32 gal. 1 qt. $2\frac{2}{5}$ gi.
6. 2 C. $70\frac{2}{5}$ cu. ft.
7. 11 days.
8. $1126\frac{2}{5}$ packages.
9. 12 days +.

Page 178.

2. 6 h. 53 min. 34 sec. +.
3. 5 h. 50 m. 6 sec. +.
4. 10 h. 10 m. 52 sec. +.
5. 5 o'clock 26 m. 40 sec. P. M.
6. 5.58 + P.M.

Page 179.

8. 75°.
9. 15° 35′.
10. 73° [illegible]2′ 10″.

Page 180.

6. 108 sq. ft.
7. $432.
8. $43\frac{1}{3}$ sq. yds.
9. $37.05.
10. 16 planks.
11. $280.
12. $2,708.33.
13. $44.52.
14. $44.15.
15. [illegible] feet.
16. [illegible] acres.
17. $220.37.
18. $3.95.

Page 181.

19. $2.50.
20. $19\frac{1}{2}$ feet.
21. $122\frac{1}{2}$ acres.
22. $2,[illegible]37½.
23. $24\frac{3}{4}$ yds.
24. $23.33⅓.
25. 165 feet.
26. $45\frac{5}{8}$ rds.

Page 182.

1. 360 cu. ft.
2. 183 cu. ft. 144 cu. in.
3. 210 cu. yds.
4. $63.80.

Page 183.

5. $541.09.
6. 320 perches.
7. 192 cu. yds.
8. 83,592,000 cu. in.
9. $3,200.
10. 35 cu. ft.

11. 128 perches.
12. $743.58.
13. $13\frac{1}{3}$ feet.
14. 25 feet.
15. $18\frac{3}{4}$ feet.

Page 184.

16. $85.54.
17. $9.72.
1. $1,346\frac{74}{77}$ gal.
2. $115\frac{1}{4}$ bush.
3. 137 bbl. 8 gal. 1 qt.
4. 370 cu. ft.

Page 185.

5. 1,000 bush.
6. $1,122\frac{6}{77}$ gal.
8. $16.36.

Page 190.

2. $8.04.
3. $14.62.
4. 16.29 ft.
5. 15 [illegible]
6. 8.305 lbs.
7. 22.35 bush.
8. $ 30.
9. $1\frac{1}{2}$ miles.
10. 36 bush.
11. $506.25.
12. 19,050 copies.
14. 300 bush.

Page 191.

15. $56\frac{1}{4}$ tons.
16. $1,575.
17. $12,150.

Page 192.

2. 80%.
3. $33\frac{1}{3}$%.
4. 20%.
5. 200%.
6. 40%.
7. $33\frac{1}{3}$%.
8. 75%.
9. 50%.
10. $83\frac{1}{3}$%.
11. 32[illegible]%.
12. 82[illegible]%.
13. $9\frac{3}{5}$%.

Page 193.

14. 76[illegible]%.
15. [illegible]%.
16. $143\frac{11}{7}$%.

Page 194.

2. 800.
3. 68.
4. 416.42 +.
5. 252.85 +.
6. 417.91 +.
7. [illegible]8.21 +.
8. [illegible]
9. 400 bush.
10. $6,482.50.
11. 504 [illegible]
12. $9,900.
13. Lost $48.
14. 38% +.
15. $4.29.

Page 198.

1. $130.
2. $15,725.
3. $1.05.
4. $52.38.
5. $[illegible]2[illegible]3.60.
6. $250.
7. Neither.
8. 50%.
9. $57.60—$4.
10. $895.25.
11. 9 cts.
12. 10%.
13. $2.70.

Page 199.

14. $[illegible].36$\frac{2}{3}$
15. 25%.
16. $2[illegible].
17. Neither.
18. 3[illegible]%.
19. 42[illegible]%.
20. $700.
21. $6.
22. $10,416.27.

Page 200.

24. 72 cts.
25. $37\frac{1}{2}$%.
26. 33[illegible]%.
27. $32.
28. 50%.
29. 10 cts.
30. Lost $2.03.
31. Lost $100.
32. Lost $18\frac{1}{2}$[illegible]%.

Page 202.

1. $95.47.
2. $4631.75.
3. $1,095.40.
4. $6,835.31.

Page 203.

5. $16,000.
6. 42 horses.
7. $[illegible]
8. $[illegible]
9. $[illegible]
10. $1[illegible]3.55.
11. 48,000 lbs.
12. 10,000 lbs.
13. 113 shares, $39.47 over.
14. $846.66.

Page 204.

15. $7.
16. $2\frac{1}{2}$%.

17. $2\frac{1}{2}$%.
18. $4,200.
19. $12,000.
20. $971.38.
21. $2,341\frac{1}{4}$ yards.
22. $131.5+.

Page 206.

1. $206.25.
2. $318
3. $1\frac{1}{4}$%.
4. $2\frac{1}{2}$%
5. $25,000.
6. $100
7. $53.50.
8. $2,709.60.

Page 207.

1. $6,800.

Page 208.

2. $\frac{1}{2}$% $18.67.
3. $14,280.
4. $210.
5. $186.
6. $\frac{4}{10}$% or 4 mills.
7. $22,500
8. $25,000.
9. $1\frac{1}{4}$%
10. $116 34.
11. $84.80.

Page 210.

1. $218.75.
2. $53.76.
3. $123.50.
4. $1,800.75.
5. $708 40.
6. 10%
7. $209.
8. $2,379 22.
9. $5.12.

Page 212.

2. $551.88.
4. 25 shares.
5. 1,600 shares.

Page 213.

7. $10,900.
8. $2,000.
11. 80
12. 80.
13. $20,850.
15. $17,000.
16. $48,000.
17. $805 88.
18. 5%.
19. 60.
20. 150.

Page 214.

21. 6%.
22. $112,000.
23. $4\frac{1}{2}$%.
24. $320.
25. Neither.
26. $19,200.
27. 5%
28. $21,600.
29. 30 shares.
30. 5%.

Page 218.

2. $6 04.
3. $10.53.
4. $14 98.
5. $10.57.
6. $48.09.
7. $44.31.
8. $47.23.
9. $121 72.
10. $144 02.
11. $356 58.
12. $423 93.
13. $398 92.
14. $425 41.
15. $1,042.74.
16. $494 96.
17. $491 95.

Page 219.

18. $364.34.
19. $854.71.
2. $7.02.
3. $17.92.
4. $104.13.

Page 220.

5. $289.17.
6. $144.26.
7. $578.94.
8. $940.71.
9. $150.
11. $4.09.
12. $5.82.
13. $83.08+.
14. $.08.

Page 222.

1. $720.
6. $234.27.
8. $800.
9. $126.

Page 223.

2. 1 yr. 4 mo. 18 da.
3. 1 year.
5. 2 yr. 2 mo. 24 da.
7. 4 yr. 5 mo. 6 da.
8. 3 yr. 3 mo. 3 da.

Page 224.

2. 6%.
4. 7%.
6. 8%.
8. $4\frac{1}{2}$%.
9. $5\frac{1}{2}$%.
10. 5%.
11. 5%.
12. 7%
13. 10%.

Page 226.

5. $1,169.86.
7. $2,659.95.
8. $1,403 83.
9. $2.09.

Page 230.

2. $39 68+.
4. $1327.41.

Page 231.

$351.53.

Page 232.

9. $545.91.

Page 233.

4. $198.36 disc.
7. $485 worth $8.81 more.
9. $3.70.
11. $1,065.06.

Page 235.

3. $2.80.
5. $541.47.
6. $443.02.
7. $3 41.
8. $4.79.

Page 236.

10. $4.54.
14. $1,800.
15. $612.56.

Page 237.

19. 60 days.
20. 90 days.
21. 6 months.
22. Oct. 19.

Page 238.

25. 6%.
26. 6%.
27. 7%.
28. 7%.
29. 6%
30. 5%.

Page 242.

3. $490 80.
4. $943 20.
5. $806.40.
6. $1906.21.
7. $835.89.
8. $2430.

Page 243.

9. $4030.68.
10. $493.70.
11. $10008.
3. $1185 19.
4. $1670 76.
5. $1311.48.
6. $598.50.
7. $905 43.
9. $802.41.

Page 244.

3. $642.60.

Page 245.

5. $894.46.
7. $424.17.
9. $832.004+.

Page 246.

2. $6\frac{3}{20}$ mo.

Page 247.

4. $3\frac{1}{4}$ mo.
6. 5 mo. 27 da.
7. 14 days.

Page 248.

3. September 14.

Page 250.

2. November 22.
3. July 19.
4. November 26.
5. June 24.

Page 251.

6. Nov. 8.

Page 254.

1. $119.
2. $279.

Page 255.

3. $139 50.
4. $2\frac{1}{2}$%
5. $1\frac{1}{3}$%.
6. $\frac{7}{8}$%
7. $7,600
8. $14,400.
9. $7 000.
10. $420
11. $377 86.
13. $6.74
14. $3,900.

Page 256.

15. $393.
16. $36.
17. $11,100.
18. $10,573.
19. $78.
20. 27 shares.
21. $13,500.
22. 30 shares.
23. $3\frac{1}{5}$%.
24. $6\frac{1}{2}$% ; $227,500.
25. $\frac{2}{5}$%
26. $2\frac{1}{2}$%.

Page 257.

27. 1%
28. 160 bbl.
29. 1,740 cords.
30. $374.
31. $216.
32. $760.
33. $4,200.
34. 48 cts.
35. $3,737 50.
36. $2,052.
37. 10%
38. $1,100.
39. $10 20
40. $381.34.

Page 258.

41. $7 20.
42. $30.
43. $3.
44. $9,600
45. $2,730
46. Loss, $211.76.
47. 50%.
48. 70%.
49. 1[illegible]%
50. $200.
51. $400
52. 6[illegible]%
53. 10%.

Page 259.

54. 50%.
55. 250%.
56. 44$\frac{4}{9}$%.
57. 83$\frac{1}{3}$%.
58. 5%.
59. $266 67.
60. $198.
61. $[illegible],300.
62. $1,800.
63. $3,500.
64. 5%.
65. $442 92 +.
66. 3 cts.

Page 262.

1. $91.
2. $3.
3. $210.
4. $312.
5. $48.
6. $113 34.
7. $27.
8. $60.
9. $180.
10. $33.75.
11. $400.
12. $9.50.
13. $78
14. $18.
15. $260
16. $322.
17. $504.

Page 267.

2. $60.
3. $3,333.34.
4. $3,000.

Page 268.

5. $36,666.67
6. $35.
7. 112 loaves
8. 50 men.
9. 60 cows.
10. $480.
11. 31$\frac{1}{4}$ feet.
12. $3.34.
13. 28 cows.
14. $5.25.
15. 5$\frac{5}{7}$ days.
16. $8,800.
17. 15 tons.
18. 54 min.
19. 960 men.

Page 269.

20. $1,200.
21. $ 84.
22. 30 spoons.
23. 418 days.
24. 3$\frac{1}{13}$ hours.
25. 12 hours.
26. $6.[illegible].
27. 54 yards.
28. 1[illegible] days.
29. 1$\frac{5}{9}$ [illegible]

Page 270.

2. 4 months.
3. $228.
4. $21.

Page 271.

5. 30 days.
6. 948$\frac{4}{27}$ bush.
7. 2 days.
8. 34[illegible] days.
9. 32,500 lbs.
10. [illegible]
11. $1,[illegible].
12. [illegible]
13. [illegible]
14. 5 days.
15. 8 men.

Page 272.

17. 480 [illegible]
18. 315 [illegible]
19. 10 da.
20. 21 da.
21. 98[illegible] flags.
23. 20 men.

Page 275.

3. A, $900 ; B, $750 ; C, $1,050 , D, $300
4. A, $800 ; B, $1,200 ; C, $500
5. Brown, $2,700 , Jones, $3,120 ; Robinson, $2,736.
6. A, $750 B $900; C, $1,000.

7. A, $13.33; B, $26.67.
8. A, $2,400; B, $1,800; C, $3,840; D, $3,900.
9. A, $[illegible] 16; B, $4,155; C, $3,564.

Page 276.

10. Wife, $4,000; son, $3,066.67; each daughter, $2,666.67.
11. A. $1050; B. $1225.
12. A. $[illegible]0; B. $[illegible]40; C, $900.
13. A, $4,242.85; B, $3,535.71; C, $5,185.71.

Page 280.

1. 10 h [illegible].
2. $9,000.
3. $1[illegible]2.
4. $3[illegible]0.
5. A's, $4,000; B's, $6,000; C's, $7,000; D's, $8,000.
6. $1,000.
7. [illegible] men.
8. [illegible] da.
9. [illegible] da.
10. 22 [illegible].
11. [illegible]

Page 281.

12. 0.50\frac{2}{3}$ per bush.
13. $324.
14. [illegible]%.
15. $[illegible].30.
16. 1[illegible].
17. A, $4.10; B, $14.26.
18. A's, $[illegible]; B's, $144; C's, $2[illegible].
19. C's, $[illegible]50; D's, $[illegible]70.
20. 112 men.
21. 57\frac{1}{7}$.
22. 2$\frac{2}{3}$ min.

Page 282.

23. 106.87\frac{1}{2}$.
24. $18.36.
25. $107.
26. $52.86.
27. [illegible].
28. $5[illegible]0.
29. $148.[illegible].
30. $[illegible]0.
31. $330.
32. $[illegible]1,8[illegible]7$\frac{1}{7}$.
33. $12[illegible].
34. $1[illegible]3.

Page 283.

35. $30.
36. 70\frac{3}{4}$.
37. $5,600.
38. $2,500.
39. 23 [illegible] sh.
40. 10 %.
41. 1,000 sheep.
42. 111.21\frac{3}{32}$.
43. 42.
44. $54.11.

Page 284.

45. $2,551$\frac{1}{2}$.
46. A's, $4,439.01; B's, $3,963.40; C's, $1,849.59. A's gain, $1,5[illegible]4.[illegible]; C's gain, $[illegible]4.53.
47. 30 of each.
48. 3[illegible]
A, $2[illegible]; B, $32; C, $4[illegible]; D, $[illegible]6.
49. 21 min. 49$\frac{1}{11}$ sec. after 4 o'cl.
50. A, $7; B, $12; C, $10.50.
51. 4 at $19; 8 at $7; 8 at $6.

Page 286.

10. .00000001
11. 1,048,576.
12. 15,[illegible]2[illegible].
13. 14,641.
14. 1,030,301.
15. [illegible]
16. [illegible]$\frac{3}{4}$
17. [illegible]
18. .25.
19. .01.
20. 5$\frac{1}{25}$.

Page 289.

23. 3.25.
24. 3.[illegible]3.
25. 15.2.
26. [illegible].36.
27. 50[illegible].
28. [illegible]$\frac{3}{4}$.
29. [illegible]$\frac{8}{9}$.
30. 3.[illegible] +.
31. 1.01.
32. .002.
1. 3,140 ft.

Page 290.

2. 60 by 180 in.
3. 55,000.
4. $120.
5. 9 ft.
6. 840 sq. yds.
7. 219 + rds.
1. 25 in.

Page 291.

2. 36 in.
3. 76 ft.
4. 77 — mi.
5. 31.7 + ft.
6. 72 ft.
7. 100 ft.
8. 91.3 + ft.
9. 53.8 + ft.
1. 1$\frac{1}{3}$.

Page 292.

2. 3 times.
3. $1\frac{9}{16}$ hr.
4. 14.12 + ft.
5. 60 ft.
6. 28 ft.
7. 75 ft.

Page 294.

3. 23.
4. 25.
8. 325.
9. 508.
10. 85.9.
11. .08.
12. $3\frac{1}{3}$.
13. $\frac{4}{7}$.
14. 1.2599 +.
15. 1.4422 +.
16. 1.8171 +.

Page 295.

1. 78 in.
2. 222 + sq. ft.
3. 3,249 sq. in.
4. 18.7 + ft.
5. 31,329 sq. in.
6. 125 ft.
7. 23 ft.
9. 24 ft.
11. 24.3 ft.
13. 12 ft.

Page 296.

1. $3\frac{3}{8}$ times.
2. $14,062.50.
3. $12\frac{4}{9}$ times.
5. 22.22 — ft.
6. 5.7 + in.
7. 1,119.7 + hhd.
9. 3.41 + ft.
10. 64.

Page 299.

7. 7.
8. $12\frac{1}{2}$ cts.
9. 6 mi.

Page 300.

3. 972.
4. $\frac{2}{3}$.
6. $655.34.

Page 303.

1. 24 sq. ft.
2. 300 sq. rds.

Page 304.

3. 612.5625 sq. ft.
4. 60 A.
5. 7.8 + ft.
6. 6 ft.
7. 4.12 + ft.
8. 72.11 + ft.
9. 21.16 + ft.
11. 55.31 + mi.

Page 305.

1. 480 sq. ft.
2. 23 A. 120 sq. rds.

Page 306.

1. 10,400 sq. ft.
2. 7150 sq. ft.
3. 32 A. 130 sq. rds.
1. 25 sq. ft.
2. 1046.8 sq. ft.

Page 307.

1. 94.248 in.
2. 5.72 + ft.
3. 141.372 ft.
1. 314.16 sq. ft.

Page 308.

2. 706.86 sq. in.
3. 314.16 sq. ft.
4. 1145.912 + sq. ft.

Page 309.

1. 240 sq. ft.
2. 272 sq. ft.
3. 942.48 sq. ft.
4. 1,099.56 sq. ft.
1. 175 cu. ft.
3. 1,413.72 cu. ft.

Page 310.

1. 420 sq. ft.
2. 180 sq. ft.
1. 612 cu. ft.
2. 369.46 + cu. ft.

Page 311.

2. 330 sq. ft.
1. 6.2832 cu. ft.
2. 34.377 + cu. ft.
1. 612 sq. ft.

Page 312.

2. 219.912 sq. ft.
1. 148 cu. ft.
2. 306 + cu. ft.

Page 313.

1. 1256.64 sq. in.
3. 3.1416 sq. ft.
3. 4188.8 cu. in.
4. 113.0976 cu. ft.
5. .5236 cu. ft.
6. 14.1372 cu. ft.
7. 1.56 + ft.

Page 314.

1. 120.09 + gal.
3. 91.93 + gal.
4. 110.32 + gal.

Page 316.

1. 77.049 M.
2. 30504.803 M.

3. 6040.37 M.
4. 7460 dm.
5. 78460 M.
6. 4280000 cm.
7. 400.32639 DM.
1. 12.17 HA.
2. 90000 da.
3. 1.848362 A.
4. 2.0066 A.
5. $67.03.

Page 317.

1. 1.846 cu. dc.
2. 392700 cu. cm.
3. 1.654 S.
4. $202.42.
5. 52 loads.

Page 318.

2. 700.2 cl.
3. .00003492 ML.
4. 16,084,000. ml.
5. 71944.
1. 3.21 DG.
2. 4.137 DG.
3. 5,630,400 cg.
4. .007001.
5. 20050904.
6. 24 cts.

Page 326.

1. MDCCCLXXIX.

Page 327.

2. $\frac{\frac{351}{200}}{135}$.
4. 9 men.
5. 1.6972+ cu. in.
7. 54.
8. 5 mi.
9. 6 h.
11. John 1 min. $3\frac{3}{4}$ sec. in advance of William.

Page 328.

12. $10.
13. $25\frac{5}{7}$ min.
14 .232 + .
15. 10 yr. 6 mo.
16. .000016.
17. 5.
18. 240 sheep.
19. 20,695.
20. 30 sheep ; 36 cows.
21. 10 wks. 2 da.
22. $6\frac{8048}{8125}$.
23. 64.
24. $100,000.

Page 329.

25. $2.70.
26. $22.
27. November 11.
29. $195.63.
30. 125%.
31. 50 yds.
32. $39.48.
33. .30716.
34. 4,108 + .
35. $612.96.
37. A, 5; B, 7; C, 3 days.

Page 330.

38. 14.
39. $60.
41. 6 days.
42. $8\frac{4}{15}$.
43. 1404 ft.
44. A, $26\frac{2}{3}$ da.; B, 80 da.
45. $\frac{43}{63}$.
46. 10 yr. 3 mo.
47. Lose 25%.
48. $1,666\frac{2}{3}$ lb.
49. $.975 +.
51. $4\frac{79}{99}$.

Page 331.

52. 60.
53. 50%.
54. $\frac{77}{111}$; $\frac{35}{111}$.
55. $\frac{7}{8}$.
56. Lost $228.57.
57. $\frac{39}{47}$.
59. .25.
60. 7%.
61. $622\frac{2}{3}$ bbl.
62. $90.
63. $121\frac{1}{2}$ sq. ft.

Page 332.

64. Lose 1%.
65. $49.87.
67. $658.06.
69. $212.16 +.
70. 64 lb.
71. 2,500 sq. ft.
73. Yard-wide carpeting $7.50 cheaper.
75. $66.67.

Page 333.

77. $600.
78. $67.50.
79. 12%.
80. $1.
81. $1,356.67.
82. 50.
83. $371.19.
84. $6.11\frac{1}{9}$.
85. $500.
86. 50 days, or July 22.

Page 334.

88. 3,750 sq. ft.
89. 15 days.
90. 28.2744 sq. ft.
91. .75.
92. B, $\frac{59}{154}$; C, $\frac{38}{154}$.
93. $975.
95. $10\frac{3}{4}$ mo.
96. $4,497.75.
97. 20 A.
98. 7.13 + rds.
99. $1,250.

Page 335.

100. $4,716.25.
101. 300 sq. ft.
102. $7,856.40 +.
103. $4.77 +.
104. 6 days.
105. Lost $34.61.
106. $\frac{1}{4}$ day.
108. $137\frac{1}{2}$%; $72\frac{8}{11}$%.

Page 336.

113. $400.
114. $75.
115. $1\frac{3}{4}$ days.
116. $25\frac{1}{2}$ ft.
118. $\frac{1044}{2025}$.
119. 80 bush.
120. $10.80; $19.20; $8.40.

Page 337.

121. $24\frac{3}{13}$ days.
122. 121.5 sq. ft.
124. $\frac{1}{4}$%; $5,050.
125. $50,000.
126. $11,362.50.
127. 10%.
128. $125.

Page 338.

130. A, B & C $2\frac{2}{3}$ da.
131. $42.06.
132. 5%.
133. A, $21; B, $28; C, $33.60; D $38.40.
135. $\frac{12}{37}$ days.
136. 10 and 2.

25 cm